

Notes

Pearson Edexcel · International A-Level · Mathematics

YMA01: course teaching notes

Version: Issue 3, April 2019; first teaching 2018; unit assessment from 2019

These are original course-owned teaching notes. Objective-level exceptions are listed in the review, and lessons are tier labelled.

P1 · Indices, surds and standard form

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
 - This lesson studies **index** 指数: The power to which a base is raised.
-

Choose the mathematical structure

- For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \leq a < 10$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

Test a tempting shortcut

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 . Do not round a surd when an exact answer is requested.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For every positive a , $a^2 + a^3$ equals a^5 . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL mathematics; official unit P1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The power to which a base is raised. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.

Choose the mathematical structure

- Factor where possible; otherwise complete the square or use the quadratic formula. A quadratic inequality needs the sign on intervals, not only the roots. The discriminant identifies repeated or missing real roots.
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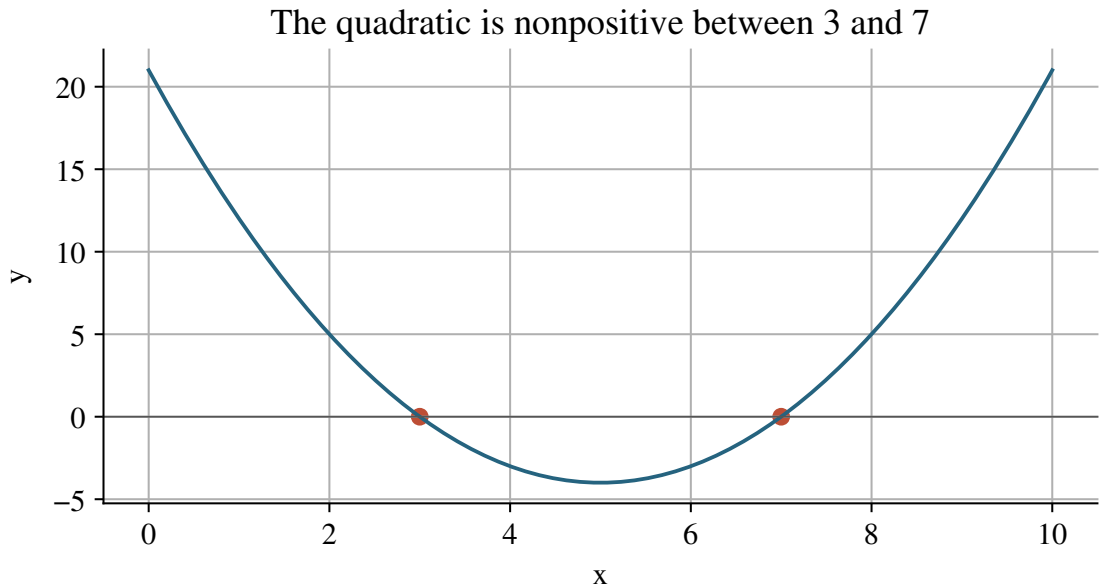
Work through a checked case

$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The condition is $x(10-x) \geq 21$, so $x^2 - 10x + 21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



Test a tempting shortcut

- Multiplying an inequality by a negative number reverses its direction. A sketch must show which side of each root satisfies the inequality. Geometry may restrict x further.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A positive discriminant means that a quadratic has no real roots. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use the vertex to interpret an optimum. Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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Key idea

The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Coordinate geometry and tangents

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
 - This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x .
-

Choose the mathematical structure

- A line through (x_1, y_1) with gradient m has $y-y_1=m(x-x_1)$. Parallel lines have equal gradients. Finite perpendicular gradients multiply to -1 . A circle has $(x-a)^2+(y-b)^2=r^2$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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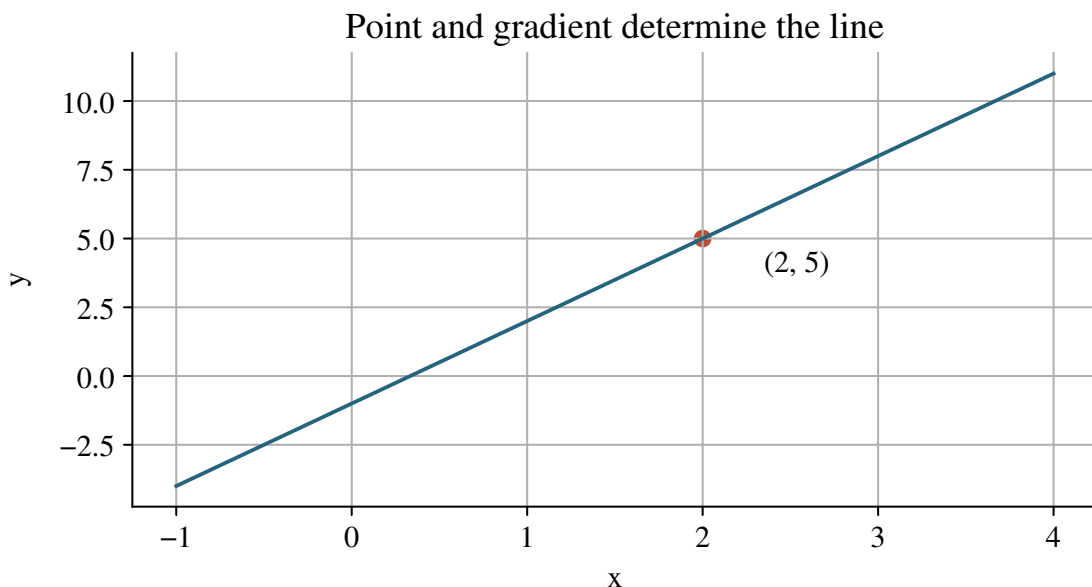
Work through a checked case

$$y - y_1 = m(x - x_1), \quad (x - a)^2 + (y - b)^2 = r^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Through (2,5) with gradient 3, $y-5=3(x-2)$, so $y=3x-1$. A perpendicular through the same point has $y-5=-(x-2)/3$. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



Test a tempting shortcut

- A vertical line has no finite gradient; do not force it into $y=mx+c$. Read the signs of a circle's centre carefully. The radius to a tangent is perpendicular to the tangent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Perpendicular nonvertical lines always have equal gradients. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections. Substitution produces a quadratic whose discriminant checks the prediction.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The change in y divided by the corresponding change in x . Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Right triangles and non-right triangles

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
 - This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.
-

Choose the mathematical structure

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$. For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

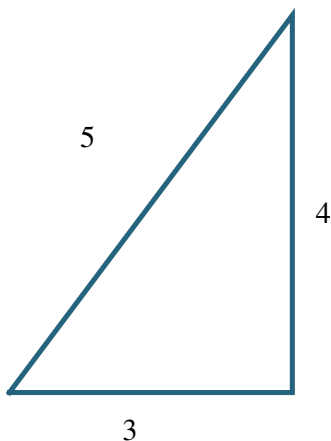
$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Test a tempting shortcut

- Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Pythagoras applies to every triangle, including triangles without a right angle. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule. Explain why the chosen triangle contains the required length or angle.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The side opposite the right angle in a right-angled triangle. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
 - This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.
-

Choose the mathematical structure

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$. Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve. List every solution in the stated interval.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

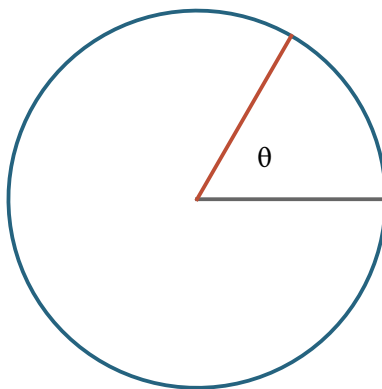
$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Test a tempting shortcut

- A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The principal inverse sine value is always the only solution in a full turn. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose an identity that reduces the equation to one trigonometric function. Keep exact familiar angles where possible, and check each solution against the original equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The angle subtended by an arc whose length equals the radius. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.

- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

Choose the mathematical structure

- For $y = ax^n$, $dy/dx = anx^{n-1}$. A stationary point satisfies $dy/dx = 0$. Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

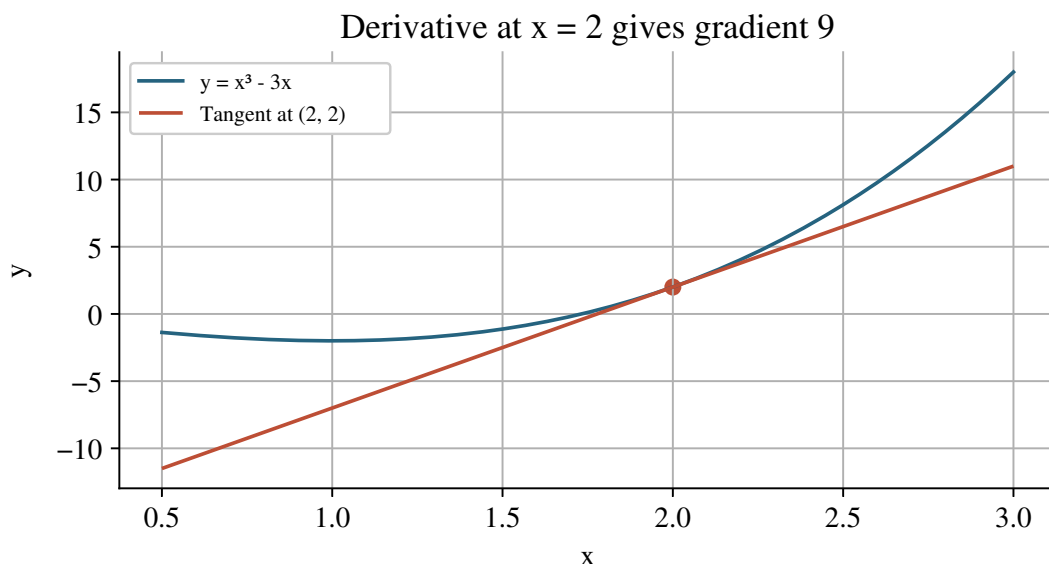
Work through a checked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad m = f'(x_0)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y = x^3 - 3x$, $dy/dx = 3x^2 - 3$. At $x = 2$, the gradient is 9 and $y = 2$. The tangent is $y - 2 = 9(x - 2)$. The normal gradient is $-1/9$.



Test a tempting shortcut

- A derivative gives a gradient, not the ordinate. Evaluate y and dy/dx separately at the supplied x -coordinate; the normal gradient is the negative reciprocal of a nonzero tangent gradient.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A tangent and normal at the same point always have the same gradient. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- P1 covers polynomial derivatives, gradients, tangents and normals. Optimization and stationary-point classification are taught in P2; they are not introduced here.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
 - This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.
-

Choose the mathematical structure

- Integrate polynomial powers by increasing the index by one and dividing by the new index. Include an arbitrary constant and use a supplied point to determine it.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

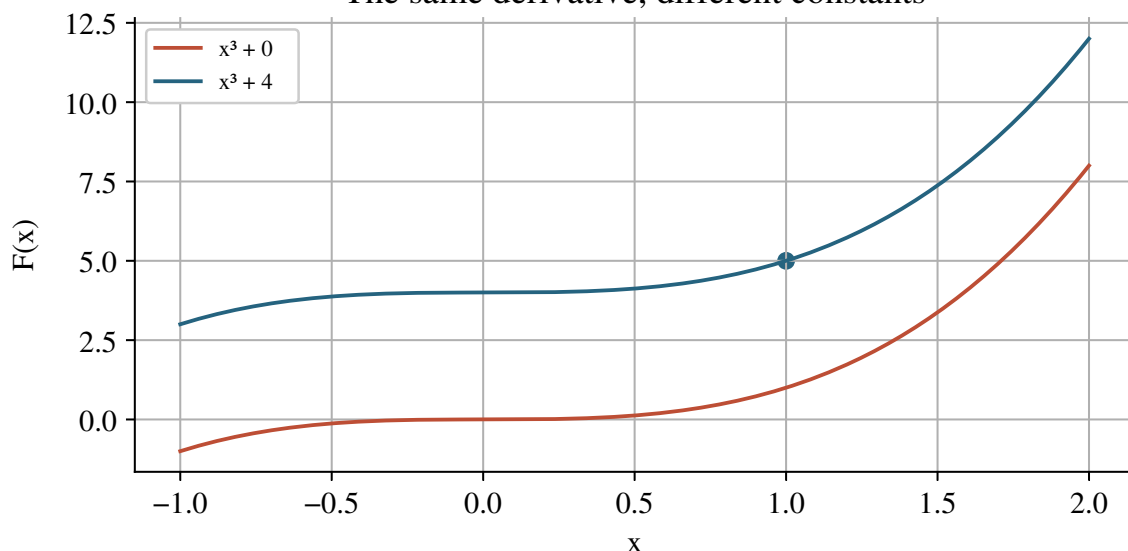
$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

An antiderivative of $3x^2$ is $x^3 + C$ because differentiating x^3 gives $3x^2$. If $F(1)=5$, then $1+C=5$, so $C=4$ and $F(x)=x^3+4$.

