

3.3 Graphs and justified conclusions

Name: _____ Class: _____ Date: _____

Total: 25 marks

Objective

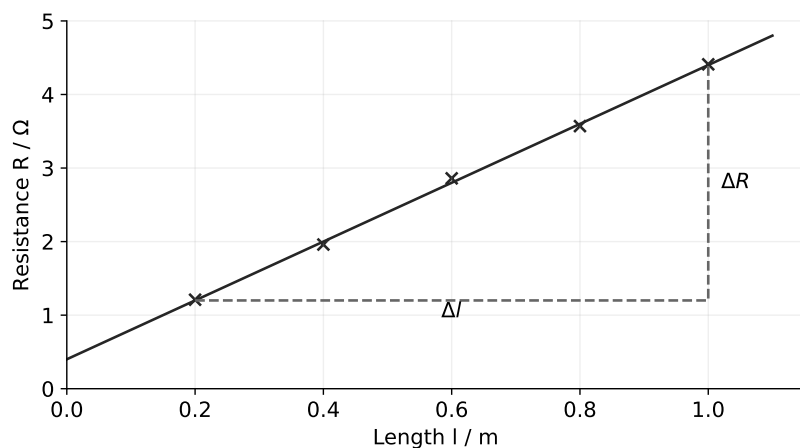
Practise the WPH13 skills in graphs and justified conclusions. These are original training tasks grounded in the specification and reviewed assessment demand.

1 Key knowledge and worked examples

■ Choose scales that show the data

Read which quantity belongs on each axis. Use clear quantity/unit labels and regular scales. Use a large part of the available grid; avoid awkward intervals that are hard to subdivide. Neither axis must always start at zero, but do not hide whether a proposed proportional relationship passes through the origin.

Plot small crosses accurately. Draw a thin best-fit line or smooth curve according to the relationship. Do not join noisy points with a zigzag unless the question requests it. A best-fit straight line balances scatter rather than passing through every point.



Use distant points on the best-fit line, not a tiny pair of neighbouring data points.

■ Turn a gradient into a physical constant

For a line $y = mx + c$, identify which part of the physical equation matches m and c . A straight line does not prove direct proportionality if its intercept is non-zero.

Worked example. A wire follows $R = (\rho/A)l + R_0$. A best-fit line passes through (0.20 m, 1.20 Ω) and (1.00 m, 4.40 Ω).

- Known: two widely separated points on the fitted line. Why: the slope equals ρ/A .

$$m = \frac{\Delta R}{\Delta l} = \frac{4.40 - 1.20}{1.00 - 0.20} \Omega \text{ m}^{-1} = 4.00 \Omega \text{ m}^{-1}$$

$$R_0 = R - ml = 1.20 \Omega - 4.00 \Omega \text{ m}^{-1} \times 0.20 \text{ m} = 0.40 \Omega$$

If $A = 1.0 \times 10^{-7} \text{ m}^2$, the resistivity is:

$$\rho = mA = 4.00 \Omega \text{ m}^{-1} \times 1.0 \times 10^{-7} \text{ m}^2 = 4.0 \times 10^{-7} \Omega \text{ m}$$

The fixed intercept may represent contact or lead resistance. Dividing one measured R by l would include that offset and give the wrong slope.

■ Scale factors belong in the gradient

Suppose an axis is labelled f/MHz . A gradient read from that graph has MHz in its units. Convert to Hz before using SI constants. A graph of f against $\sin \theta$ has a dimensionless horizontal axis; its gradient therefore has frequency units. Use degree mode if the measured angle is in degrees.

If a supplied equation is $f = (v/\lambda) \sin \theta$, a graph of f against $\sin \theta$ has gradient v/λ . Rearrange $v = \lambda \times \text{gradient}$. Do not assume every frequency graph has gradient equal to speed.

■ Curves, maxima and specific improvements

A broad range first locates a maximum. Then take more closely spaced readings around that region. Repeating one point does not locate the maximum between existing points. Keep the other conditions steady, and consider scatter when quoting the best angle or resistance.

For a motion experiment, a set square can improve a horizontal-distance measurement. For a lamp experiment, a dark enclosure can reduce changing background illumination. For a resonance experiment, approach the loudest sound from both higher and lower frequencies and repeat. Each improvement targets a named limitation.

A value with uncertainty describes an interval. Compare that interval with the proposed value, then state what the evidence supports. Agreement within uncertainty does not prove that a material or model is uniquely identified.

Worked example. A measured density is 8.90 g cm^{-3} with percentage uncertainty 0.9%. Could it agree with a proposed value 8.94 g cm^{-3} ?

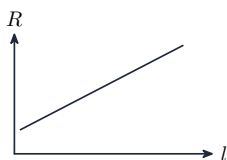
$$\Delta\rho = \frac{0.9}{100} \times 8.90 \text{ g cm}^{-3} = 0.080 \text{ g cm}^{-3}$$

The interval is about 8.82 to 8.98 g cm⁻³. The proposed value lies inside it, so the measurement is consistent with that value. It does not prove the object has that composition.

If two model predictions both lie inside the interval, the data does not distinguish them. Reduce the dominant uncertainty or add a different measurement before claiming a unique identification.

