

1.8 Solid materials: Hooke, stress–strain, strain energy

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS stiffness • stress • strain • Young modulus • elastic limit
• plastic deformation • elastic strain energy

Objective

Build the skills to answer WPH11 questions on **solid materials**: Hooke's law, stress and strain, the Young modulus, the named points on the graphs, and elastic strain energy. This sheet covers Unit 1 statements 27–32 (core practical 3).

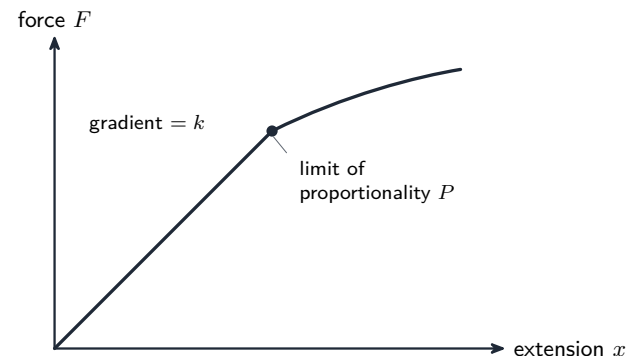
You must be able to:

- use $\Delta F = k \Delta x$ and read the stiffness k from a force–extension graph
- define stress $\sigma = \frac{F}{A}$ and strain $\varepsilon = \frac{\Delta x}{x}$, and compute the Young modulus $E = \frac{\sigma}{\varepsilon}$
- name and use the limit of proportionality, elastic limit, yield point and breaking stress
- distinguish elastic from plastic deformation
- calculate elastic strain energy from $\Delta E_{\text{el}} = \frac{1}{2} F \Delta x$ and from areas under graphs

1 Key knowledge and worked examples

■ Hooke's law and stiffness

While a material behaves proportionally, $\Delta F = k \Delta x$: the force is proportional to the extension, and the **stiffness** k is the gradient of the force–extension graph. The straight part ends at the **limit of proportionality**.



Worked example (deduce-and-compare, June 2025 style). A spring's force–extension line passes through (20 mm, 55 N). A table lists three springs with stiffnesses 1700, 2700 and 6100 N m^{−1}. Which spring was tested?

- Known: one point on the straight line through the origin. Why: $k = \frac{\Delta F}{\Delta x}$ is the gradient.

$$k = \frac{\Delta F}{\Delta x} = \frac{55 \text{ N}}{20 \times 10^{-3} \text{ m}} = 2750 \text{ N m}^{-1}$$

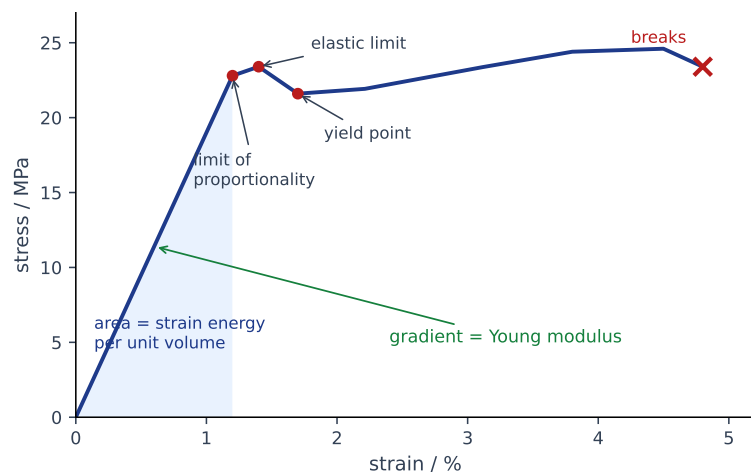
- 2750 N m^{−1} is closest to 2700 N m^{−1}, so the spring is the second one. Readings from a real graph carry a tolerance – say the conclusion **and** the comparison.