The same derivative, different constants



Test a tempting shortcut

- An indefinite integral needs a constant. Integrating each term changes its power; copying the derivative rule gives the wrong result.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every indefinite integral has only one possible antiderivative. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- P1 uses indefinite polynomial integration and a point to determine the constant. Definite integrals, areas and the trapezium rule are P2 content.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

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Key idea

A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
 - This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.
-

Choose the mathematical structure

- A deductive proof starts from stated definitions or assumptions. A counterexample refutes an all-values claim. Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

Odd + odd: $(2a + 1) + (2b + 1)$

Factor 2: $2(a + b + 1)$

The bracket is an integer $\rightarrow$ the sum is even

Test a tempting shortcut

- Do not assume the conclusion while proving it. In contradiction, identify the impossible consequence and reject the initial contrary assumption. Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Checking the first ten integers proves a claim for every positive integer. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- AQA proof includes deduction, exhaustion and contradiction. IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction. Keep the method matched to the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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Key idea

A single valid case that disproves a universal claim. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
 - This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
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Choose the mathematical structure

- Factor where possible; otherwise complete the square or use the quadratic formula. A quadratic inequality needs the sign on intervals, not only the roots. The discriminant identifies repeated or missing real roots.
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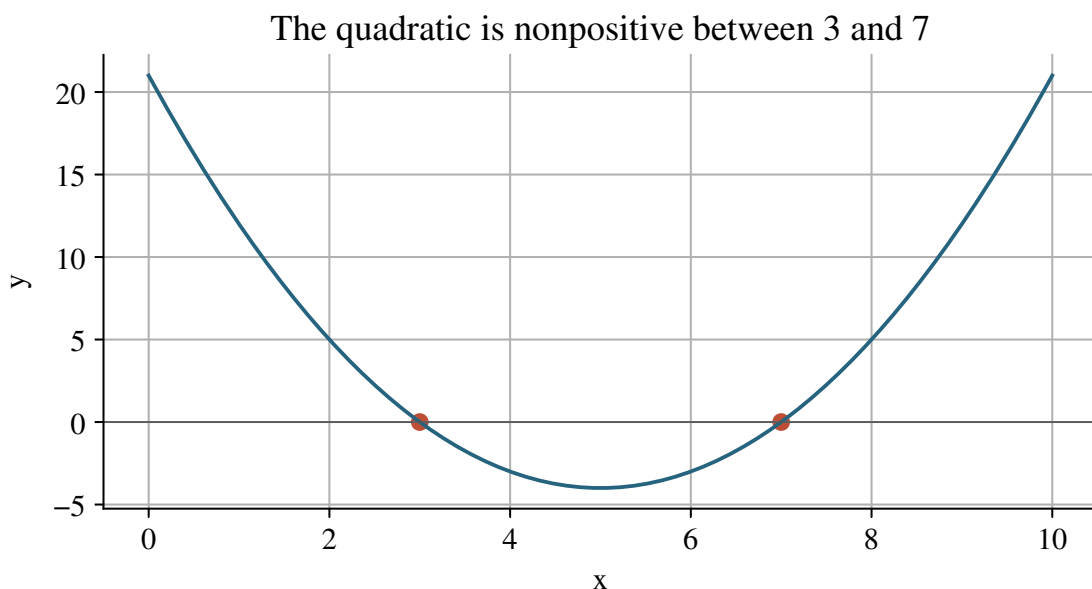
Work through a checked case

$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The condition is $x(10-x) \geq 21$, so $x^2 - 10x + 21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



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- Multiplying an inequality by a negative number reverses its direction. A sketch must show which side of each root satisfies the inequality. Geometry may restrict x further.
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A positive discriminant means that a quadratic has no real roots. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use the vertex to interpret an optimum. Check an endpoint and a point between the roots; the algebra and graph should tell the same story.
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The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Coordinate geometry and tangents

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
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Choose the mathematical structure

- A line through (x_1, y_1) with gradient m has $y-y_1=m(x-x_1)$. Parallel lines have equal gradients. Finite perpendicular gradients multiply to -1 . A circle has $(x-a)^2+(y-b)^2=r^2$.
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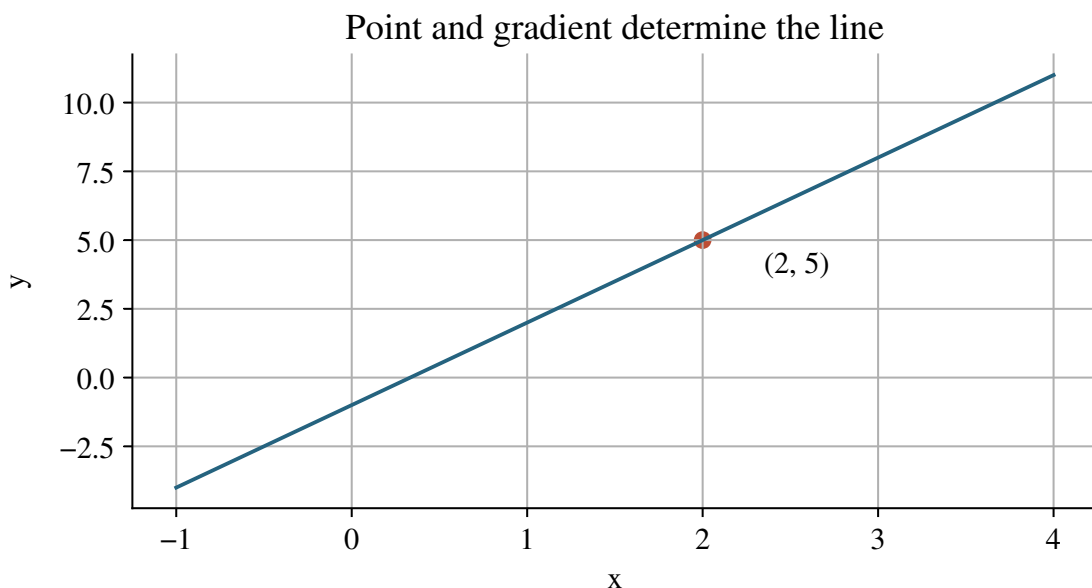
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Worked example

Through (2,5) with gradient 3, $y-5=3(x-2)$, so $y=3x-1$. A perpendicular through the same point has $y-5=-(x-2)/3$. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



Test a tempting shortcut

- A vertical line has no finite gradient; do not force it into $y=mx+c$. Read the signs of a circle's centre carefully. The radius to a tangent is perpendicular to the tangent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

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Perpendicular nonvertical lines always have equal gradients. This claim is false. Explain which definition or assumption it violates.

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The change in y divided by the corresponding change in x . Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Sequences, series and recurrence

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
 - This lesson studies **common ratio** 公比: The constant multiplier between consecutive terms of a geometric sequence.
-

Choose the mathematical structure

- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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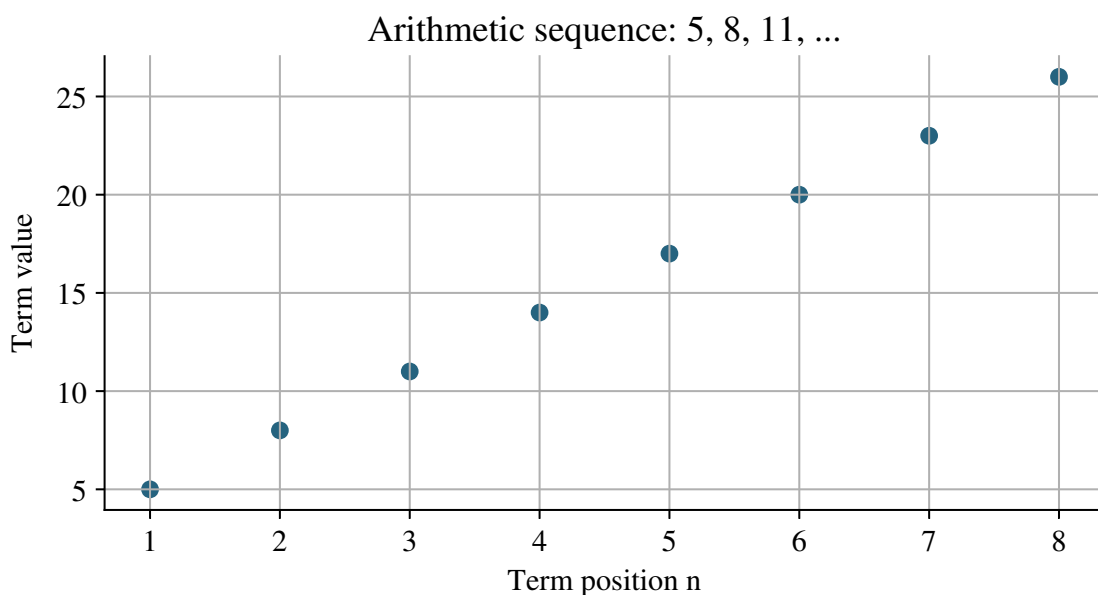
Work through a checked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10+21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1-1/2) = 24$. For $u_{n+1} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



Test a tempting shortcut

- The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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Key idea

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Binomial expansion and valid approximations

How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** 二项式系数: The number of ways to choose a specified number of objects from a set.

Choose the mathematical structure

- For positive integer n , $(a+b)^n$ is a finite binomial expansion. For noninteger n , expand $(1+x)^n$ as $1+nx+\frac{n(n-1)}{2}x^2+\dots$ with $|x|<1$. Factor out constants before using this form.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$(1+2x)^5=1+10x+40x^2+80x^3+80x^4+32x^5$. For $(1+x)^{-1}$, the first three terms are $1-x+x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1\approx 0.909091$.

$$(1+2x)^5$$

Coefficients: 1, 10, 40, 80, 80, 32

The x^2 coefficient includes $(2x)^2$

Test a tempting shortcut

- An expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$. Positive-integer expansions are finite and do not have that convergence restriction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every binomial expansion is a finite polynomial, including powers that are not positive integers. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State the range of validity alongside an approximation. Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x .
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The number of ways to choose a specified number of objects from a set. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.

- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

Choose the mathematical structure

- For $y=Ae^{kt}$, k is a proportional rate. Taking logs gives $\ln y = \ln A + kt$. Logarithms require positive arguments, and $\log(x+y)$ is not $\log x + \log y$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

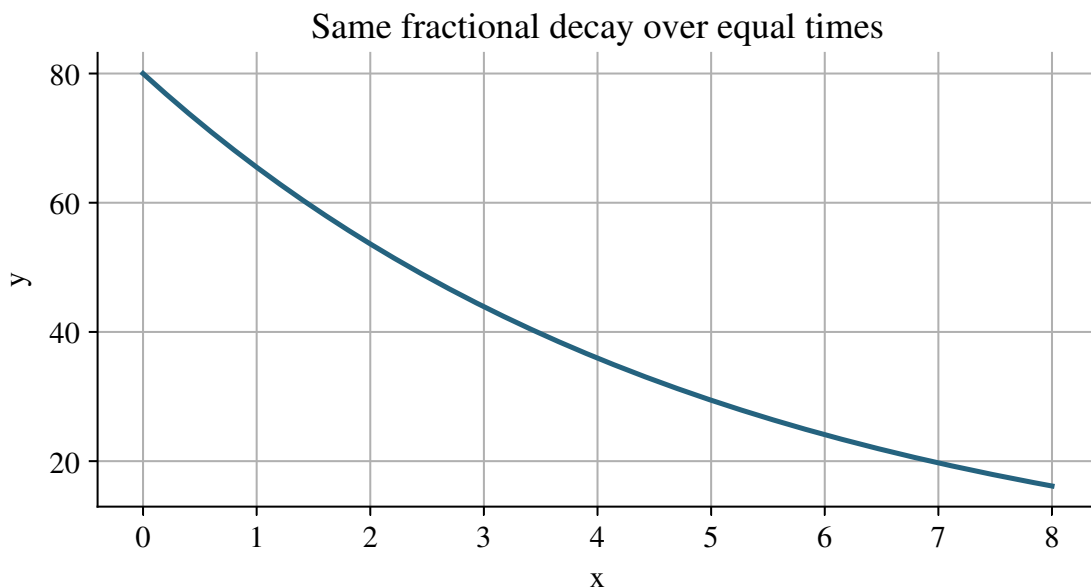
Work through a checked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If $y=80e^{(-0.2t)}$, $y=40$ gives $e^{(-0.2t)}=1/2$. Hence $t=\ln 2/0.2 \approx 3.466$. For $3^x=20$, $x=\ln 20/\ln 3$. Plotting $\ln y$ against t linearises this exponential model.



Test a tempting shortcut

- A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative k describes decay; a negative starting amount usually contradicts the context.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For positive x and y , $\ln(x+y) = \ln x + \ln y$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Compare actual observations with the model. Systematic departures may indicate changing conditions. In a report, explain what the rate and initial value mean, rather than giving a bare equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The time for a decaying quantity to fall to half its initial value. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
 - This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.
-

Choose the mathematical structure

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$. Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve. List every solution in the stated interval.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

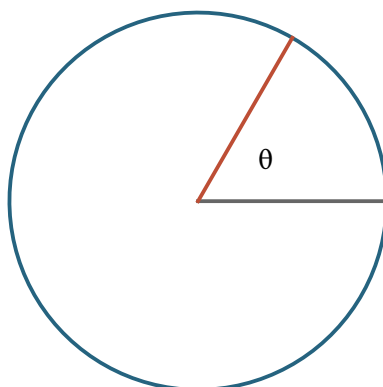
$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Test a tempting shortcut

- A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x = 0$; check that case first.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The principal inverse sine value is always the only solution in a full turn. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose an identity that reduces the equation to one trigonometric function. Keep exact familiar angles where possible, and check each solution against the original equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The angle subtended by an arc whose length equals the radius. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

Choose the mathematical structure

- For $y = ax^n$, $dy/dx = anx^{n-1}$. A stationary point satisfies $dy/dx = 0$. Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

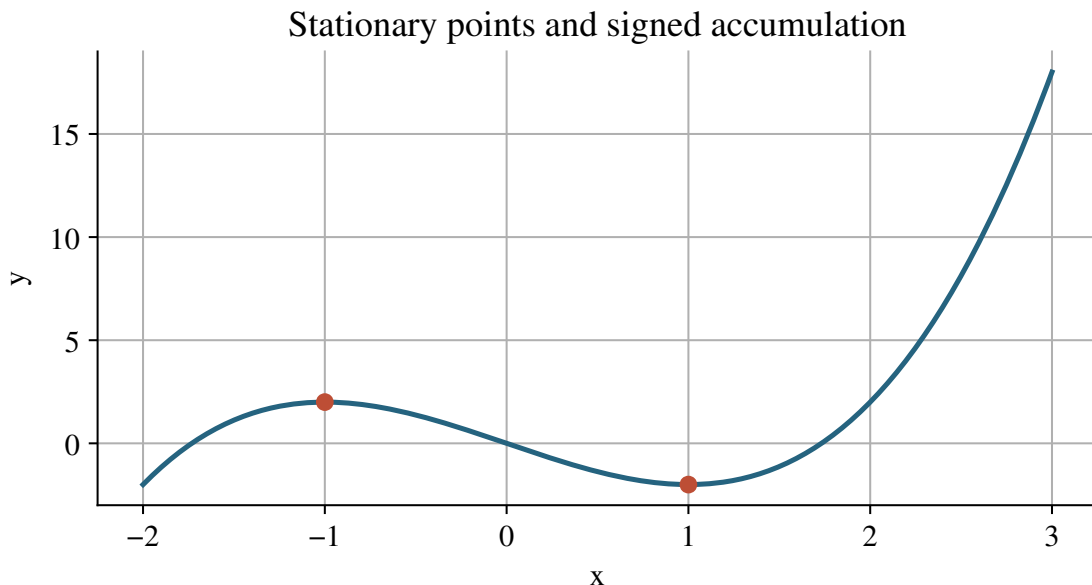
Work through a checked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=x^3-3x$, $\frac{dy}{dx}=3x^2-3$. At $x=1$, the gradient is 0 and $y=-2$. The second derivative is $6x$, positive at $x=1$, so this is a local minimum. At $x=-1$, $y=2$ and the second derivative is negative, giving a local maximum.