2 Practice

2.1 A fitted graph of resistance against length is straight with a positive intercept. Explain why this is not proof of direct proportionality and suggest the physical origin of the intercept. [3]



2.2 A best-fit resistance-length line passes through (0.20 m, 1.30 Ω) and (1.00 m, 4.50 Ω). Find gradient and intercept. Wire area is $1.2 \times 10^{-7} \text{ m}^2$. Find resistivity. [4]

best-fit line points
(0.20 m, 1.30 Ω)
(1.00 m, 4.50 Ω)
wire area $1.2 \times 10^{-7} \text{ m}^2$

2.3 A measured value is 8.90 g cm⁻³ with 0.9% uncertainty. Determine its interval and deduce whether it is consistent with 8.94 g cm⁻³. [3]

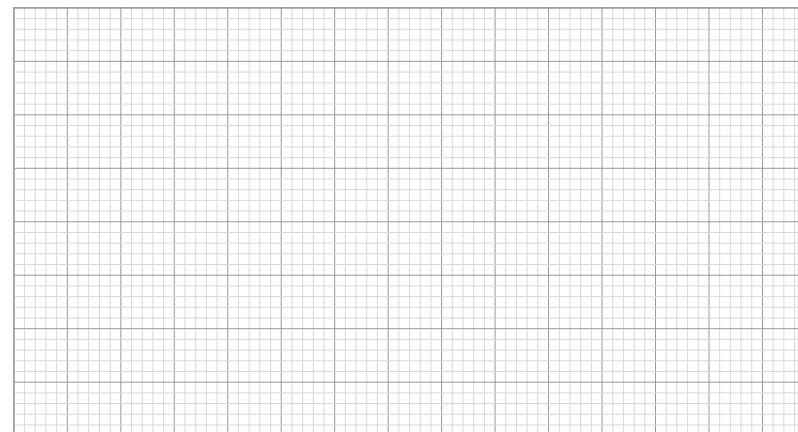
measured density
8.90 g cm⁻³
uncertainty 0.9%

proposed value
8.94 g cm⁻³

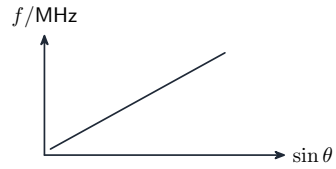
3 Exam-style questions

3.1 Plot resistance against length from the data below. Draw a best-fit straight line and use a large triangle to estimate its gradient. State suitable units and explain why the triangle should be large. [6]

Length / m	Resistance / Ω
0.20	1.31
0.40	2.08
0.60	2.92
0.80	3.67
1.00	4.51

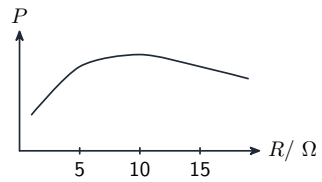


3.2 A supplied model is $f = (v/\lambda)\sin\theta$. A graph of f/MHz against $\sin\theta$ has gradient $2.30 \times 10^3 \text{ MHz}$. The light wavelength is 650 nm. Calculate v and explain the danger of ignoring the frequency-axis unit. [4]



supplied gradient: 2.30×10^3 MHz

3.3 A power-resistance graph suggests a maximum near $10\ \Omega$; current data use $5\ \Omega$ steps. Explain how to improve the maximum-location estimate. At the maximum, half the source power heats its internal resistance. Give one advantage and one disadvantage of this operating point for a transmitter. [5]



Solutions

Original-task allocations: B1 = one point, M1 = one method mark, A1 = one accuracy mark.

2.1 Direct proportionality requires a straight line through the origin. A positive intercept gives non-zero extrapolated resistance at zero measured wire length. Fixed contact/lead resistance is a possible cause.

2.2 $m = \Delta R / \Delta l = (4.50 - 1.30) / (1.00 - 0.20) = 4.00 \, \Omega \, \text{m}^{-1}$. $c = R - ml = 1.30 - 4.00 \times 0.20 = 0.50 \, \Omega$. $\rho = mA = 4.00 \times 1.2 \times 10^{-7} = 4.8 \times 10^{-7} \, \Omega \, \text{m}$.

2.3 $\Delta\rho = (0.9/100) \times 8.90 = 0.0801 \, \text{g cm}^{-3}$. Interval about 8.82 to 8.98 g cm^{-3} . 8.94 is inside, so the measurement is consistent with it; this does not uniquely prove composition.

3.1 Correct orientation, quantity/unit labels and regular scales filling a useful part of the grid. All five points accurately plotted. Thin straight best-fit line balancing scatter. Large triangle using well-separated points on the fitted line. Gradient about $4.0 \, \Omega \, \text{m}^{-1}$, accepting about 3.9–4.1 from a sensible fit. A larger triangle reduces percentage coordinate-reading uncertainty in the changes.

3.2 The SI gradient is $2.30 \times 10^3 \times 10^6 = 2.30 \times 10^9 \, \text{Hz}$. The model gives $v = \lambda \times \text{gradient}$. $v = 650 \times 10^{-9} \times 2.30 \times 10^9 = 1.50 \times 10^3 \, \text{m s}^{-1}$. Ignoring MHz produces an answer too small by a million.

3.3 Take additional readings at smaller resistance intervals around the indicated maximum. Keep source conditions steady, limiting current/heating and checking repeated readings for drift. Use the fitted curve/scatter rather than declaring one point exact. Advantage: maximum external power available for transmission. Disadvantage: large internal heating/only 50% source efficiency, wasting energy and needing cooling.