

1.2 Vectors and projectile motion

Name: _____ Class: _____ Date: _____

Total: 26 marks

KEY WORDS scalar • vector • equilibrium • resultant • projectile
• component

Objective

Build the skills to answer WPH11 questions on **vectors** and **projectile motion**: resolve a force into components, combine components into a resultant, draw a scale vector diagram, and treat horizontal and vertical motion as independent. This sheet covers Unit 1 statements 4–7.

You must be able to:

- tell scalars from vectors and read vector notation ($\vec{F}$)
- resolve a vector into perpendicular components by drawing and by calculation
- find the resultant of two coplanar vectors at right angles by calculation, and at any angle by a scale drawing
- explain why horizontal and vertical motion are independent, and use that in projectile calculations
- finish a *show that* or *deduce* question with the required comparison

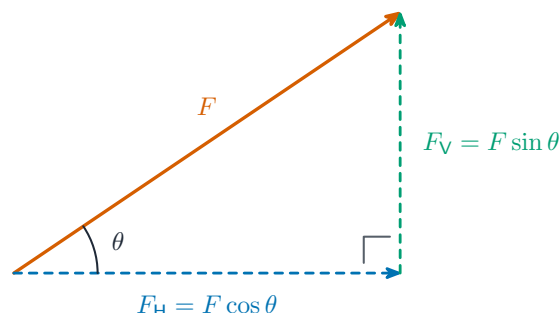
1 Key knowledge and worked examples

■ Scalars and vectors

A **scalar** has size only: distance, speed, mass, energy, work, temperature. A **vector** has size **and direction**: displacement, velocity, acceleration, force, momentum, weight. The paper writes vectors with an arrow, $\vec{F}$; your answer can underline the letter.

Because displacement is the straight line from start to finish, a winding path always gives distance larger than the magnitude of the displacement.

■ Resolving one vector into components



The component of F **along** the direction that makes angle θ with it is $F \cos \theta$; the component **perpendicular** to that direction is $F \sin \theta$. Draw the right triangle first, then read off which side holds the cosine.

Worked example. A rope pulls a sled with $T = 80$ N at 35° above the horizontal. Find the horizontal and vertical components.

- Known: T , θ from the horizontal. Why: the triangle has θ between the rope and the horizontal, so the horizontal side is the cosine.

$$T_h = T \cos \theta = 80 \text{ N} \times \cos 35^\circ = 66 \text{ N}$$

$$T_v = T \sin \theta = 80 \text{ N} \times \sin 35^\circ = 46 \text{ N}$$

- Check: the components rebuild the rope – $\sqrt{65.5^2 + 45.9^2} \text{ N} = 80 \text{ N} \checkmark$ (keep full digits until the end).

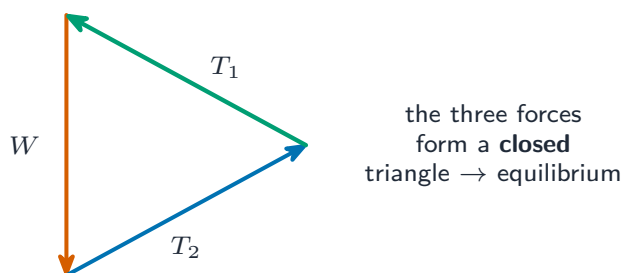
■ Combining vectors into a resultant

Perpendicular components rebuild the vector:

$$R = \sqrt{F_x^2 + F_y^2} \quad \tan \theta = \frac{F_y}{F_x}$$

Combine before squaring: the resultant of 500 N up and 200 N sideways is $\sqrt{(500)^2 + (200)^2} \text{ N} = 539 \text{ N}$, not $500 + 200$ and not $\sqrt{500^2} + \sqrt{200^2}$.

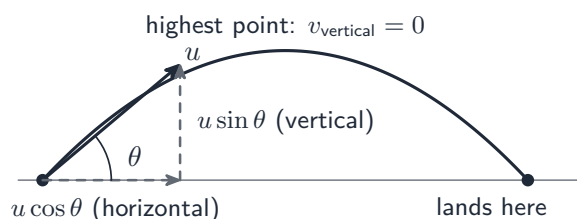
At any other angle, add the vectors **nose to tail** and measure from a scale drawing, or resolve each vector into components and add the components. Three forces in **equilibrium** close into a triangle: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$.



■ Projectile motion: two independent motions

A projectile moving freely under gravity has **no horizontal force**, so its horizontal velocity is constant, while its vertical velocity changes with $a = -g$. The two motions share only the time.

- Two balls leaving the same tabletop – one fast, one slow, or one heavy, one light – land after the **same** time, because the fall time comes only from the vertical motion.
- Resolve the launch velocity first: $u_h = u \cos \theta$, $u_v = u \sin \theta$.



Worked example. A ball leaves a 1.25 m table horizontally at 3.0 m s^{-1} . Find the fall time and the horizontal distance to the landing point.

- Known: $u_v = 0$, $s_v = -1.25 \text{ m}$, $a = -g$; $u_h = 3.0 \text{ m s}^{-1}$. Why: vertical suvat gives the time; the horizontal motion just keeps u_h .

$$s_v = u_v t + \frac{1}{2} a t^2 \Rightarrow -1.25 \text{ m} = 0 - \frac{1}{2} \times 9.81 \text{ m s}^{-2} \times t^2$$

$$t = \sqrt{\frac{2 \times 1.25 \text{ m}}{9.81 \text{ m s}^{-2}}} = 0.50 \text{ s}$$

$$s_h = u_h \times t = 3.0 \text{ m s}^{-1} \times 0.50 \text{ s} = 1.5 \text{ m}$$

- Check: half a second to fall just over a metre – consistent with $s = \frac{1}{2} g t^2$ for one second giving about 5 m.