Test a tempting shortcut

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there. At advanced level, connect the derivative to rates and optimization with a valid domain.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
 - This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.
-

Choose the mathematical structure

- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$. A definite integral is signed accumulation. Split at sign changes when total area or distance is required.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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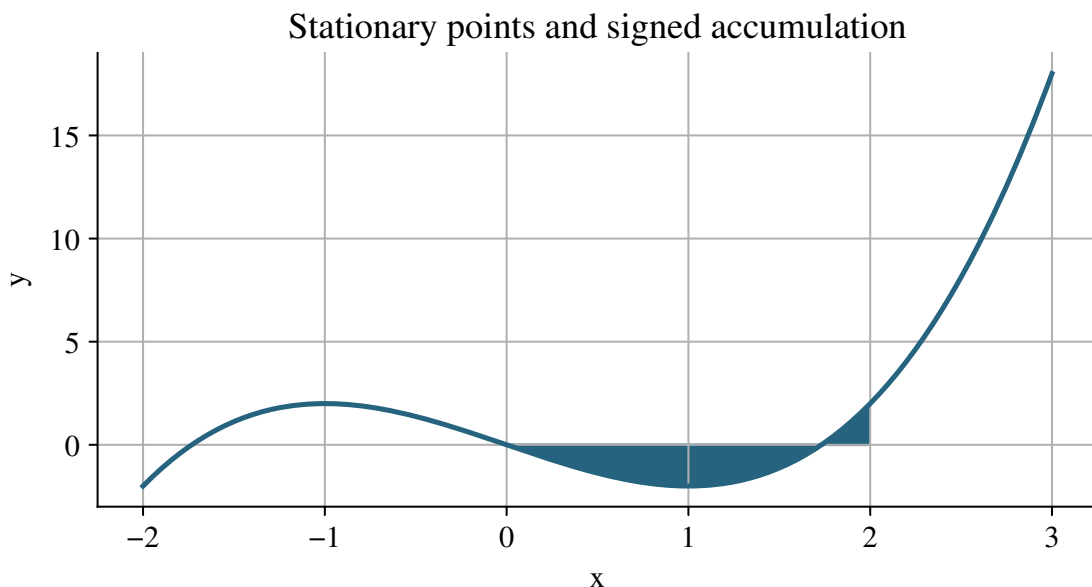
Work through a checked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $v(t) = 3t^2 - 3$ over $0 \leq t \leq 2$, displacement $= [t^3 - 3t]_0^2 = 2$. Since v changes sign at $t = 1$, distance $= -[t^3 - 3t]_0^1 + [t^3 - 3t]_1^2 = 2 + 4 = 6$. The constants cancel only for a definite integral.



Test a tempting shortcut

- The integral of $1/x$ is $\ln|x| + C$, not the power rule with $n = -1$. Signed area can be zero even when the total enclosed area is positive.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A definite integral always equals the total positive area, even when the graph crosses the axis. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Identify what the integral means and include the correct units. A rate measured per second integrates to the underlying quantity, not to another rate. Differentiate an antiderivative to check it.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Root finding and numerical integration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
 - This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.
-

Choose the mathematical structure

- For equal strip width h , the trapezium estimate is $h/2$ times the sum of the two endpoint heights plus twice the internal heights.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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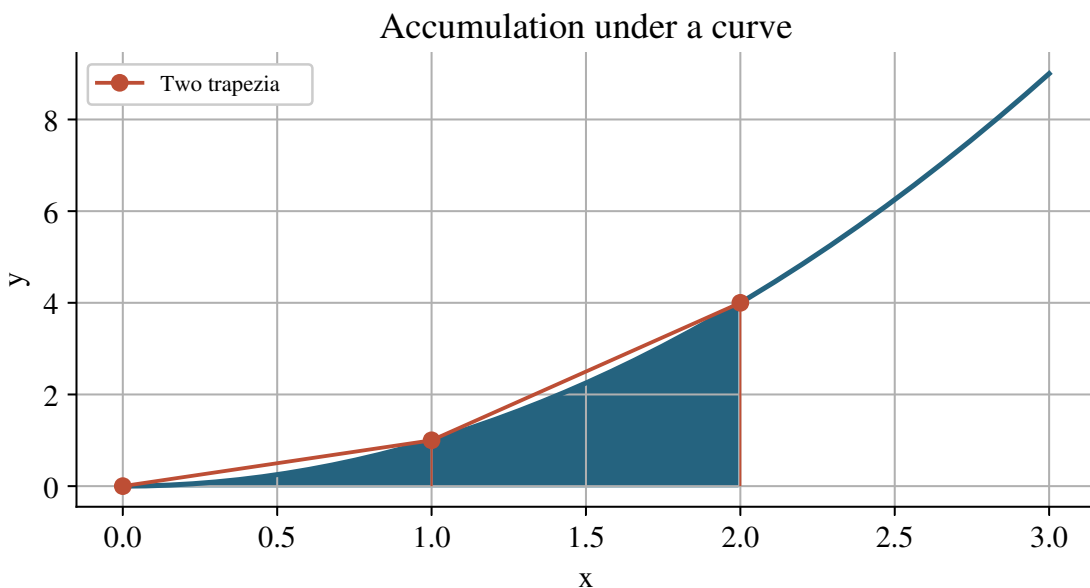
Work through a checked case

$$A \approx \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=x^2$ on $[0,2]$ with two equal strips, $h=1$ and the heights are 0,1,4. The estimate is $(1/2)(0+2\times 1+4)=3$. The exact area is $8/3$, so this convex curve gives an overestimate.



Test a tempting shortcut

- Use equal strip widths and include internal heights twice. Curvature decides whether the estimate is above or below the exact area; being increasing alone does not.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

An increasing function always gives a trapezium overestimate. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- P2 introduces the trapezium rule. Newton–Raphson and other root-finding methods belong to other named units; they are not part of this P2 lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Domains, inverses and composition

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
 - This lesson studies **domain** 定义域: The set of allowed inputs to a function.
-

Choose the mathematical structure

- State the domain and range. For an inverse, first ensure the function is one-to-one on its domain. Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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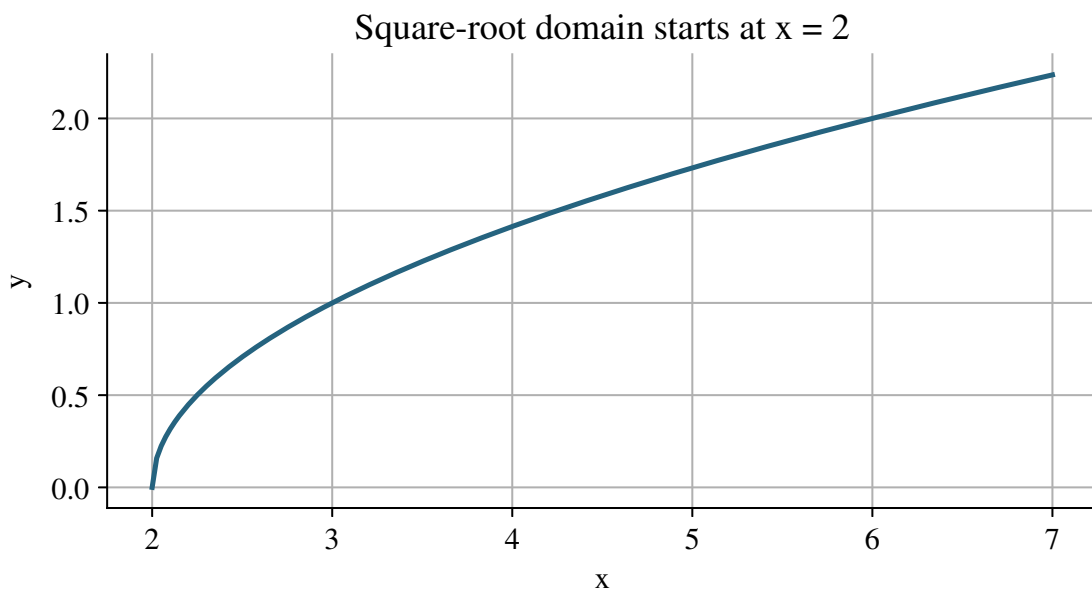
Work through a checked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus $f^{-1}(x)=x^2+2$ with $x \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



Test a tempting shortcut

- Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every quadratic function on all real numbers has an inverse function. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check $f(f^{-1}(x))=x$ on the inverse domain. Use a sketch to test whether a horizontal line meets the original graph more than once.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The set of allowed inputs to a function. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
 - This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.
-

Choose the mathematical structure

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$. Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve. List every solution in the stated interval.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

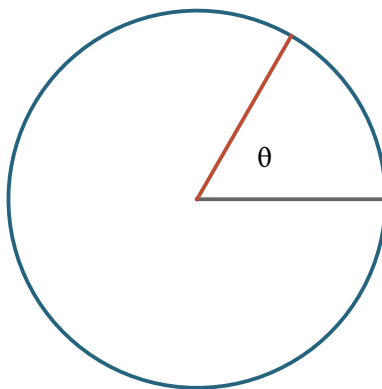
$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Test a tempting shortcut

- A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x = 0$; check that case first.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The principal inverse sine value is always the only solution in a full turn. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose an identity that reduces the equation to one trigonometric function. Keep exact familiar angles where possible, and check each solution against the original equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The angle subtended by an arc whose length equals the radius. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.

- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

Choose the mathematical structure

- For $y=Ae^{kt}$, k is a proportional rate. Taking logs gives $\ln y=\ln A+kt$. Logarithms require positive arguments, and $\log(x+y)$ is not $\log x+\log y$.
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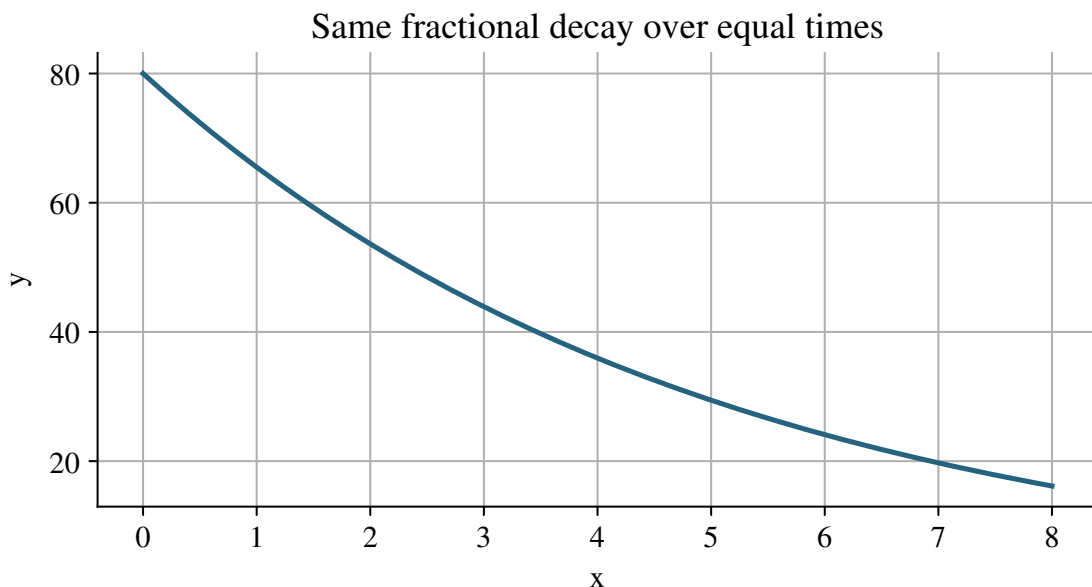
Work through a checked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If $y=80e^{(-0.2t)}$, $y=40$ gives $e^{(-0.2t)}=1/2$. Hence $t=\ln 2/0.2 \approx 3.466$. For $3^x=20$, $x=\ln 20/\ln 3$. Plotting $\ln y$ against t linearises this exponential model.



Test a tempting shortcut

- A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative k describes decay; a negative starting amount usually contradicts the context.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For positive x and y , $\ln(x+y) = \ln x + \ln y$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Compare actual observations with the model. Systematic departures may indicate changing conditions. In a report, explain what the rate and initial value mean, rather than giving a bare equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The time for a decaying quantity to fall to half its initial value. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.
-

Choose the mathematical structure

- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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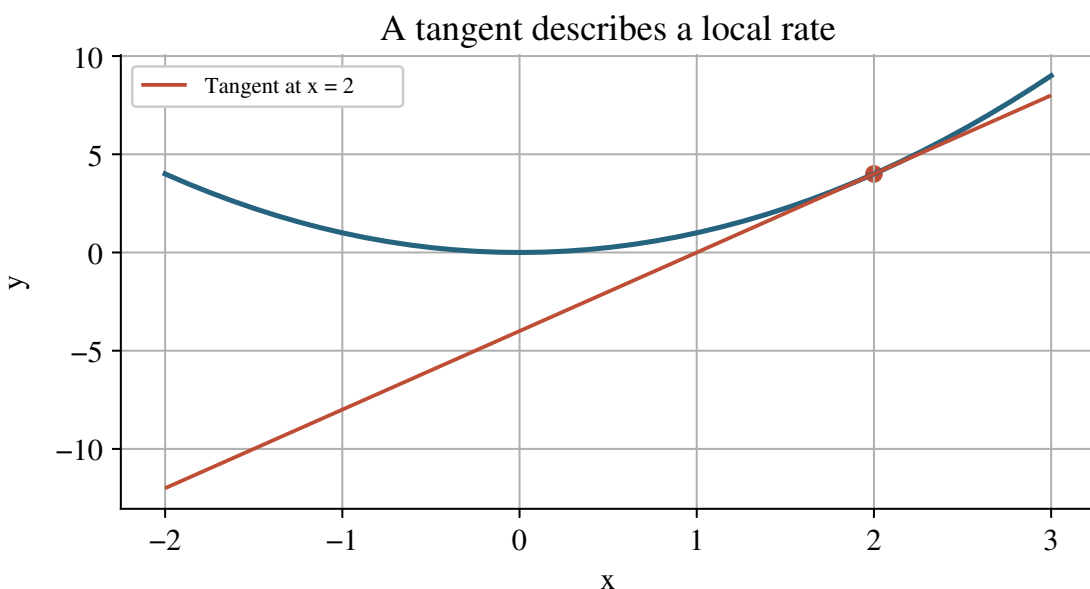
Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Test a tempting shortcut

- A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The derivative of $u(x)v(x)$ is always u prime times v prime. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

Choose the mathematical structure

- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$. A definite integral is signed accumulation. Split at sign changes when total area or distance is required.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

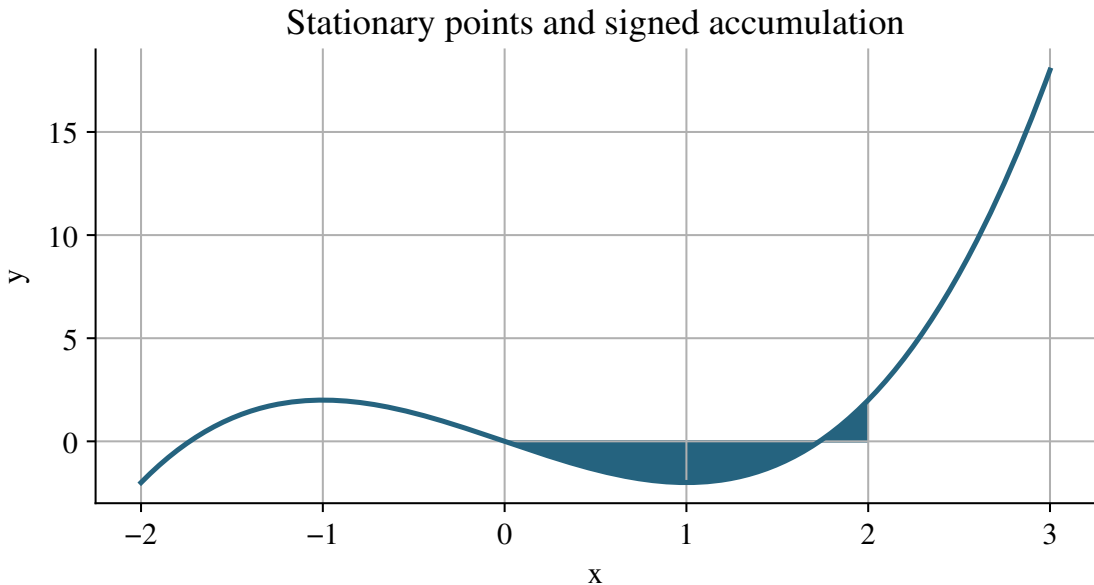
Work through a checked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $v(t)=3t^2-3$ over $0 \leq t \leq 2$, displacement $= [t^3-3t]_0^2=2$. Since v changes sign at $t=1$, distance $= -[t^3-3t]_0^1 + [t^3-3t]_1^2 = 2+4=6$. The constants cancel only for a definite integral.



