Statistics 2

Edexcel IAL S2 (WST02/01); binomial and Poisson, continuous variables, uniform, sampling and tests

Formulae to memorise

Family	**Must know**
binomial	$E = np$, $\mathrm{Var} = np(1-p)$
Poisson	$E = \mathrm{Var} = \lambda$; rates scale with time
continuous	$f = F'$, $E(X) = \int xf\,dx$
uniform	mean $\frac{a+b}{2}$, variance $\frac{(b-a)^2}{12}$

Binomial or Poisson?

Binomial	Poisson
n independent trials	singly, independently
constant probability p	at a constant average rate

Worked: moments, then approximations

1. **Solve** $np = 3,\ np(1-p) = 2.55$: $p = 0.15,\ n = 20$
2. **Binomial** $B(40, 0.2)$: $P(X > 8) = 1 - P(X \leq 8) = 0.407$
3. **Poisson** $B(80, 0.1) \approx \mathrm{Po}(8)$: $P(X \geq 12) = 0.112$

Decision

Approximate only when n is large and p small.

Reading a CDF

Median: $F(m) = \frac{1}{2}$

- $F(x) = P(X \leq x)$, $f = F'$.
- Total area 1.

Worked: mode and skew

1. **Density** $f(x) = \frac{12x - 4x^3}{5}$
2. **Peak** $f'(x) = \frac{12 - 12x^2}{5} = 0$: $x = 1$
3. **Compare** mean 0.64 $<$ **median** 0.671 $<$ **mode** 1

Decision

Negative skew

A piecewise-linear density

$(2k-7)^2 = 0$: $k = 3.5$

- Normalise to pin k.
- No calculator.

Worked: mean and variance

1. **First piece** $\frac{2}{21} \cdot \frac{3.5^3}{3}$
2. **Second piece** $\frac{2}{15} \int_{3.5}^{6} (6x - x^2)\, dx$
3. **Mean** $E(X) = \frac{19}{6}$

Decision

$$\mathrm{Var}(X) = \frac{277}{24} - \frac{361}{36} = \frac{109}{72}$$

Uniform: two facts pin a and b

$b - a = 16$, $a + b = 12$: $a = -2$, $b = 14$

- Probabilities are lengths.
- Keep one root throughout.

Worked: the rest of the uniform

1. **Coding** $E(cX+1) = 6c+1 = 3$: $c = \frac{1}{3}$
2. **Variance** $\frac{16^2}{12} = \frac{64}{3}$
3. **Second moment** $E(X^2) = \frac{64}{3} + 36 = \frac{172}{3}$

Decision

$P(\frac{3}{2}X - b > a) = P(X > 8) = \frac{3}{8}$

Census or sample?

Census

measures everybody

complete, but costly, slow, destructive

Sample

measures part

cheaper, faster, sampling error

A sampling distribution

$1 - (\frac{283}{343})^n > 0.95$: $n = 16$

- Three beads, black scores 4, white 1.
- $P(\text{black}) = \frac{2}{7}$.

The testing routine

> Conclude in context: "insufficient evidence that ... has changed".

- One tail or two, as asked.
- Two tails split the level.

A two-tailed Poisson test

Critical region $X \leq 3$ or $X \geq 18$; 16 is outside

- $H_0 : \lambda = 10$, $H_1 : \lambda \neq 10$, 3%.
- Each tail at most 1.5%.

Worked: a binomial test

1. **Fewer than r** $B(40, 0.3)$: $P(X \le 6) = 0.0238$, $P(X \le 7) = 0.0553$: $r = 7$
2. **Normal** $B(200, 0.3) \approx N(60, 42)$: $\frac{t-0.5-60}{\sqrt{42}} = -1.62$, $t = 50$
3. **Test** $P(X \ge 11) = 0.0978 < 0.10$

Decision

Reject H_0: evidence the proportion has increased.

Check yourself

Question	**Answer**
Poisson mean and variance?	both λ
Median of a continuous X?	$F(m) = \frac{1}{2}$
Uniform variance?	$\frac{(b-a)^2}{12}$
Is $\overline{X} - \mu$ a statistic?	no: μ is unknown
"At least t" under a Normal?	$t - 0.5$