

1.7 Fluids: density, upthrust and viscous drag

Name: _____ Class: _____ Date: _____ **Total: 19 marks**

KEY WORDS density • upthrust • viscous drag • viscosity
• laminar flow • terminal velocity

Objective

Build the skills to answer WPH11 questions on **fluids** : density, upthrust, and viscous drag with Stokes’ law, including the falling-ball viscosity experiment. This sheet covers Unit 1 statements 23–26 (core practical 2).

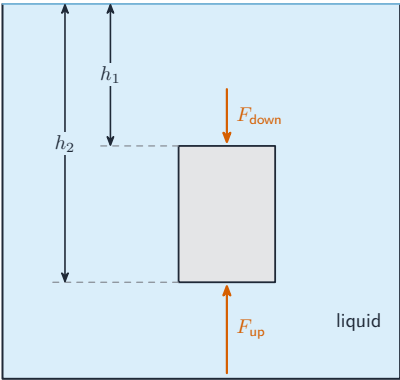
You must be able to:

- use $\rho = \frac{m}{V}$, including the volume of a sphere $V = \frac{4}{3}\pi r^3$
- state that upthrust equals the **weight of fluid displaced**
- use Stokes’ law $F = 6\pi\eta r v$ and state its conditions
- explain the falling-ball method for viscosity, including why terminal velocity matters
- predict how temperature changes a viscosity measurement

1 Key knowledge and worked examples

■ Density and upthrust

- **Density** : $\rho = \frac{m}{V}$, unit kg m^{-3} .
- **Upthrust** = weight of the fluid displaced. Not the volume. Not the mass. The *weight*.
- Upthrust exists because pressure increases with depth: the upward force on the bottom of a submerged object beats the downward force on its top.



Worked example. A steel sphere has radius $7.0 \times 10^{-3} \text{ m}$. The density of steel is $7.8 \times 10^3 \text{ kg m}^{-3}$. Calculate its mass.

- Known: r, ρ . Why: $\rho = \frac{m}{V}$ needs the volume first – a sphere, so $V = \frac{4}{3}\pi r^3$.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (7.0 \times 10^{-3} \text{ m})^3 = 1.44 \times 10^{-6} \text{ m}^3$$

$$m = \rho V = 7.8 \times 10^3 \text{ kg m}^{-3} \times 1.44 \times 10^{-6} \text{ m}^3 = 1.1 \times 10^{-2} \text{ kg}$$

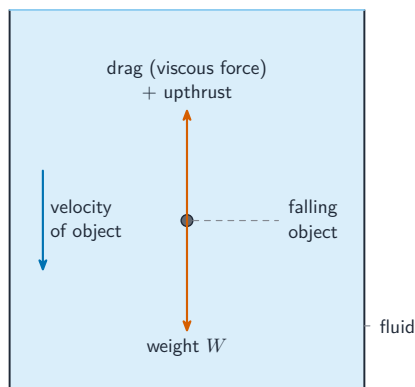
- Check: about 11 grams for a ball-bearing – sensible.

■ Stokes’ law

For a **small sphere** moving **slowly** through a fluid in **laminar flow** (no turbulence), the viscous drag is

$$F = 6\pi\eta r v$$

with η the viscosity of the fluid, r the radius, v the speed. For this oil, viscosity **falls as the liquid warms** – warmer oil drags less, so the same ball falls faster.



■ Core practical 2: the falling-ball method

Drop a ball-bearing into a tall measuring cylinder of the liquid. Mark two lines below the surface; start timing where the ball already falls at **terminal velocity**

. At terminal velocity:

$$\text{weight} = \text{upthrust} + \text{drag} \quad \Rightarrow \quad 6\pi\eta rv = \frac{4}{3}\pi r^3(\rho_{\text{ball}} - \rho_{\text{liquid}})g$$

You need: the ball's radius and mass (or density), the liquid's density, the marked distance and the time. Do not time from the surface – the ball is still accelerating there. To test the model, calculate the predicted Stokes drag and compare it with $W - U$ at the measured terminal speed. A substantial discrepancy indicates that the assumed Stokes conditions do not apply; substituting a speed into the formula does not prove the model is valid.

Worked example (January 2025 Q20 style). A sphere falls 0.45 m of oil at terminal velocity in 7.3 s. Its radius is 0.90×10^{-3} m; the oil's viscosity is 0.21 Pa s. Calculate the viscous drag.