Test a tempting shortcut

- The integral of $1/x$ is $\ln|x|+C$, not the power rule with $n=-1$. Signed area can be zero even when the total enclosed area is positive.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A definite integral always equals the total positive area, even when the graph crosses the axis. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Identify what the integral means and include the correct units. A rate measured per second integrates to the underlying quantity, not to another rate. Differentiate an antiderivative to check it.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Fixed-point iteration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
 - This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.
-

Choose the mathematical structure

- Rearrange the equation as $x=g(x)$, choose a starting value and iterate. A limit L must satisfy $L=g(L)$. Near a fixed point, $|g'(L)|<1$ provides a local convergence check; a sign bracket can validate the reported rounded root.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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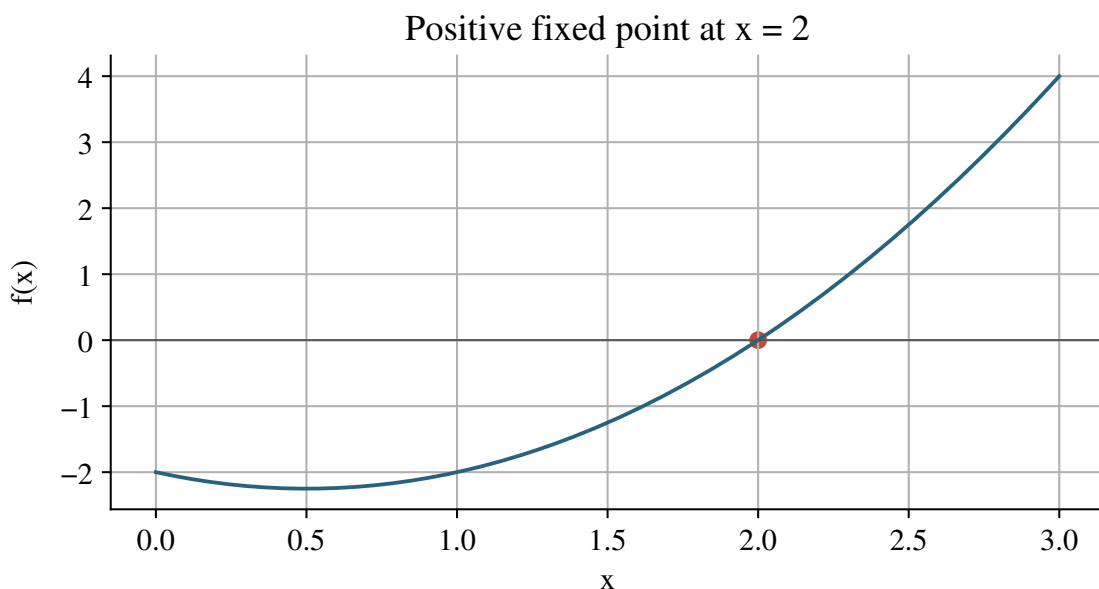
Work through a checked case

$$x_{n+1} = \sqrt{2 + x_n}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $x^2 - x - 2 = 0$ choose $x_{\text{next}} = \sqrt{2 + x}$, starting at $x_0 = 1$. Then $x_1 = \sqrt{3} \approx 1.73205$, $x_2 \approx 1.93185$ and the positive fixed point is 2. At 2, $g' = 1/(2\sqrt{4}) = 1/4$, so small local errors shrink. The square-root rearrangement seeks the positive root; it does not give the negative root -1.



Test a tempting shortcut

- Different rearrangements can have different convergence behaviour. Check the iterates, domain and original equation; a square-root update cannot reach a negative fixed point.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every rearrangement of an equation converges to the same root from every starting value. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- P3 uses equation rearrangement and numerical iteration. This lesson excludes Newton and numerical integration, which belong to other units.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
 - This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.
-

Choose the mathematical structure

- A deductive proof starts from stated definitions or assumptions. A counterexample refutes an all-values claim. Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

Odd + odd: $(2a + 1) + (2b + 1)$

Factor 2: $2(a + b + 1)$

The bracket is an integer $\rightarrow$ the sum is even

Test a tempting shortcut

- Do not assume the conclusion while proving it. In contradiction, identify the impossible consequence and reject the initial contrary assumption. Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Checking the first ten integers proves a claim for every positive integer. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- AQA proof includes deduction, exhaustion and contradiction. IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction. Keep the method matched to the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A single valid case that disproves a universal claim. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Equations, identities and rearrangement

When do two plans cost the same?

- Two mobile plans cost $20+3x$ and $44+x$ yuan for x GB. When do they cost the same?
 - This lesson studies **identity** 恒等式: An equality that holds for every allowed value of its variable.
-

Choose the mathematical structure

- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs. Preserve equality by applying the same operation to both sides. State restrictions before dividing by a variable.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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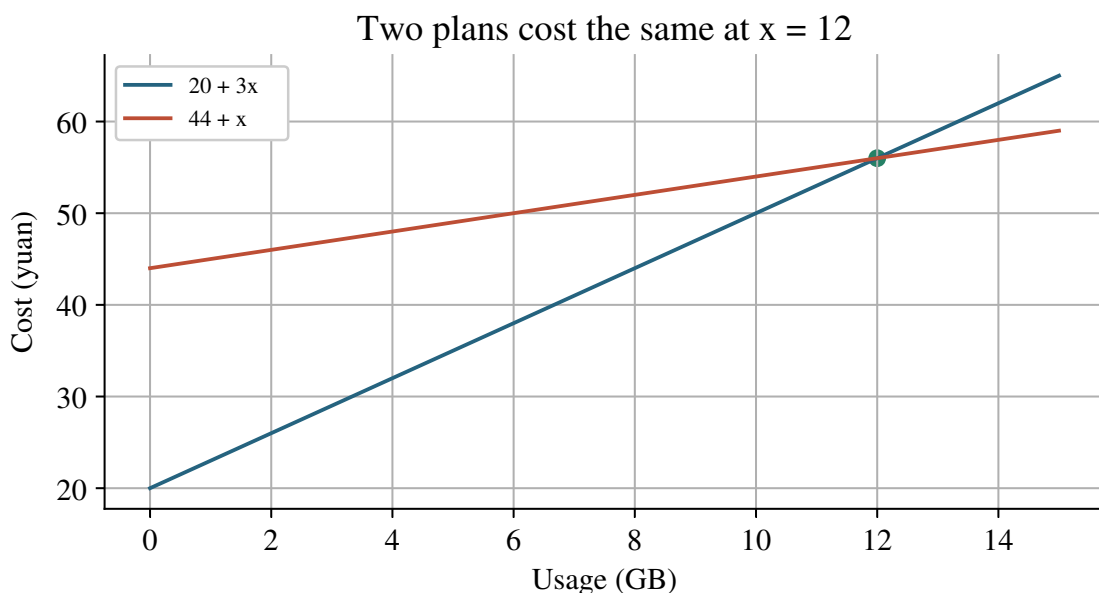
Work through a checked case

$$C_1 = 20 + 3x, \quad C_2 = 44 + x, \quad C_1 = C_2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$20+3x=44+x$ gives $2x=24$ and $x=12$. Both plans then cost 56. In $A=\pi r^2$, divide by π and take the positive square root to obtain $r=\sqrt{A/\pi}$, because r is a length.



Test a tempting shortcut

- Cancelling a term is not the same as cancelling a factor. In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$. Check a rearrangement by substitution.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Set up the equation from units and the meaning of the unknown. A negative or fractional solution may be algebraically correct but impossible for a count.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An equality that holds for every allowed value of its variable. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Binomial expansion and valid approximations

How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** 二项式系数: The number of ways to choose a specified number of objects from a set.

Choose the mathematical structure

- For positive integer n , $(a+b)^n$ is a finite binomial expansion. For noninteger n , expand $(1+x)^n$ as $1+nx+\frac{n(n-1)}{2}x^2+\dots$ with $|x|<1$. Factor out constants before using this form.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$(1+2x)^5 = 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$. For $(1+x)^{-1}$, the first three terms are $1 - x + x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1 \approx 0.909091$.

$$(1 + 2x)^5$$

Coefficients: 1, 10, 40, 80, 80, 32

The x^2 coefficient includes $(2x)^2$

Test a tempting shortcut

- An expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$. Positive-integer expansions are finite and do not have that convergence restriction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every binomial expansion is a finite polynomial, including powers that are not positive integers. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State the range of validity alongside an approximation. Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x .
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The number of ways to choose a specified number of objects from a set. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Conics, parametric curves and tangent reasoning

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
- This lesson studies **parametric equation** 参数方程: A representation in which coordinates are expressed using a common parameter.

Choose the mathematical structure

- For $x=a \cos t, y=b \sin t$, eliminating t gives $x^2/a^2 + y^2/b^2 = 1$. For parametric curves, $dy/dx = (dy/dt)/(dx/dt)$, where $dx/dt \neq 0$. A zero denominator may signal a vertical tangent.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

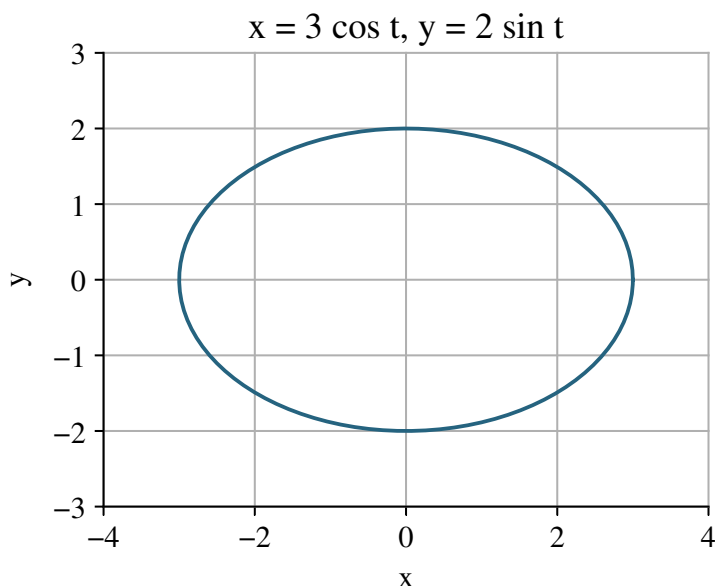
Work through a checked case

$$x = a \cos t, \quad y = b \sin t, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $x=3\cos t, y=2\sin t$ at $t=\pi/4$, the gradient is $(2\cos t)/(-3\sin t)=-2/3$. The point is $(3/\sqrt{2}, \sqrt{2})$. The area inside the ellipse is $\pi ab=6\pi$; its semiaxes are 3 and 2.



Test a tempting shortcut

- Eliminating a parameter may lose a domain restriction or direction of travel. A vertical tangent cannot be assigned a finite dy/dx . Distinguish semiaxes from full widths.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Eliminating a parameter always preserves every restriction and direction automatically. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For advanced coordinate geometry, use the defining equation and a consistent parameter. For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A representation in which coordinates are expressed using a common parameter. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.
-

Choose the mathematical structure

- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

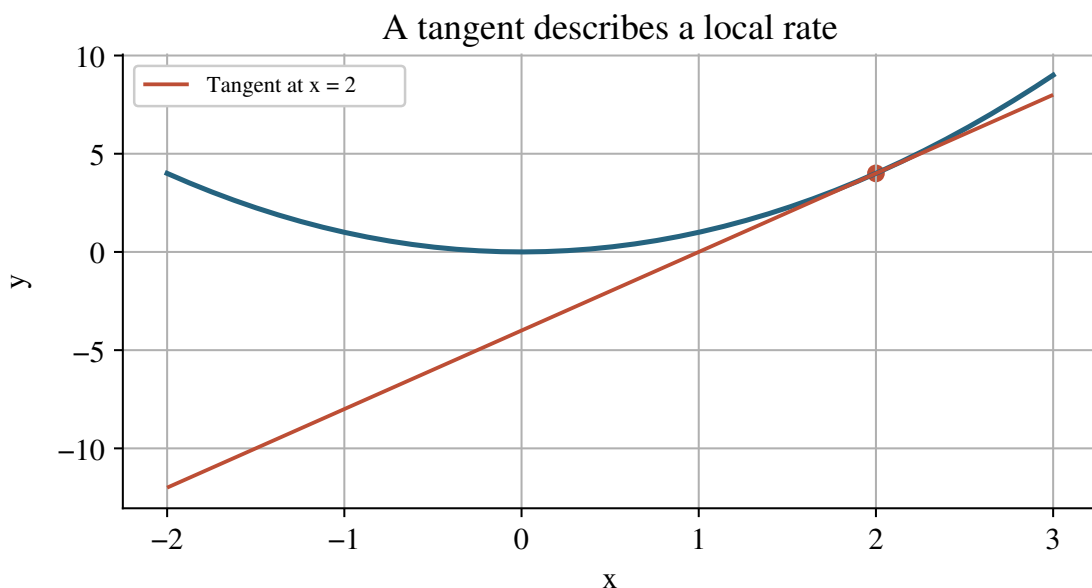
Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Test a tempting shortcut

- A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The derivative of $u(x)v(x)$ is always u prime times v prime. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
 - This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.
-

Choose the mathematical structure

- Use substitution when an inner derivative appears as a factor. By parts, $\int u v' = uv - \int u' v$. For a rational function, divide first if needed, then decompose into partial fractions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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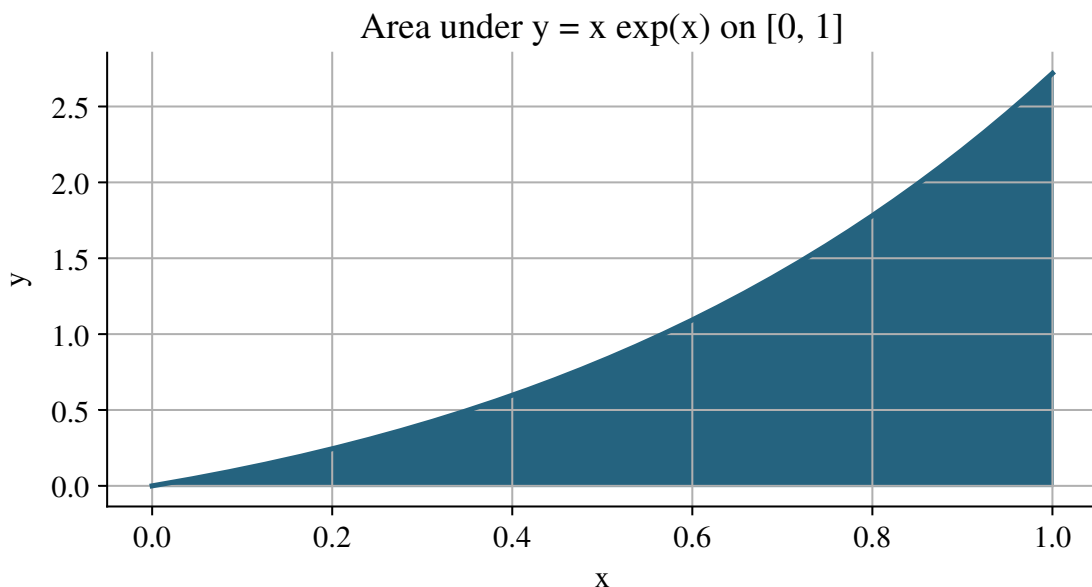
Work through a checked case

$$\int uv' dx = uv - \int u'v dx$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For integral $\int x e^x dx$, take $u=x$ and $v' = e^x$. Then the integral is $x e^x - e^x + C$. For integral $\int \frac{2x}{x^2+1} dx$, substitute $w=x^2+1$, $dw=2x dx$; the answer is $\ln(x^2+1) + C$.



