

Notes

Pearson Edexcel · International A-Level · Further Mathematics

YFM01: course teaching notes

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These are original course-owned teaching notes. Objective-level exceptions are listed in the review, and lessons are tier labelled.

FP1 · Complex arithmetic and quadratic roots

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
 - This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.
-

Choose the mathematical structure

- Write $z=a+bi$ with $i^2=-1$. Add components, multiply brackets and use a conjugate to make a division denominator real. Non-real roots of a real-coefficient quadratic occur in conjugate pairs.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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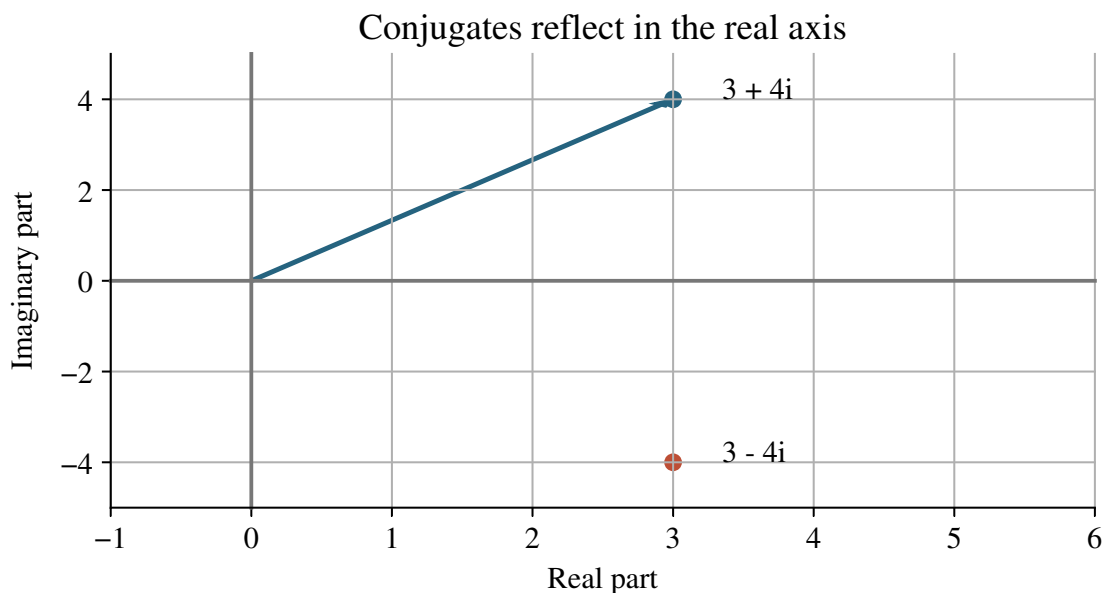
Work through a checked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



Test a tempting shortcut

- The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 uses complex arithmetic, modulus, Argand representation and quadratic roots. De Moivre powers and roots of unity are reserved for FP2.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL further mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Parabolas and rectangular hyperbolas

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
- This lesson studies **parabola** 抛物线: The locus of points equally distant from a fixed focus and a fixed directrix.

Choose the mathematical structure

- For $y^2=4ax$, the focus is $(a,0)$, directrix $x=-a$ and vertex $(0,0)$. For $xy=c^2$, the coordinate axes are asymptotes. Read the parameter from the coefficient rather than assuming it equals $4a$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

