Mechanics 2

Edexcel IAL M2 (WME02/01); variable acceleration, centres of mass, energy, collisions and rigid bodies

Formulae to memorise

Family	**Must know**
energy	KE $\frac{1}{2}mv^2$, PE mgh, work Fs, power Fv
calculus	$v = \int a\,dt$, $s = \int v\,dt$
restitution	$e = \frac{v_B - v_A}{u_A - u_B}$, $0 \leq e \leq 1$
from M1	momentum mv, impulse, $F = \mu R$, moments

Worked: integrate, with a trap

1. **Integrate** $v = k(3t - \frac{t^3}{3})$, constant zero by rest
2. **Find k** $3.375k = 13.5$: $k = 4$
3. **Again** $s = 6t^2 - \frac{t^4}{3}$

Decision

Turns at $t = 3$, $s = 27$: total distance 54 m

Distance is not displacement

> The return leg is a second 27, not a subtraction.

- Find where $v = 0$.
- Add both legs.

Worked: differentiate, then the impulse

1. **Velocity** $\mathbf{v} = 2t^{-1/2}\mathbf{i} - 3\mathbf{j}$; at $t = 4$: $\mathbf{i} - 3\mathbf{j}$
2. **Impulse** $3(\mathbf{v}' - (\mathbf{i} - 3\mathbf{j})) = 6\mathbf{i} - 3\mathbf{j}$
3. **New velocity** $\mathbf{v}' = 3\mathbf{i} - 4\mathbf{j}$

Decision

Speed $5\ \mathrm{m\,s^{-1}}$

Worked: an S-shaped wire

Piece	Length	From AB	Moment
arc AB	$2\pi a$	$\frac{4a}{\pi}$	$8a^2$
arc CD	πa	$-\frac{2a}{\pi}$	$-2a^2$
straights	$2a$	0	0
total	$(3\pi + 2)a$		$6a^2$

Where the wire balances

$\bar{y} = \frac{6a}{3\pi+2}$ from AB

- Semicircular wire: $\frac{2r}{\pi}$.
- Semicircular lamina: $\frac{4r}{3\pi}$.

Worked: hung from A

1. **At A** $\tan\angle OAG = \dfrac{6a/(3\pi+2)}{2a} = \dfrac{3}{3\pi+2}$
2. **Geometry** $OA = OE$: $\angle OAE = 45^\circ$
3. **Combine** $\theta = 45^\circ - \angle OAG$

Decision

$\tan\theta = \dfrac{3\pi-1}{3\pi+5}$

Energy and power

Work-energy

total work = change in KE

resistance: $-Rs$

Power

$P = Fv$

constant speed: $v = \frac{P}{F}$

A car on a slope

At $10\ \mathrm{m\,s^{-1}}$: $2000 - 1250 = 750a$, $a = 1$

- 750 kg, $\sin\alpha = \frac{1}{7}$, 20 kW.
- Resistance 200 N.

Worked: engine off

1. **KE at A** $\frac{1}{2}(750)(10^2) = 37500$ J
2. **Losses** $750g \cdot d\sin\alpha + 200d = 1250d$
3. **Equate** $37500 = 1250d$

Decision

$d = 30$ m

Worked: a projectile closed by energy

1. **At 11 m** $200 = \frac{1}{2}(V^2 + 4V^2) + 110$: $V = 6$
2. **Hits at 90°** plane at 60°: velocity 30° below horizontal
3. **Speed** $\frac{12}{\cos 30^\circ} = 8\sqrt{3}$; $200 = \frac{1}{2}(192) + 10H$

Decision

$H = 10.4 \approx 10$ m

Worked: the full collision

1. **Momentum** $2u = 2v_P + v_Q$
2. **Restitution** $eu = v_Q - v_P$
3. **Solve** $v_Q = \frac{2(1+e)u}{3}$, $v_P = \frac{(2-e)u}{3}$

Decision

KE lost $\frac{mu^2}{3}(1 - e^2)$

The impulse fixes e

$m \cdot \frac{2(1+e)u}{3} = \frac{8}{9}mu$: $e = \frac{1}{3}$

- Impulse from one body's momentum change.
- A wall multiplies speed by its e.

Successive impacts

> Solve each collision completely before the next.

- Keep a signed diagram per stage.
- Equate only what the question ties.

A rod against a smooth rail

About A: $S(1.5a) = mga\cos\theta$

- Zero force and zero **moment**.
- Pick the unknown-killing pivot.

Worked: an equation in tan θ

1. **Limit** $\frac{4}{7} \cdot \frac{mg}{3}(3 - 2\cos^2\theta) = \frac{2mg}{3}\cos\theta\sin\theta$
2. **Divide** $\frac{4}{7}(3\sec^2\theta - 2) = 2\tan\theta$
3. **Use** $\sec^2\theta = 1 + \tan^2\theta$

Decision

$6\tan^2\theta - 7\tan\theta + 2 = 0$

Check yourself

Question	**Answer**
Particle turns back: total distance?	add both legs
Wire pieces weighted by?	length
Constant speed, power P?	$v = \frac{P}{F}$
KE loss factor?	$1 - e^2$
Rod on a rail: moments about?	the ground contact