Test a tempting shortcut

- Choosing u and v prime well matters: the remaining integral should become simpler. In a definite substitution, either change the limits or return to x before applying the original limits.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The integral of a product equals the product of its separate integrals. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check by differentiating. For volumes of revolution around the x -axis use $V = \pi \int y^2 dx$; do not confuse the square of a function with the integral of the function.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An integration method based on the derivative of a product. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.

- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

Choose the mathematical structure

- Separate the variables in $dy/dx=ky$, integrate both sides and apply an initial condition. Include any equilibrium solution excluded when dividing by y .
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

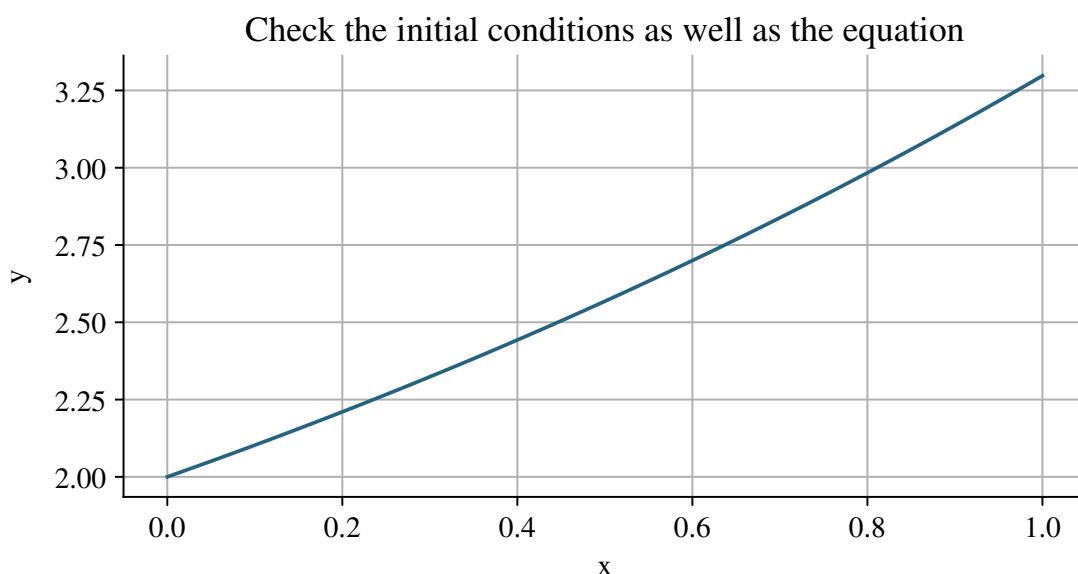
Work through a checked case

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $dy/dx=0.5y$ and $y(0)=2$, separation gives $\ln|y|=0.5x+C$, hence $y=Ae^{(0.5x)}$. The initial value gives $A=2$. Differentiate the result to check the equation and substitute $x=0$ to check the initial condition.



Test a tempting shortcut

- An initial condition determines the integration constant; it is not a replacement for integrating. Dividing by y can omit an equilibrium solution $y=0$. Euler accuracy depends on step size and the equation.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

An initial condition never changes the constant in a differential-equation solution. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- This unit uses first-order separation and initial conditions. Euler numerical integration and second-order complementary functions are excluded from this lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A specified value that selects a particular solution from a family. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Spatial vectors, lines and angles

Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
 - This lesson studies **direction vector** 方向向量: A nonzero vector giving the direction of a line.
-

Choose the mathematical structure

- A line has $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ with position vector $\mathbf{a}$ and nonzero direction $\mathbf{b}$. To find an intersection, equate all components and solve for both line parameters. For an angle use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where scalar products are in the named syllabus.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

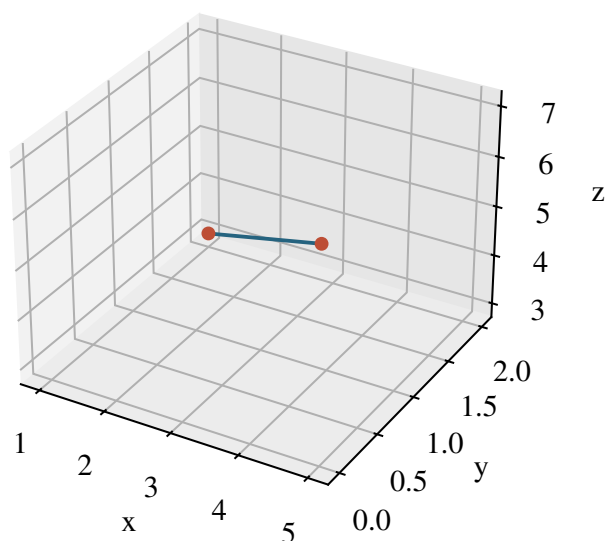
$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}, \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $\mathbf{a} = (1, 2, 3)$, $\mathbf{b} = (2, -1, 2)$, $\lambda = 2$ gives $\mathbf{r} = (5, 0, 7)$. The magnitude of $\mathbf{b}$ is $\sqrt{4+1+4}=3$. With $\mathbf{c} = (1, 0, -1)$, $\mathbf{b} \cdot \mathbf{c} = 2-2=0$, so $\mathbf{b}$ and $\mathbf{c}$ are perpendicular.

$$\mathbf{r} = (1, 2, 3) + \lambda(2, -1, 2)$$



Test a tempting shortcut

- Satisfying two component equations does not guarantee the third. Nonparallel lines may be skew. A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Any two nonparallel lines in three dimensions must intersect. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State whether a requested angle is acute, directed or an angle between lines. Use the syllabus-specific plane or distance method only when it is actually required for that unit.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A nonzero vector giving the direction of a line. Choose the relationship, show the method, check its assumptions and interpret the result.

M1 · Kinematics and Newton's laws

Which force accelerates the trolley?

- A trolley accelerates while friction opposes its motion. The engine's force alone does not equal mass times acceleration.
 - This lesson studies **resultant force** 合力: The vector sum of all external forces acting on the modelled object.
-

Choose the mathematical structure

- Choose a positive direction and draw a force diagram. Use resultant $F=ma$. For constant acceleration, $v=u+at$ and $s=ut+\frac{1}{2}at^2$. A particle model ignores size; a smooth surface ignores friction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

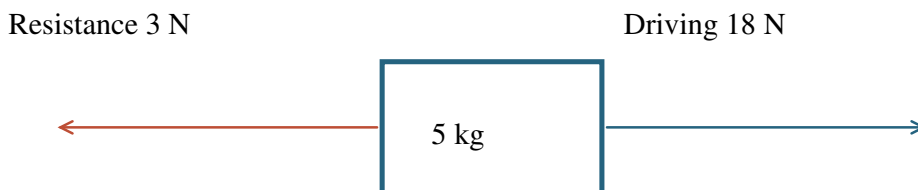
$$F_{\text{resultant}} = ma, \quad s = ut + \frac{1}{2}at^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A 5 kg trolley has driving force 18 N and resistance 3 N. The resultant is 15 N, so $a = 15/5 = 3 \text{ m/s}^2$. Starting from rest for 4 s gives $v = 0 + 3 \times 4 = 12 \text{ m/s}$ and $s = 0 \times 4 + 3 \times 4^2 / 2 = 24 \text{ m}$.

Identify the resultant before using $F = ma$



Test a tempting shortcut

- The normal reaction need not equal weight on a slope. Connected-particle models need one equation per particle and a consistent acceleration relation. The constant-acceleration formulae cannot be used for arbitrary variable acceleration.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The constant-acceleration formulae apply unchanged when acceleration varies with time. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Explain what each idealisation permits and what it leaves out. For a velocity-time graph, gradient represents acceleration and signed area represents displacement.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit M1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The vector sum of all external forces acting on the modelled object. Choose the relationship, show the method, check its assumptions and interpret the result.

M1 · Vectors and transformation geometry

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
 - This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.
-

Choose the mathematical structure

- Add corresponding vector components and subtract position vectors to find a displacement. A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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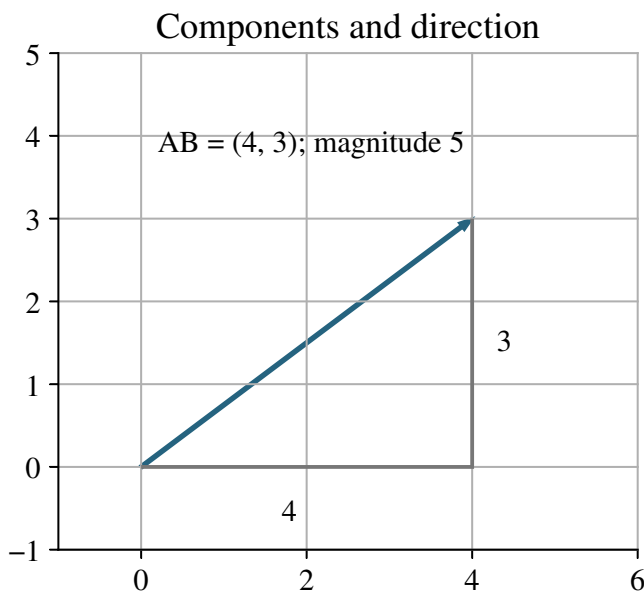
Work through a checked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With $\mathbf{a}=(4,1)$ and $\mathbf{b}=(1,3)$, $\mathbf{a}+\mathbf{b}=(5,4)$. From $A=(1,2)$ to $B=(5,5)$, displacement $\overrightarrow{AB}=(4,3)$. Its magnitude is $\sqrt{4^2+3^2}=5$.



Test a tempting shortcut

- The order of subtraction matters: $\mathbf{BA} = -\mathbf{AB}$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

$\overrightarrow{AB}$ and $\overrightarrow{BA}$ always have the same components. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use vector components, scalar multiples and magnitudes for displacements and forces. This unit does not require the scalar product.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit M1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The vector sum representing the combined displacement or force. Choose the relationship, show the method, check its assumptions and interpret the result.

M1 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
 - This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.
-

Choose the mathematical structure

- For equilibrium, both the resultant force and resultant moment are zero. Take moments about a useful pivot to remove unknown forces through that point. A combined centre of mass is a mass-weighted mean position.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

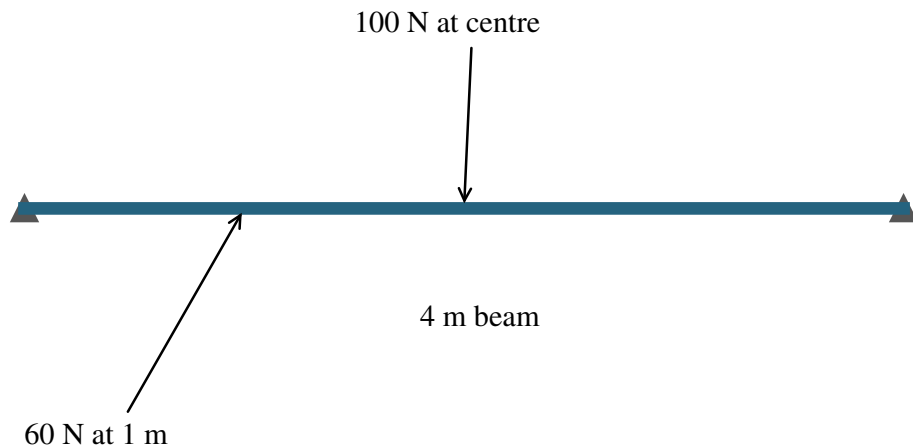
Work through a checked case

$$M = Fd_{\perp}, \quad \sum M = 0, \quad \sum \mathbf{F} = \mathbf{0}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give $4R_{\text{right}} = 100 \times 2 + 60 \times 1 = 260$, so $R_{\text{right}} = 65$ N. Vertical equilibrium gives $R_{\text{left}} = 95$ N.



Test a tempting shortcut

- Use the perpendicular lever arm, not a diagonal distance to the point of application. A zero resultant force alone does not prevent rotation. A support that would need a negative normal reaction may lose contact.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Zero resultant force by itself is enough for a rigid body's equilibrium. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution. A removed region has negative mass in that calculation, not a new physical negative material.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit M1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Force multiplied by the perpendicular distance from its line of action to a pivot. Choose the relationship, show the method, check its assumptions and interpret the result.

M1 · Inclined planes, friction and connected particles

Is friction already at its maximum?