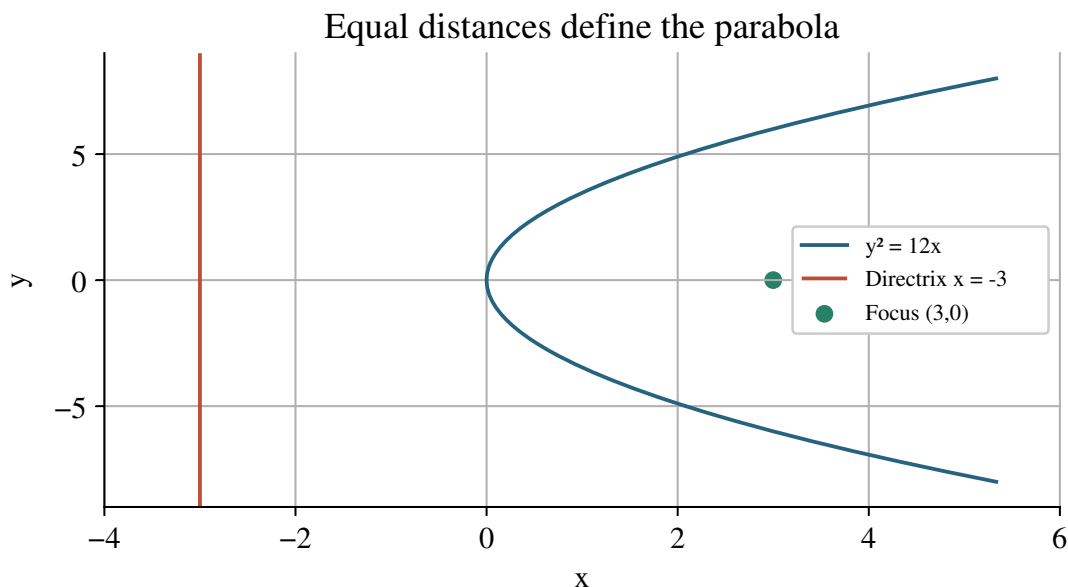
Work through a checked case

$$y^2 = 12x, \quad xy = 9$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y^2=12x$, $a=3$: the focus is $(3,0)$ and directrix $x=-3$. At $x=3$, $y=\pm 6$; at $(3,6)$, distance to the focus and directrix is 6. For $xy=9$, the point $(3,3)$ lies on the hyperbola and $x=0, y=0$ are its asymptotes.



Test a tempting shortcut

- The coefficient of x is $4a$, not a . Include both signs when solving y^2 . An asymptote is approached; it is not a finite intercept of $xy=c^2$.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For $y^2=4ax$, the focus x -coordinate is $4a$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 coordinate-geometry support for parabolas and rectangular hyperbolas. Ellipse area and ellipse parametrisation are excluded.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The locus of points equally distant from a fixed focus and a fixed directrix. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Matrix operations, inverses and plane transformations

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
 - This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
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Choose the mathematical structure

- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero. Transform column vectors by left multiplication. The rightmost matrix acts first in a product.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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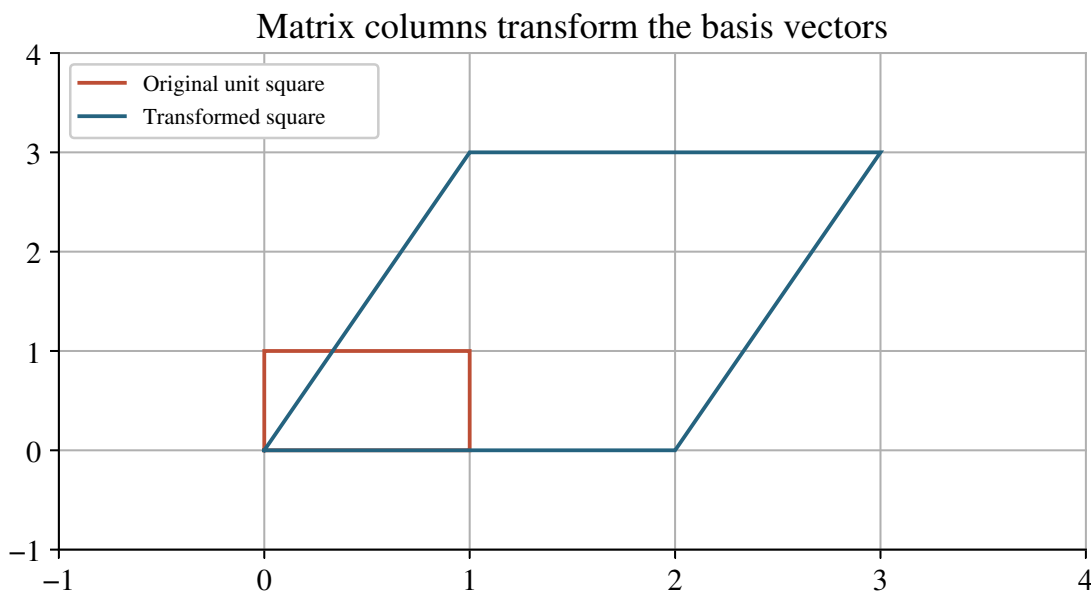
Work through a checked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. Check by multiplying A by its inverse to obtain the identity matrix.