■ Stress, strain and the Young modulus

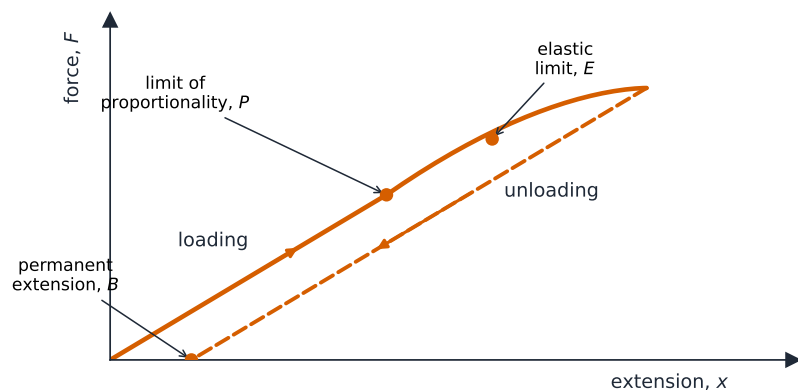
Force and extension describe the **sample**. Stress and strain describe the **material**:

$$\sigma = \frac{F}{A} \quad \varepsilon = \frac{\Delta x}{x} \quad E = \frac{\sigma}{\varepsilon}$$

The Young modulus is the gradient of the straight part of a **stress–strain** graph – it is the same for every sample of the material, whatever its size.



- **elastic deformation** : the sample returns to its original length when the force is removed.
- **plastic deformation** : a permanent extension remains.
- The **elastic limit** separates them; a material with a distinct **yield point** stretches substantially with little extra force near yield; other ductile materials yield gradually; the **breaking stress** is the stress at fracture.



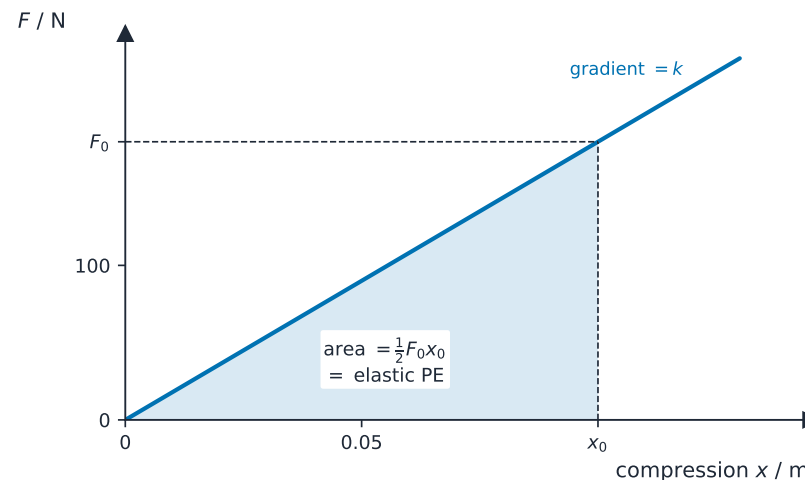
Hooke up to limit of proportionality · plastic beyond yield

■ Elastic strain energy

Stretching stores energy. For a linear spring:

$$\Delta E_{\text{el}} = \frac{1}{2} F \Delta x$$

–the area of the triangle under the force–extension line. For a non-linear graph, estimate the area by counting squares. The area under a loading *stress–strain* graph is the work done **per unit volume**, since $\frac{F}{A} \times \frac{\Delta x}{x} = \frac{F \Delta x}{V}$. In the elastic region this energy is recoverable; beyond the elastic limit, the loading work is not all recoverable.



■ Core practical 3: the Young modulus of a wire

Hang known weights on a long wire and measure the extension with a marker against a scale. Use a **long** wire (larger extensions to measure), measure the diameter with a micrometer in several places, add loads in steps and stay within the proportional region. Then E comes from the gradient of the stress–strain line.