2 Practice

2.1 Sort into scalars and vectors: speed, velocity, work, weight, distance, momentum, displacement, energy. [2]

2.2 A force of 60 N acts at 25° to the horizontal. Calculate its horizontal and vertical components. [2]

2.3 Two perpendicular forces act on a point: 30 N north and 40 N east. Calculate the magnitude of the resultant and its angle to north. [3]

2.4 A picture hangs from a hook by a wire. The wire pulls the picture with two tensions T_1 and T_2 and the picture has weight W .

(a) Write the vector equation that shows the picture is in equilibrium. [1]

(b) Explain what a scale drawing of the three forces must look like. [2]

2.5 Two coins leave the edge of the same bench: coin P slides off at 1.0 m s^{-1} , coin Q is flicked off at 4.0 m s^{-1} . Explain why both coins land at the same time, and say which lands further from the bench. [2]

3 Exam-style questions

3.1 A boat is pulled north by 420 N and east by 160 N. Which expression gives the magnitude of the resultant force? [1]

- ☐ **A** $\sqrt{420^2} + \sqrt{160^2}$
B $420 + 160$
C $\sqrt{420^2 + 160^2}$
D $\sqrt{420^2 - 160^2}$

3.2 A stone is thrown horizontally from a cliff at the same moment as a second stone is dropped from the same height. Ignore air resistance. Which statement is correct? [1]

- ☐ **A** the thrown stone lands first because it moves faster
B the dropped stone lands first because it travels less far
C both land together because their vertical motions are identical
D both land together because their masses are equal

3.3 Traffic lights of weight W hang from the middle of a wire. Each half of the wire is at angle θ to the horizontal, with tension T in each half.

(a) Resolve each tension vertically and show that $W = 2T \sin \theta$. [2]

(b) The wire is tightened so that θ gets smaller. Explain what happens to the tension needed to support the same weight. [2]

3.4 A launcher fires a ball at 25 m s^{-1} at 40° above the horizontal from ground level.

(a) Show that the vertical component of the launch velocity is about 16 m s^{-1} . [1]

(b) Calculate the time the ball spends in the air. [3]

(c) Calculate the horizontal distance travelled before landing. [2]

(d) The launch speed stays the same but the angle increases. Explain why the horizontal distance can decrease even though the ball stays in the air longer. [2]

Solutions

2.1 Scalars: speed, work, distance, energy. Vectors: velocity, weight, momentum, displacement.

2.2 Horizontal: $F_h = F \cos \theta = 60 \text{ N} \times \cos 25^\circ = 54 \text{ N}$. Vertical: $F_v = F \sin \theta = 60 \text{ N} \times \sin 25^\circ = 25 \text{ N}$.

2.3 $R = \sqrt{F_x^2 + F_y^2} = \sqrt{30^2 + 40^2} \text{ N} = 50 \text{ N}$; $\tan \theta = \frac{40}{30}$ so $\theta = 53^\circ$ east of north.

2.4

(a) $\vec{W} + \vec{T}_1 + \vec{T}_2 = \vec{0}$. (b) Drawn nose to tail, the three force arrows form a **closed triangle**; any side's length gives its force on the chosen scale.

2.5 Both coins have the same initial vertical velocity (zero) and fall the same height under g , and the vertical motion is independent of the horizontal motion – so the times are equal; coin Q travels further because its horizontal velocity is larger for the same time.

3.1 C. The forces are perpendicular, so the resultant is the hypotenuse. A adds the roots separately; B adds without squaring; D subtracts the squares.

3.2 C. The vertical motion decides the landing time and is identical for both stones. A and B confuse horizontal speed with vertical fall; D names mass, which does not appear in free fall.

3.3 (a) Each wire half pulls up with $T \sin \theta$; equilibrium gives $W = T \sin \theta + T \sin \theta = 2T \sin \theta$. (b) $T = \frac{W}{2 \sin \theta}$: as θ gets smaller, $\sin \theta$ gets smaller, so the tension must **increase** for the same weight.

3.4 (a) $u_v = u \sin \theta = 25 \text{ m s}^{-1} \times \sin 40^\circ = 16.1 \text{ m s}^{-1} \approx 16 \text{ m s}^{-1}$. (b) Known: $u_v = 16.1 \text{ m s}^{-1}$, and at landing $v_v = -16.1 \text{ m s}^{-1}$ (same level). Use $v = u + at$:

$$t = \frac{v_v - u_v}{-g} = \frac{-16.1 \text{ m s}^{-1} - 16.1 \text{ m s}^{-1}}{-9.81 \text{ m s}^{-2}} = 3.3 \text{ s (M1, A1)}$$

(c) $u_h = u \cos \theta = 25 \text{ m s}^{-1} \times \cos 40^\circ = 19.2 \text{ m s}^{-1}$; $s_h = u_h \times t = 19.2 \text{ m s}^{-1} \times 3.3 \text{ s} = 63 \text{ m}$.

(d) A larger angle raises u_v , so the flight is longer, but it lowers u_h ; since distance = $u_h \times t$, the smaller horizontal speed can win, and the distance falls.