- A crate on a slope is at rest. Friction can adjust to balance the force down the slope; it need not already be at its maximum.
 - This lesson studies **limiting friction** 极限摩擦力: The maximum static friction before slipping, equal to μ times the normal reaction in the model.
-

Choose the mathematical structure

- Resolve parallel and perpendicular to the plane. Weight components are $mg \sin \theta$ and $mg \cos \theta$. Static friction satisfies $F \leq \mu R$ and equals μR only at the limiting case. Write separate equations for connected particles.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

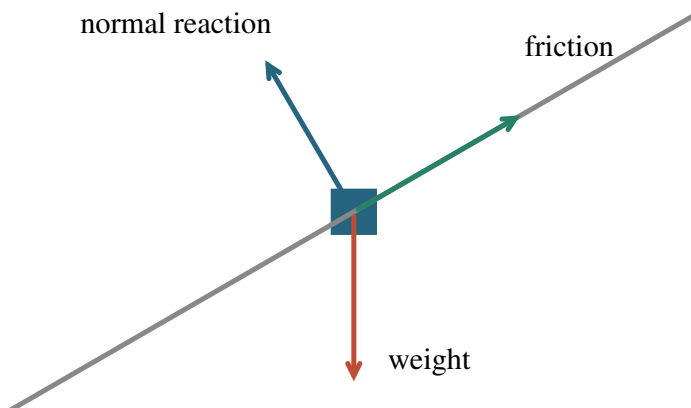
$$R = mg \cos \theta, \quad F \leq \mu R$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A 5 kg crate on a 30° slope has $R = 5 \times 9.8 \times \cos 30^\circ \approx 42.44$ N. The downhill weight component is 24.5 N. With $\mu = 0.6$, maximum friction ≈ 25.46 N, so equilibrium is possible with friction 24.5 N.

Resolve weight along and normal to the slope



Test a tempting shortcut

- Friction opposes motion or the tendency to move, not always the coordinate direction. A taut light inextensible string over a smooth pulley gives equal tension and a common acceleration magnitude; each assumption has a job.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Static friction is always exactly μR even when the body is not about to slip. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Start with force diagrams and a proposed direction of motion. If the calculated direction conflicts with the friction assumption, revisit the model instead of keeping inconsistent signs.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit M1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The maximum static friction before slipping, equal to μ times the normal reaction in the model. Choose the relationship, show the method, check its assumptions and interpret the result.

M2 · Kinematics and Newton's laws

Which force accelerates the trolley?

- A trolley accelerates while friction opposes its motion. The engine's force alone does not equal mass times acceleration.
 - This lesson studies **resultant force** 合力: The vector sum of all external forces acting on the modelled object.
-

Choose the mathematical structure

- Choose a positive direction and draw a force diagram. Use resultant $F=ma$. For constant acceleration, $v=u+at$ and $s=ut+\frac{1}{2}at^2$. A particle model ignores size; a smooth surface ignores friction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

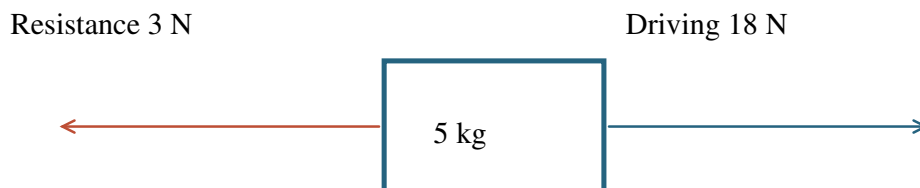
$$F_{\text{resultant}} = ma, \quad s = ut + \frac{1}{2}at^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A 5 kg trolley has driving force 18 N and resistance 3 N. The resultant is 15 N, so $a=15/5=3 \text{ m/s}^2$. Starting from rest for 4 s gives $v=0+3\times 4=12 \text{ m/s}$ and $s=0\times 4+3\times 4^2/2=24 \text{ m}$.

Identify the resultant before using $F = ma$



Test a tempting shortcut

- The normal reaction need not equal weight on a slope. Connected-particle models need one equation per particle and a consistent acceleration relation. The constant-acceleration formulae cannot be used for arbitrary variable acceleration.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The constant-acceleration formulae apply unchanged when acceleration varies with time. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Explain what each idealisation permits and what it leaves out. For a velocity-time graph, gradient represents acceleration and signed area represents displacement.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit M2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The vector sum of all external forces acting on the modelled object. Choose the relationship, show the method, check its assumptions and interpret the result.

M2 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
 - This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.
-

Choose the mathematical structure

- For equilibrium, both the resultant force and resultant moment are zero. Take moments about a useful pivot to remove unknown forces through that point. A combined centre of mass is a mass-weighted mean position.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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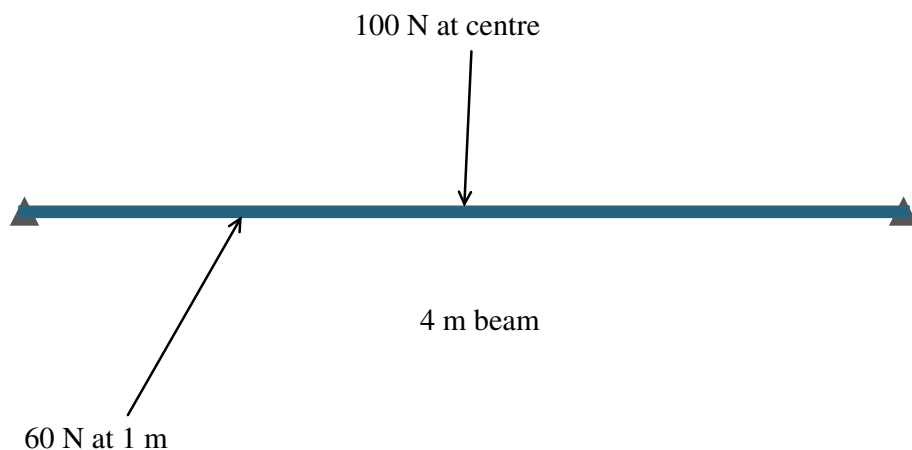
Work through a checked case

$$M = Fd_{\perp}, \quad \sum M = 0, \quad \sum \mathbf{F} = \mathbf{0}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give $4R_{\text{right}} = 100 \times 2 + 60 \times 1 = 260$, so $R_{\text{right}} = 65$ N. Vertical equilibrium gives $R_{\text{left}} = 95$ N.



Test a tempting shortcut

- Use the perpendicular lever arm, not a diagonal distance to the point of application. A zero resultant force alone does not prevent rotation. A support that would need a negative normal reaction may lose contact.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Zero resultant force by itself is enough for a rigid body's equilibrium. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution. A removed region has negative mass in that calculation, not a new physical negative material.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit M2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Force multiplied by the perpendicular distance from its line of action to a pivot. Choose the relationship, show the method, check its assumptions and interpret the result.

M2 · Work, energy, power and collisions

What does a slower rebound tell us?

- A ball rebounds slower after hitting a wall. Momentum and kinetic energy answer different parts of the collision story.
 - This lesson studies **coefficient of restitution** 恢复系数: The ratio of relative separation speed to relative approach speed along the line of impact.
-

Choose the mathematical structure

- Work is force times displacement in its direction. Kinetic energy is $mv^2/2$ and gravitational potential energy is mgh . Momentum is conserved for an isolated collision; restitution gives the relative speed relation, not conservation of kinetic energy.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$E_k = \frac{1}{2}mv^2, \quad W = Fs, \quad P = \frac{dE}{dt}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A 2 kg particle moving at 6 m/s has kinetic energy $2 \times 6^2 / 2 = 36$ J. If a constant opposing force of 3 N stops it, $\text{work} = 3s = 36$, giving $s = 12$ m. A wall bounce with approach speed 5 and $e = 0.6$ has rebound speed 3 m/s in the opposite direction.



Test a tempting shortcut

- Conserve momentum only when the external impulse is negligible. Kinetic energy is generally lost in an inelastic collision. Power is a rate of energy transfer, not an amount of energy.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every collision conserves kinetic energy. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State the direction signs before writing collision equations. For a connected motion problem, choose the whole system for energy when internal tensions cancel; account for frictional work explicitly.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit M2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The ratio of relative separation speed to relative approach speed along the line of impact. Choose the relationship, show the method, check its assumptions and interpret the result.

M2 · Projectile motion and variable acceleration

Why does horizontal motion share vertical time?

- A ball is launched at an angle. Its horizontal and vertical motion share time but follow different equations.
 - This lesson studies **projectile** 抛体: A particle moving under gravity after launch in a model that neglects air resistance.
-

Choose the mathematical structure

- Resolve initial velocity into horizontal $u \cos\theta$ and vertical $u \sin\theta$. With no air resistance, horizontal acceleration is zero and vertical acceleration is $-g$. Use the same time in both components.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

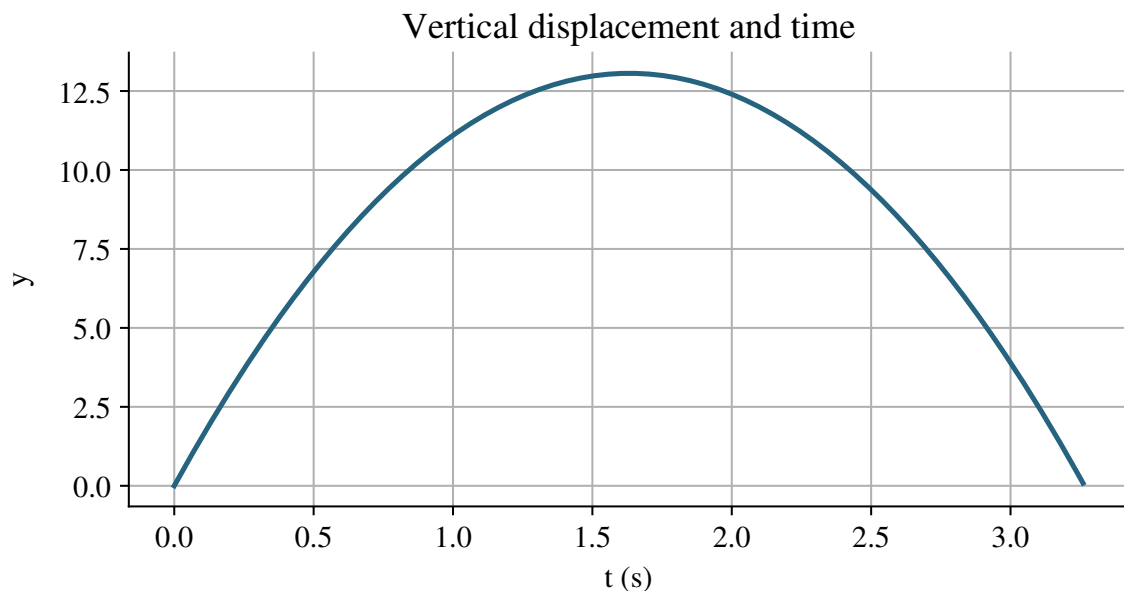
Work through a checked case

$$x = u_x t, \quad y = u_y t - \frac{1}{2} g t^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For components $u_x=12$, $u_y=16$ and $g=9.8$, at $t=2$ s the horizontal displacement is $x=12 \times 2=24$ m and vertical displacement is $y=16 \times 2 - 9.8 \times 2^2 / 2 = 12.4$ m. At the highest point, $v_y=0$ but v_x remains 12.



Test a tempting shortcut

- Zero vertical velocity at the highest point does not mean zero total speed. Launch and landing heights need not be equal. Do not use a range formula that assumes equal heights without checking them.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The total speed of a projectile is zero at its highest point. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For variable acceleration, integrate $a(t)$ to get $v(t)$ and use the initial velocity to find the constant, then integrate for displacement. Confirm units and the physical time interval.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit M2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A particle moving under gravity after launch in a model that neglects air resistance. Choose the relationship, show the method, check its assumptions and interpret the result.

S1 · Data summaries, histograms and interpretation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** 频率密度: Frequency divided by class width, used as histogram height.

Choose the mathematical structure

- Compare an appropriate average and spread in context. A histogram uses area for frequency, so $\text{height} = \text{frequency} / \text{class width}$. Grouped estimates assume representative values within intervals.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

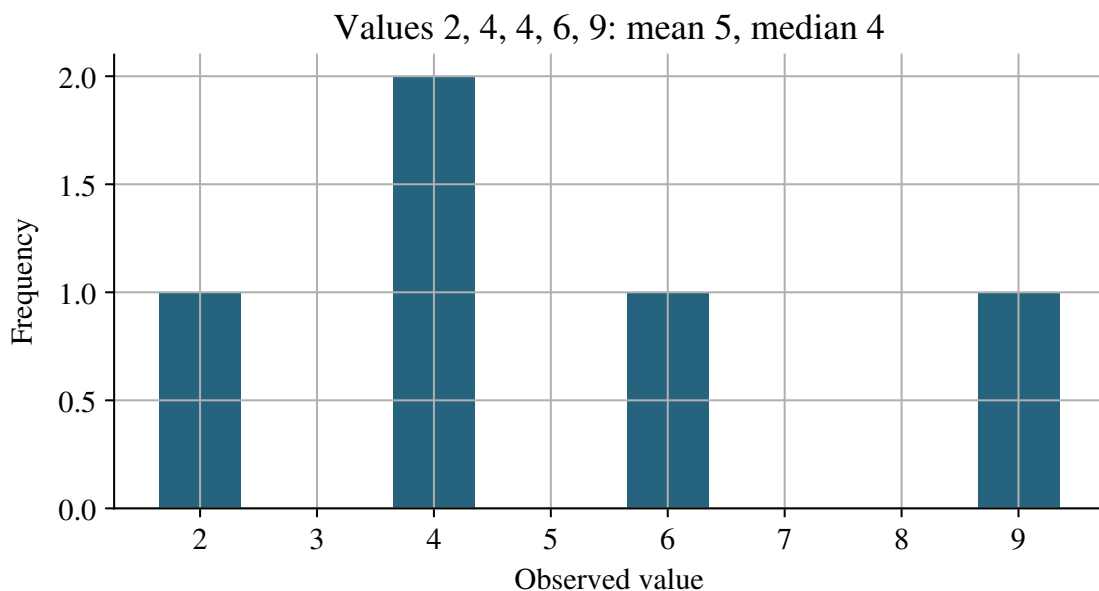
Work through a checked case

$$\bar{x} = \frac{\sum x_i}{n}, \quad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A class from 10 to 20 with frequency 30 has density $30/10=3$. A class from 20 to 40 with frequency 20 has density $20/20=1$. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.



Test a tempting shortcut

- The tallest histogram bar need not contain the most observations. A grouped mean is an estimate. Correlation does not prove causation, and extrapolation extends beyond the observed range.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A histogram bar's height always equals its frequency, even with unequal class widths. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a display that fits the data type. Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit S1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Frequency divided by class width, used as histogram height. Choose the relationship, show the method, check its assumptions and interpret the result.

S1 · Probability, trees and conditional reasoning

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.

- This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.