2 Practice

2.1 A spring extends 4.0 cm under 10 N. Calculate its stiffness. [2]

2.2 Define (a) stress and (b) strain, giving the unit of each. [2]

2.3 A wire of cross-sectional area $2.0 \times 10^{-6} \text{ m}^2$ carries 50 N. Calculate the stress. [2]

2.4 A spring obeys Hooke's law with $k = 250 \text{ N m}^{-1}$. Calculate the elastic strain energy stored when it is stretched by 6.0 cm. *[Hint: find F first.]* [3]

3 Exam-style questions

3.1 A spring is stretched beyond its elastic limit and the force is then removed. What happens? [1]

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- A the force was proportional to the extension throughout
- B the spring returns to its original length
- C plastic deformation began before the elastic limit
- D the spring keeps a permanent extension

3.2 The stress on a sample is increased until it breaks. What does the gradient of the straight part of the stress-strain graph give? [1]

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- A the area under the graph
- B the Young modulus
- C the breaking stress
- D the yield point

3.3 A climbing rope sample has original length 40.0 m and stretches to 41.4 m under load.

(a) Calculate the strain in the sample. [2]

(b) The rope's cross-sectional area is $1.1 \times 10^{-4} \text{ m}^2$ and the stress at this strain is 95 MPa. Calculate the force on the rope. [3]

(c) Calculate the Young modulus of the rope material. [2]

3.4 A brick in a wall is compressed. Its force–compression graph is a straight line through the origin passing through (50 nm, 100 N).

(a) Calculate the stiffness of the brick in compression. [2]

(b) Determine the increase in elastic strain energy in the brick as the force rises from 100 N to 200 N. [3]

(c) State what is meant by **elastic** deformation. [1]

Solutions

2.1 $k = \frac{\Delta F}{\Delta x} = \frac{10 \text{ N}}{0.040 \text{ m}} = 250 \text{ N m}^{-1}.$

2.2 (a) stress = force per cross-sectional area, unit Pa; (b) strain = extension per original length, no unit.

2.3 $\sigma = \frac{F}{A} = \frac{50 \text{ N}}{2.0 \times 10^{-6} \text{ m}^2} = 2.5 \times 10^7 \text{ Pa}.$

2.4 $F = k \Delta x = 250 \text{ N m}^{-1} \times 0.060 \text{ m} = 15 \text{ N}; \Delta E_{\text{el}} = \frac{1}{2} F \Delta x = \frac{1}{2} \times 15 \text{ N} \times 0.060 \text{ m} = 0.45 \text{ J}.$

3.1 D. Beyond the elastic limit the deformation is plastic, so a permanent extension remains. A fails at the limit of proportionality; B describes elastic deformation; C starts plasticity too early.

3.2 B. The gradient of stress over strain is the Young modulus. The breaking stress is a point on the stress axis, the yield point a point on the curve, and the area is the strain energy per unit volume – not the gradient.

3.3 (a) $\varepsilon = \frac{\Delta x}{x} = \frac{41.4 \text{ m} - 40.0 \text{ m}}{40.0 \text{ m}} = 0.035.$ (b) $\sigma = \frac{F}{A} \Rightarrow F = \sigma A: F = 95 \times 10^6 \text{ Pa} \times 1.1 \times 10^{-4} \text{ m}^2 = 1.0 \times 10^4 \text{ N}.$ (c) $E = \frac{\sigma}{\varepsilon} = \frac{95 \times 10^6 \text{ Pa}}{0.035} = 2.7 \times 10^9 \text{ Pa}.$

3.4 (a) $k = \frac{\Delta F}{\Delta x} = \frac{100 \text{ N}}{50 \times 10^{-9} \text{ m}} = 2.0 \times 10^9 \text{ N m}^{-1}.$ (b) The increase is the area of the trapezium between the two forces: $\Delta E_{\text{el}} = \frac{1}{2} F_2 x_2 - \frac{1}{2} F_1 x_1$ with $x_1 = 50 \text{ nm}$, $x_2 = 100 \text{ nm}$; $\Delta E_{\text{el}} = \frac{1}{2} (200 \text{ N} \times 100 \text{ nm} - 100 \text{ N} \times 50 \text{ nm}) = \frac{1}{2} (2.0 \times 10^{-5} - 5.0 \times 10^{-6}) \text{ J} = 7.5 \times 10^{-6} \text{ J}.$ (c) The material returns to its original size and shape when the force is removed.