

Solutions

Original-task marks: M1 = method, A1 = accuracy, B1 = stated reasoning point.

2.1 Rebound reverses the velocity sign.

$$\Delta p = m(v - u) = 0.30 \text{ kg} \times (-3.0 - 5.0) \text{ m s}^{-1} = -2.4 \text{ kg m s}^{-1}$$

Impulse is negative, opposite the approach. Use impulse per contact time.

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{-2.4 \text{ kg m s}^{-1}}{0.040 \text{ s}} = -60 \text{ N}$$

Average force is 60 N opposite approach.

2.2 Integrate the triangular pulse.

$$J = \frac{1}{2} F_{\text{max}} \Delta t = \frac{1}{2} \times 60 \text{ N} \times 0.060 \text{ s} = 1.8 \text{ N s}$$

Impulse +1.8 N s. Use $J = m(v - u)$ with $u = 0$.

$$v = \frac{J}{m} = \frac{1.8 \text{ N s}}{0.40 \text{ kg}} = 4.5 \text{ m s}^{-1}$$

Final speed. The force is below its maximum for almost all the interval; its average here is half the peak.

2.3 Rearrange momentum to eliminate velocity.

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{p}{m} \right)^2 = \frac{p^2}{2m}$$

At equal energy, $p \propto \sqrt{m}$, so heavier:lighter is 2 : 1.

3.1 Resolve the momentum in both axes; equal masses cancel.

$$v_{2x} = u - v_1 \cos \theta = 3.0 \text{ m s}^{-1} - 2.4 \text{ m s}^{-1} \cos 30.0^\circ = 0.922 \text{ m s}^{-1}$$

x component.

$$v_{2y} = -v_1 \sin \theta = -2.4 \text{ m s}^{-1} \sin 30.0^\circ = -1.20 \text{ m s}^{-1}$$

y component. Recombine perpendicular components.

$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{0.922^2 + 1.20^2} \text{ m s}^{-1} = 1.51 \text{ m s}^{-1}$$

$$\beta = \tan^{-1} \left(\frac{|v_{2y}|}{v_{2x}} \right) = \tan^{-1} \left(\frac{1.20}{0.922} \right) = 52.5^\circ$$

Speed and direction 52.5° below $+x$. Compare total kinetic energies.

$$E_{k,b} = \frac{1}{2} m u^2 = \frac{1}{2} \times 0.20 \text{ kg} \times (3.0 \text{ m s}^{-1})^2 = 0.900 \text{ J}$$

$$E_{k,a} = \frac{1}{2} m (v_1^2 + v_2^2) = \frac{1}{2} \times 0.20 \text{ kg} \times (2.4^2 + 1.513^2) \text{ m}^2 \text{s}^{-2} = 0.805 \text{ J}$$

Kinetic energy decreases: inelastic.

3.2 Measure trolley mass and use light gates with a measured interrupt length for velocities immediately before and after. Zero the force sensor and use consistent positive directions for force and velocity. Integrate the entire force–time pulse, including signed areas. Compare that impulse with $m(v - u)$ over repeated trials. Choose high enough sampling rate to resolve the short pulse; a missed peak, sensor offset or friction can cause systematic disagreement. Secure the track and catch the trolley to prevent falls.

3.3 Track sphere centres over several frames before and after contact and divide calibrated x and y displacements by the time interval. Put a length scale in the collision plane. Use a known frame rate and camera perpendicular to the plane to reduce perspective error. Calculate kinetic energies from both components and measured masses, propagating or estimating uncertainty. A 4% increase may lie within the combined energy uncertainty, especially because velocity is squared. For a passive collision, investigate calibration/tracking errors or an additional energy source before claiming a physical increase; agreement of momentum to 3% does not by itself validate energy to 4%.