Choose the mathematical structure

- Multiply along a tree branch and add disjoint branches. With replacement, the composition stays fixed. Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

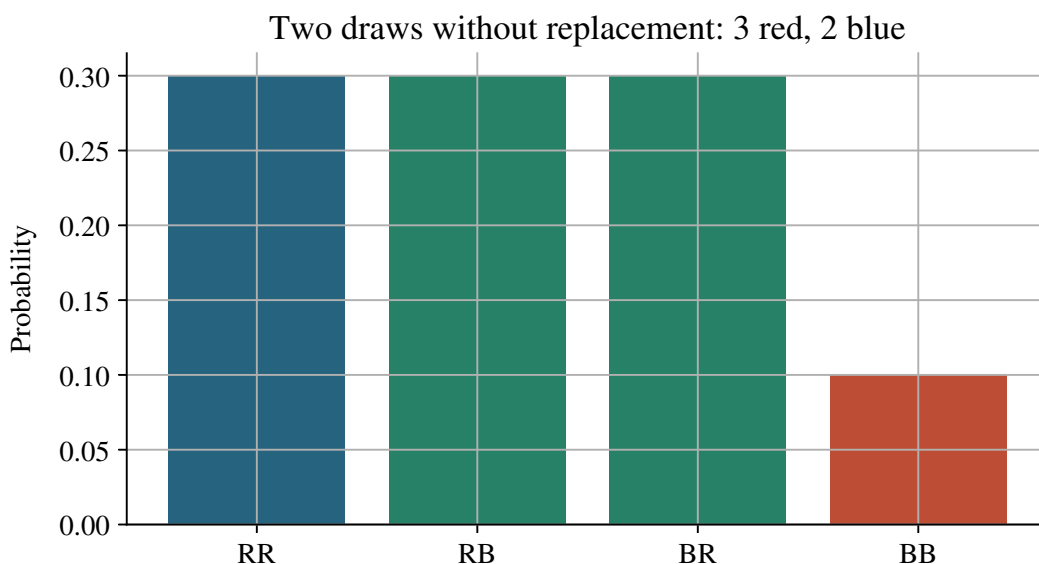
Work through a checked case

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



Test a tempting shortcut

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Mutually exclusive events with positive probabilities must be independent. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- A two-way table makes the restricted denominator visible. Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit S1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The probability of an event after restricting the sample space to a stated condition. Choose the relationship, show the method, check its assumptions and interpret the result.

S1 · Regression, financial models and residuals

Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
 - This lesson studies **residual** 残差: The observed value minus the value predicted by a fitted model.
-

Choose the mathematical structure

- A linear model $y=a+bx$ has intercept a and slope b with contextual units. Inspect residuals and the data range. For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

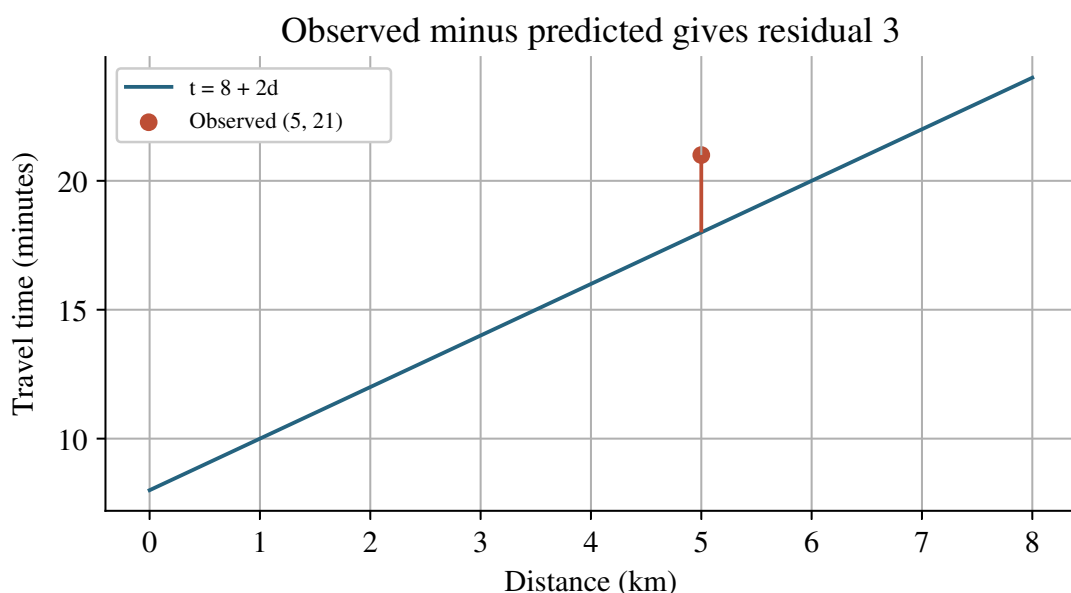
Work through a checked case

$$\hat{t} = 8 + 2d, \quad e = t - \hat{t}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The fitted model $t=8+2d$ predicts 18 minutes at $d=5$ km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes $1000 \times 1.05^3 = 1157.625$ after 3 years.



Test a tempting shortcut

- A good fit does not prove a causal mechanism. A correlation coefficient measures linear association, not the gradient. Calculator output must be translated into a model, checked and interpreted.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A high correlation permits reliable extrapolation to any distance. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- In IB AI, record the data source, domain, assumptions and calculator method. Compare an alternative model and judge predictions against residuals. A financial answer must state payment timing and whether interest is compounded.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL mathematics; official unit S1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The observed value minus the value predicted by a fitted model. Choose the relationship, show the method, check its assumptions and interpret the result.

S1 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.

Choose the mathematical structure

- For a discrete random variable, sum the probabilities to 1 and calculate $E(X) = \sum xP(X=x)$. For the normal model use $z = (x - \mu)/\sigma$ and calculate an appropriate area.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

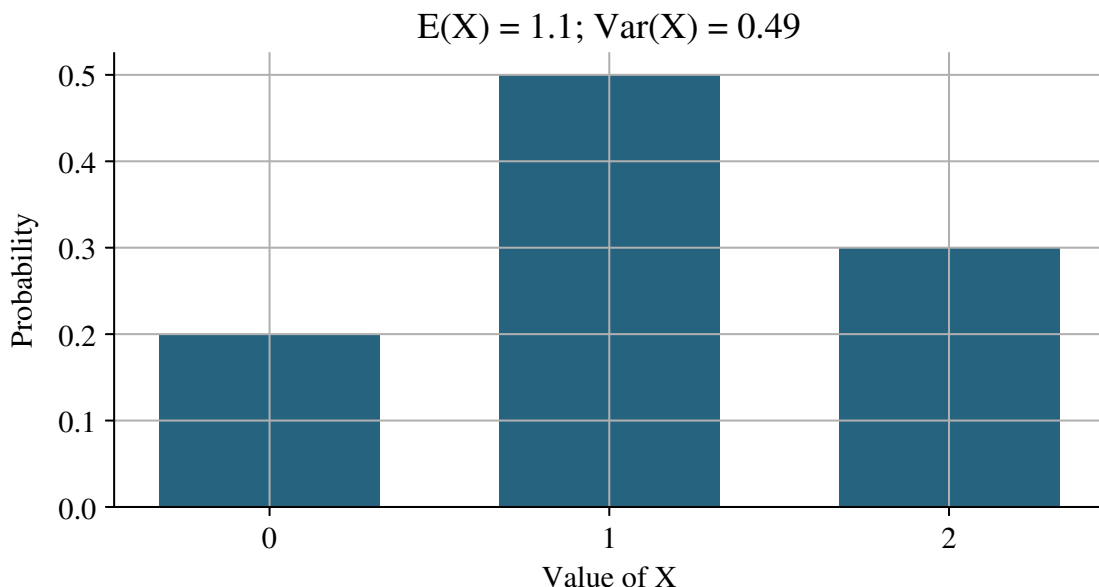
Work through a checked case

$$E(X) = \sum xP(X = x), \quad z = \frac{x - \mu}{\sigma}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If X takes 0,1,2 with probabilities 0.2,0.5,0.3, $E(X)=0\times 0.2+1\times 0.5+2\times 0.3=1.1$. Also $E(X^2)=0+0.5+1.2=1.7$, so $\text{Var}(X)=1.7-1.1^2=0.49$. For $\mu=100, \sigma=15$, a value of 130 has $z=2$.



Test a tempting shortcut

- Not every count is binomial: changing p or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A random variable with three possible values must be binomial. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- S1 includes discrete random variables and the normal distribution. Named binomial and Poisson models are S2 content.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit S1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The probability-weighted mean of a random variable. Choose the relationship, show the method, check its assumptions and interpret the result.

S2 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
 - This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.
-

Choose the mathematical structure

- A binomial model needs fixed n , independent trials, two outcomes and constant p . $E(X)=np$ and $\text{Var}(X)=np(1-p)$. For a normal model use $z=(x-\mu)/\sigma$. A Poisson model describes counts with a constant rate and appropriate independence assumptions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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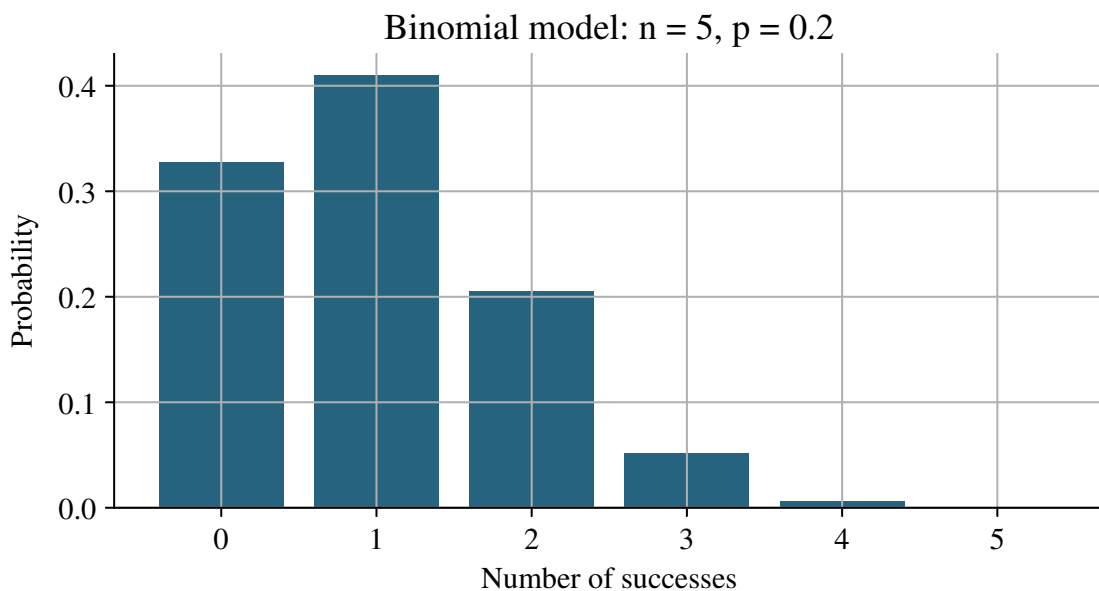
Work through a checked case

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad X \sim B(n, p)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For X binomial(5,0.2), $P(X=0)=0.8^5=0.32768$, $E(X)=1$ and $\text{Var}(X)=0.8$. For a normal quantity with $\mu=100, \sigma=15$, the value 130 has $z=2$. A Poisson mean of 3 per hour gives mean 6 over two hours.



Test a tempting shortcut

- Not every count is binomial: changing p or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every count of successes has a binomial distribution regardless of dependence. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Write the event as an inequality before using calculator distribution functions. Distinguish $P(X < k)$, $P(X \leq k)$ and a tail complement. State assumptions in context.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit S2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The probability-weighted mean of a random variable. Choose the relationship, show the method, check its assumptions and interpret the result.

S2 · Sampling and a large data set

Who is missing from the data?

- A weather database has a missing entry and several stations. Treating each row as identical can distort a comparison.
 - This lesson studies **sampling frame** 抽样框: The list or population definition from which a sample is selected.
-

Choose the mathematical structure

- Identify the population, sampling unit and frame. Distinguish random, systematic, stratified, quota and opportunity sampling. Missing data is not zero; verify units, dates and variable definitions before comparing samples.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

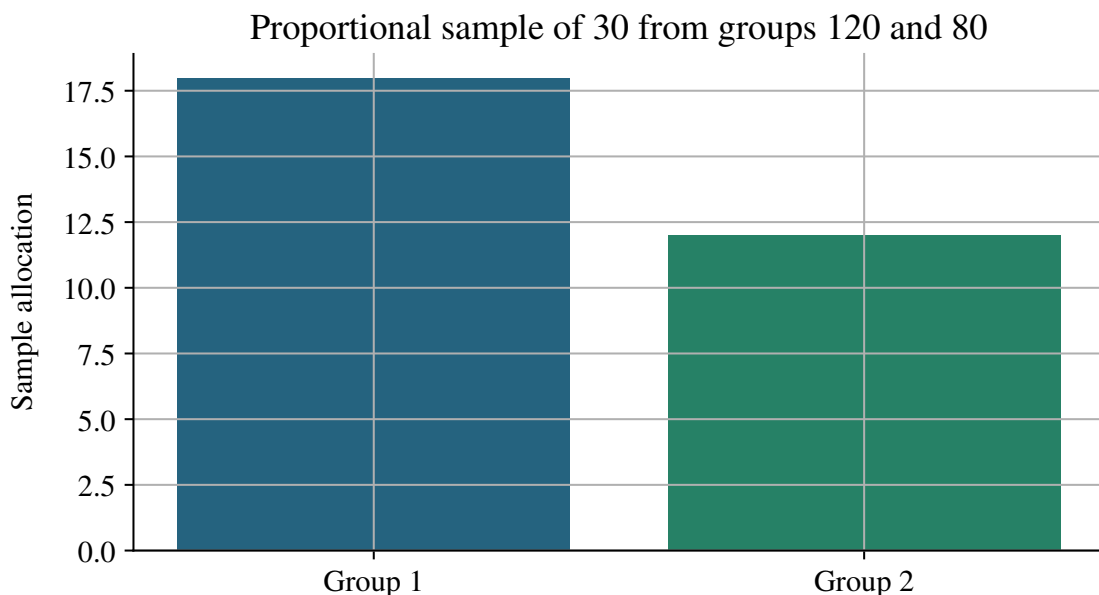
Work through a checked case

$$n_i = n \frac{N_i}{N}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A population has 120 students in one group and 80 in another. A proportional stratified sample of 30 needs $30 \times 120/200 = 18$ from the first and 12 from the second. Random selection is then needed within each group.



Test a tempting shortcut

- A large biased sample remains biased. Stratification is not the same as selecting whoever is available from each group. AQA large-data-set familiarity requires the actual supplied data and metadata, not invented weather values.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Doubling a biased sample automatically removes its selection bias. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For AQA, use the official large data set in a supervised spreadsheet task: identify a variable, justify a comparison, inspect missing values, create a display and explain a limitation. Save the decisions with the analysis.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit S2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The list or population definition from which a sample is selected. Choose the relationship, show the method, check its assumptions and interpret the result.