- Known: s, t, r, η . Why: at terminal velocity the speed is constant, so $v = \frac{s}{t}$; drag follows Stokes' law (conditions stated).

$$v = \frac{s}{t} = \frac{0.45 \text{ m}}{7.3 \text{ s}} = 0.0616 \text{ m s}^{-1}$$

$$F = 6\pi\eta rv = 6\pi \times 0.21 \text{ Pa s} \times 0.90 \times 10^{-3} \text{ m} \times 0.0616 \text{ m s}^{-1} = 2.2 \times 10^{-4} \text{ N}$$

- Check: a fraction of a millinewton – the ball-bearing's weight is of the same order, as the force balance requires.

2 Practice

2.1 A block of aluminium has mass 54 g and volume 20 cm^3 . Calculate its density in kg m^{-3} . [2]

2.2 A cube displaces 0.040 kg of water. State the upthrust on the cube. (*Take $g = 9.81 \text{ N kg}^{-1}$.*) [2]

2.3 State the three conditions needed for Stokes' law to apply. [2]

2.4 In the falling-ball experiment, explain why the timing must start below the surface of the oil, after the ball has reached terminal velocity. [2]

3 Exam-style questions

3.1 A cube of volume V placed in a liquid displaces a weight W of the liquid. What is the magnitude of the upthrust on the cube? [1]

☐

- A $V \times W$
 B V
 C $V \times \rho$
 D W

3.2 A student warms the oil and repeats a falling-ball measurement. The time for the ball to fall the marked distance is shorter. Which statement explains this? [1]

- ☐
- A** warm oil is more viscous, so drag increases

B warm oil is less viscous, so terminal velocity is higher

C warm oil is denser, so upthrust increases

D the ball expands, so its weight increases

3.3 A ball-bearing of diameter 1.60 mm falls 25.0 cm through vegetable oil at terminal velocity in 1.8 s. The viscous drag at that speed is 8.4×10^{-5} N.

(a) Calculate the viscosity of the oil. [3]

(b) State two further measurements needed to check the force balance directly. [2]

3.4 A submarine hanging from a crane cable moves upwards at constant speed, starting fully submerged and ending fully out of the water. Explain how the tension in the cable changes during the lift. Assume drag is negligible. [4]

Solutions

2.1 $\rho = \frac{m}{V}$: $\rho = \frac{0.054 \text{ kg}}{20 \times 10^{-6} \text{ m}^3} = 2.7 \times 10^3 \text{ kg m}^{-3}$.

2.2 Upthrust = weight of displaced fluid = $mg = 0.040 \text{ kg} \times 9.81 \text{ N kg}^{-1} = 0.39 \text{ N}$.

2.3 Small sphere; low speed / laminar flow, no turbulence; (and viscosity depends on temperature, so the temperature must be steady).

2.4 Just below the surface the ball is still accelerating, so the speed is not constant; the measured time must be at constant terminal velocity, or $v = \frac{s}{t}$ gives the wrong value.

3.1 D. Upthrust equals the weight of the displaced liquid – the weight, not its volume or its mass. A and C confuse volume with weight; B is a volume, not a force.

3.2 B. Viscosity falls as the oil warms, so drag is less at the same speed and the terminal velocity is higher, so the time is shorter. A reverses the temperature effect; C is wrong on density; D confuses expansion with weight.

3.3 (a) $v = \frac{s}{t} = \frac{0.250 \text{ m}}{1.8 \text{ s}} = 0.139 \text{ m s}^{-1}$; $F = 6\pi\eta rv \Rightarrow \eta = \frac{F}{6\pi rv}$; $\eta = \frac{8.4 \times 10^{-5} \text{ N}}{6\pi \times 0.80 \times 10^{-3} \text{ m} \times 0.139 \text{ m s}^{-1}}$
0.040 Pa s. (b) The ball's mass (or weight) and the oil's density. The diameter is already given, so the displaced volume is known: use $mg = \rho_{oil}Vg + F_{drag}$ to check the force balance.

3.4 Fully submerged at constant speed: tension $T = W - U$, with U the upthrust. As the submarine emerges, the displaced volume falls, so the upthrust falls, and the tension rises. Once fully out of the water there is no upthrust, so $T = W$ and stays there.