Test a tempting shortcut

- Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every square matrix has an inverse. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 uses 2×2 matrix operations, inverses and plane transformations. Eigenvalues and 3×3 diagonalisation belong to FP3, not this unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Bisection and Newton root finding

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
 - This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.
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Choose the mathematical structure

- For a continuous function with a sign change, bisect a root bracket and keep the half with opposite endpoint signs. Newton updates x to $x - f(x)/f'(x)$, provided the derivative is nonzero.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

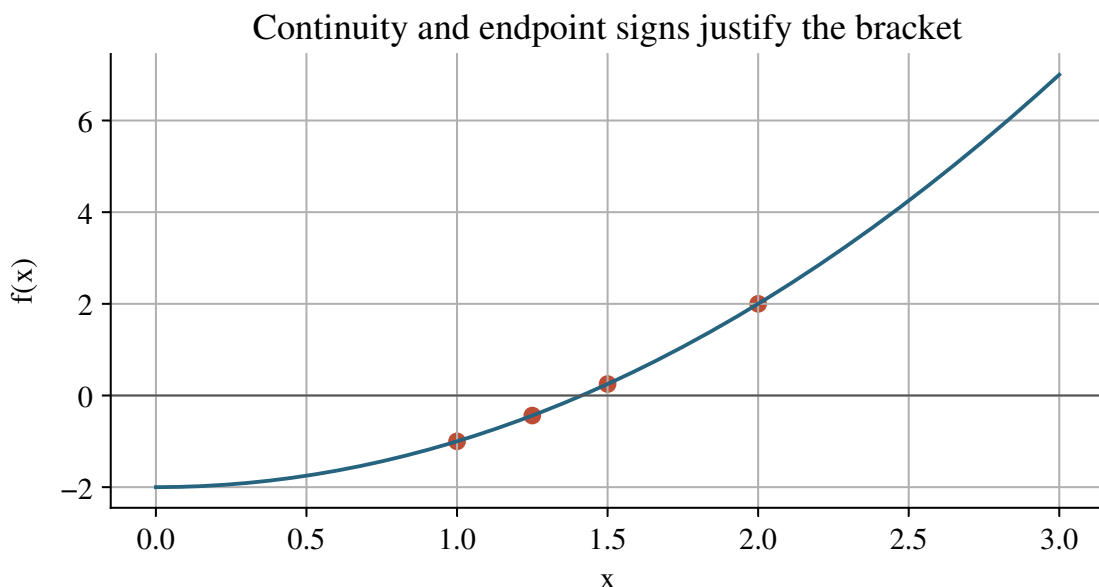
Work through a checked case

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $f(x)=x^2-2$, $f(1)=-1$ and $f(2)=2$. At midpoint 1.5, $f=0.25$, so the new bracket is $[1,1.5]$. At midpoint 1.25, $f=-0.4375$, so the next bracket is $[1.25,1.5]$. Newton from 1.5 gives 1.4166667.