S2 · Hypothesis testing and contextual conclusions

Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
 - This lesson studies **significance level** 显著性水平: The chosen probability threshold for rejecting a null hypothesis.
-

Choose the mathematical structure

- State H_0 and H_1 in population parameters before inspecting the outcome. Calculate the appropriate tail probability under H_0 . Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

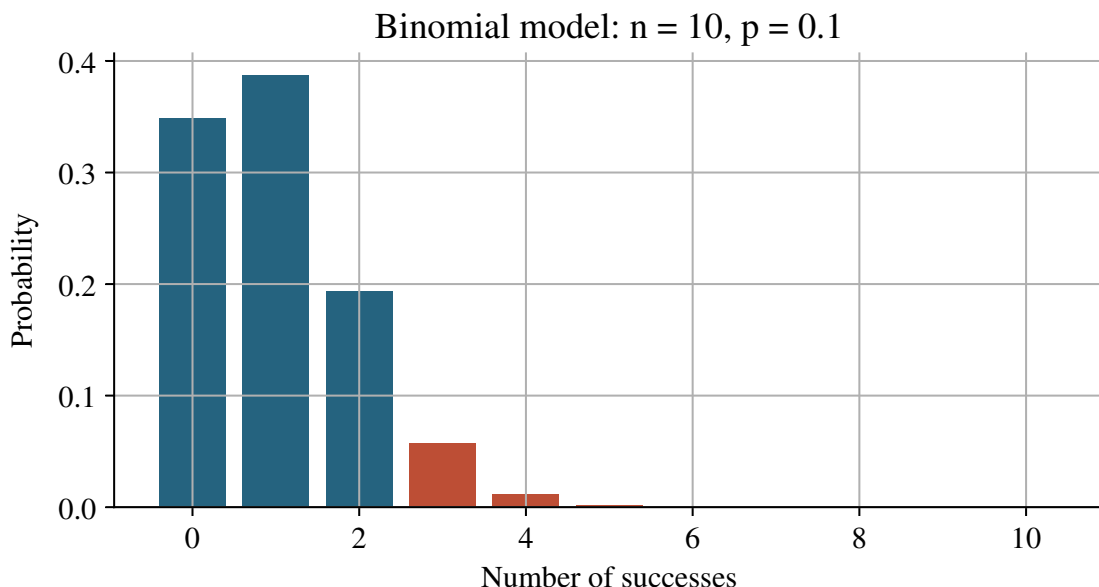
Work through a checked case

$$H_0 : p = p_0, \quad H_1 : p > p_0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $H_0:p=0.1$ and $H_1:p>0.1$ with $n=10$, observing at least 3 defects has probability $1-(0.9^{10}+10\times0.1\times0.9^9+45\times0.1^2\times0.9^8)\approx0.070191$. At 5%, this is insufficient evidence that the defect rate exceeds 10%.



Test a tempting shortcut

- Failing to reject H_0 is not proof that H_0 is true. Choose the tail from H_1 , not from whichever tail gives a small result. Statistical significance does not measure the practical size of an effect.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Failing to reject a null hypothesis proves it is true. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Finish with a sentence about the population and the original claim. State the model's assumptions and consider whether the sampling procedure supports them.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit S2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The chosen probability threshold for rejecting a null hypothesis. Choose the relationship, show the method, check its assumptions and interpret the result.

S2 · Continuous distributions and density functions

Is a height the same as a probability?

- A waiting time can take any value in an interval. Its probability comes from area, not the graph's height at one instant.
-

- This lesson studies **probability density** 概率密度: A nonnegative function whose integral over an interval gives its probability.

Choose the mathematical structure

- A density f must be nonnegative and integrate to 1 over its support. The cumulative distribution $F(x)$ is the integral up to x . For a continuous variable $P(X=x)=0$ and $E(X)$ is the integral of $xf(x)$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

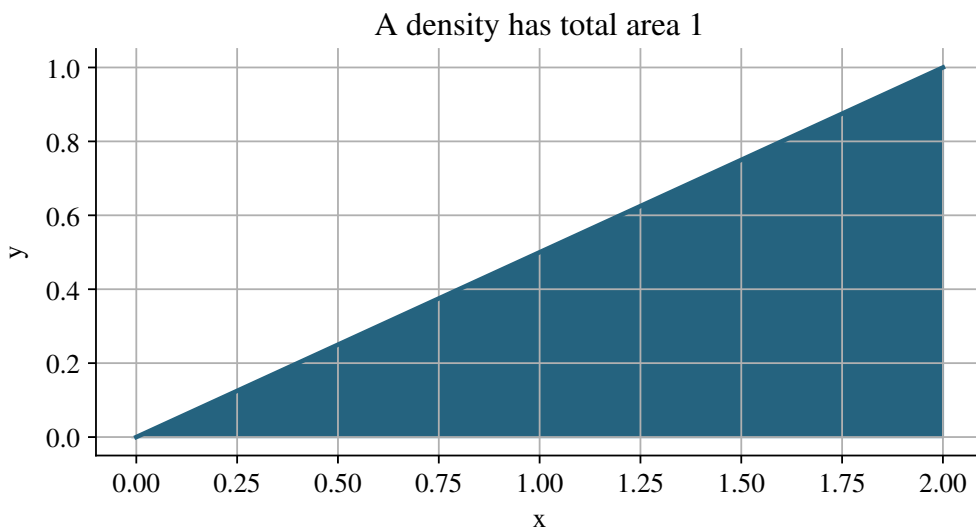
Work through a checked case

$$\int f(x) dx = 1, \quad E(X) = \int xf(x) dx$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Let $f(x)=kx$ on $0 \leq x \leq 2$ and zero elsewhere. Normalisation gives $\int_0^2 kx dx = 2k = 1$, so $k=1/2$. Then $P(X \leq 1) = \int_0^1 x/2 dx = 1/4$ and $E(X) = \int_0^2 x^2/2 dx = 4/3$.



Test a tempting shortcut

- Density can exceed 1 without being invalid; total area must equal 1. A cumulative distribution cannot decrease. Include the zero-density region outside the support when defining a complete model.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A continuous variable has $P(X=x)$ equal to the density height $f(x)$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a uniform distribution, density is reciprocal interval length. Find medians or percentiles using the cumulative probability, and use a support-aware integral for moments.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit S2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A nonnegative function whose integral over an interval gives its probability. Choose the relationship, show the method, check its assumptions and interpret the result.

D1 · Algorithms, networks and linear programming

Cheapest network or shortest route?

- A school must connect buildings with cable. The shortest route between two buildings and the cheapest whole network are different problems.
 - This lesson studies **minimum spanning tree** 最小生成树: A connected cycle-free network joining every vertex with the smallest possible total edge weight.
-

Choose the mathematical structure

- Kruskal selects edges in increasing weight while avoiding cycles. Dijkstra updates shortest tentative distances from a start. Linear programming optimizes a linear objective over a feasible region; inspect vertices and integer restrictions when required.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

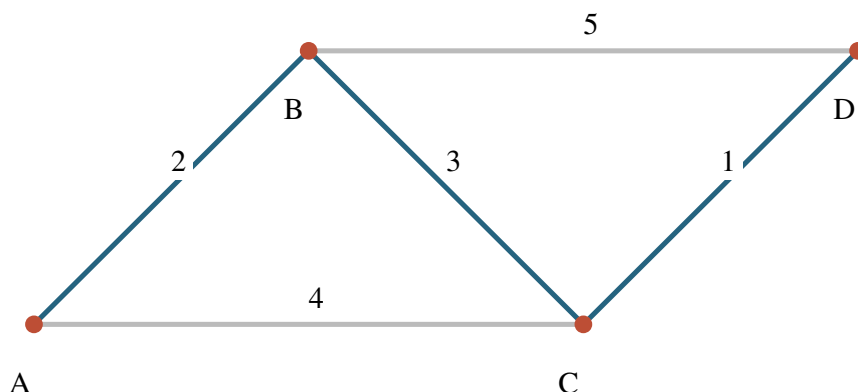
$$Z = 3x + 2y, \quad x + y \leq 4, \quad x \leq 2, \quad x, y \geq 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For vertices A,B,C,D and edges AB=2,BC=3,AC=4,CD=1,BD=5, Kruskal selects CD,AB,BC for total 6. The shortest A-to-D route is A-B-C-D, also 6, but that equality is incidental. Maximize $3x+2y$ with $x+y \leq 4, x \leq 2, x, y \geq 0$: the best vertex is (2,2), value 10.

Minimum spanning tree: weight 6



Test a tempting shortcut

- A spanning tree has no cycles and joins all vertices. The largest single edge is not automatically excluded from every optimal solution. A shortest-path algorithm cannot replace a minimum-spanning-tree algorithm.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A shortest path between two vertices must also be a minimum spanning tree of the whole network. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For critical paths, calculate earliest and latest event times and identify zero-float activities. State units and interpret the optimum. The existing archive has no D1 pairs, so these original tasks do not establish a reviewed D1 past-paper bank.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit D1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A connected cycle-free network joining every vertex with the smallest possible total edge weight. Choose the relationship, show the method, check its assumptions and interpret the result.

D1 · Critical paths and scheduling

Must every activity wait for every other?

- A school event has activities that can run in parallel. Adding every duration overestimates the minimum completion time.
- This lesson studies **float** 浮动时间: The time an activity can be delayed without delaying completion under the network model.

Choose the mathematical structure

- For an activity network, calculate earliest event times forward and latest event times backward. Total float for activity i to j is $L_j - E_i - d_{ij}$. A critical activity has zero total float; more than one critical path may exist.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

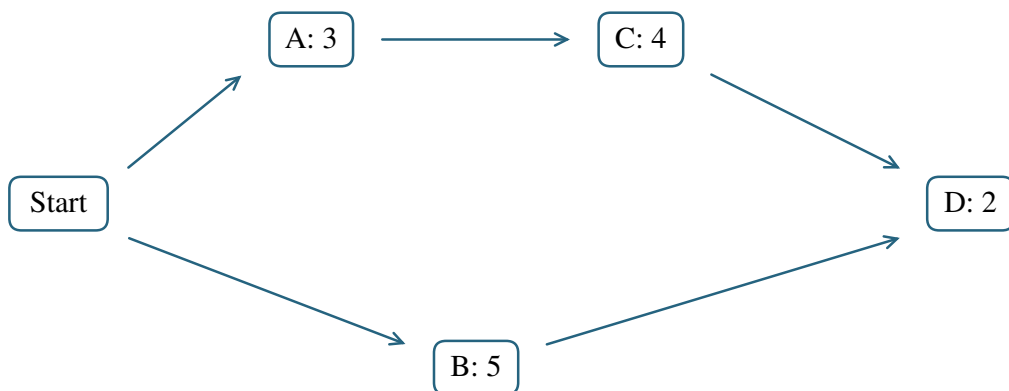
$$F_{ij} = L_j - E_i - d_{ij}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Activities A=3 and B=5 start together. C=4 follows A, and D=2 follows both B and C. The earliest completion of C is 7; D must wait until $\max(5,7)=7$ and finishes at 9. Path A-C-D is critical; B has float 2.

Parallel activities meet before D starts



Test a tempting shortcut

- Do not add durations of independent parallel activities. A zero-float activity belongs to a critical path, but there may be several such paths. Resource limits can require a schedule longer than the network's theoretical minimum.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The minimum project duration is always the sum of every activity duration. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Draw precedence relationships before assigning times. Distinguish activity duration from event time. Explain what a delay does to the completion date, and record any resource assumptions.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit D1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The time an activity can be delayed without delaying completion under the network model. Choose the relationship, show the method, check its assumptions and interpret the result.

D1 · Sorting, packing and network algorithms

Is a fast packing method always optimal?

- A packing method quickly fills boxes, but a fast valid arrangement need not use the smallest number of boxes.
 - This lesson studies **algorithm** 算法: A finite set of ordered instructions that solves a defined class of problems.
-

Choose the mathematical structure

- Trace the named algorithm exactly, including its tie rules. In first-fit packing, place each item in the first available bin that can hold it. First-fit decreasing sorts before applying first-fit. A heuristic may be valid without being optimal.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

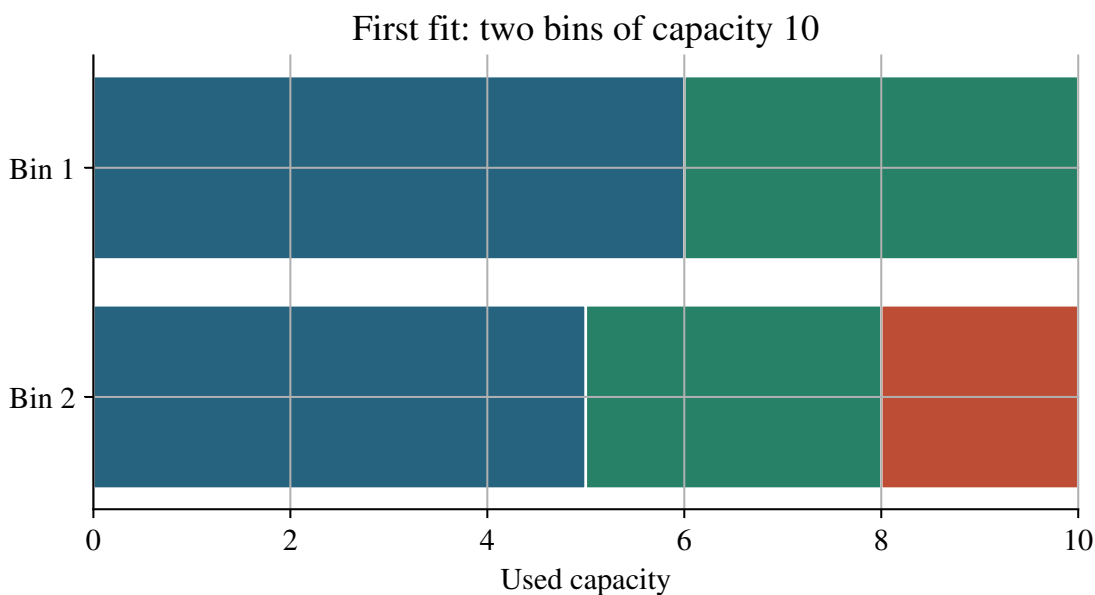
Work through a checked case

$$B_{\min} \geq \left\lceil \frac{\sum s_i}{C} \right\rceil$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With bin capacity 10 and items 6,5,4,3,2 in that order, first-fit places 6 and 4 in bin 1, then 5,3,2 in bin 2. It uses 2 bins. The total size is 20, so the lower bound is $\text{ceil}(20/10)=2$; this arrangement is optimal for this instance.



Test a tempting shortcut

- An example of success does not prove a heuristic is always optimal. Keep intermediate lists in a sorting trace; do not jump from input to a sorted final list. A shortest-path update must retain predecessor information if a route is requested.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A packing heuristic that works well on one example must always be optimal. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For Dijkstra, choose the smallest unsettled tentative label and update its neighbours. For route-inspection problems, distinguish a closed route from an open one and identify odd vertices before pairing them.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL mathematics; official unit D1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A finite set of ordered instructions that solves a defined class of problems. Choose the relationship, show the method, check its assumptions and interpret the result.