Test a tempting shortcut

- A sign change must occur on an interval where the function is continuous. Newton can fail if its derivative is zero, or if its iterates leave the useful domain.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every sign change proves a root, including one across a discontinuity. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 numerical methods concern roots: bisection, interpolation and Newton methods. Numerical integration is not included in this lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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Use this in your course

- edexcel IAL further mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Finite sums and induction

Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
 - This lesson studies **sigma notation** 求和符号: Notation that adds indexed terms over a stated finite range.
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Choose the mathematical structure

- For $k=1$ to n , $\text{sum } k = n(n+1)/2$, $\text{sum } k^2 = n(n+1)(2n+1)/6$ and $\text{sum } k^3 = [n(n+1)/2]^2$. Differences between consecutive terms can telescope. In induction, verify the starting case and show the next case follows from the assumed case.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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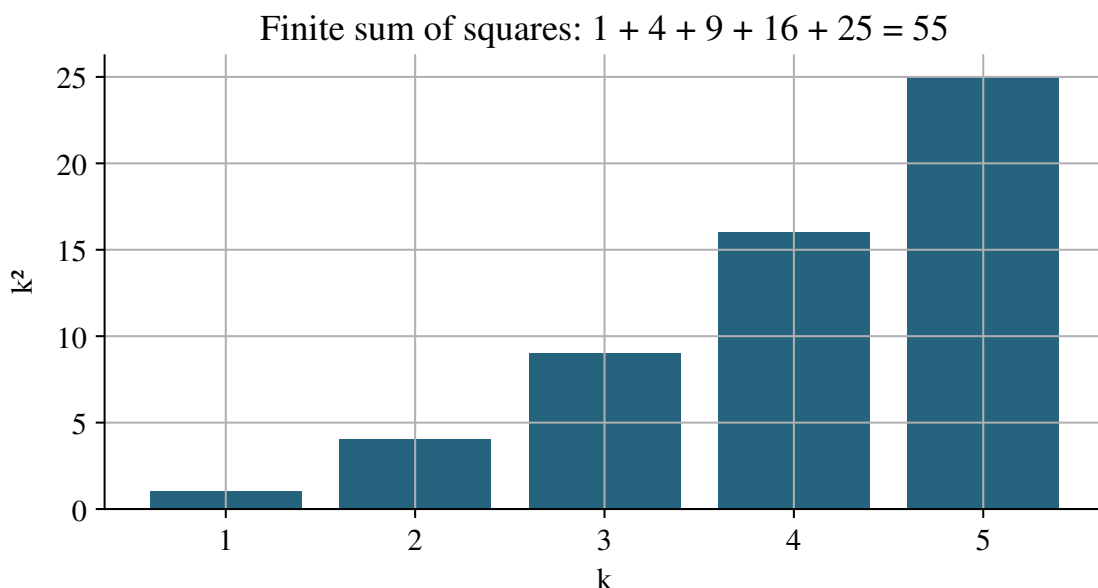
Work through a checked case

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $n=5$, $\text{sum } k=15$, $\text{sum } k^2=1+4+9+16+25=55$ and $\text{sum } k^3=1+8+27+64+125=225$. In proving a sum formula, adding the $(n+1)$ th term to the assumed expression must produce the formula with n replaced by $n+1$.



Test a tempting shortcut

- The sum of squares is not the square of the sum. State the index range and check the first case; an inductive step by itself is not a complete induction proof.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The sum of squared terms always equals the square of their sum. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 finite series and induction support only. Maclaurin and Taylor approximation belong to later further-pure units and are excluded.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Notation that adds indexed terms over a stated finite range. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
- This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.

Choose the mathematical structure

- A deductive proof starts from stated definitions or assumptions. A counterexample refutes an all-values claim. Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

Odd + odd: $(2a + 1) + (2b + 1)$

Factor 2: $2(a + b + 1)$

The bracket is an integer $\rightarrow$ the sum is even

Test a tempting shortcut

- Do not assume the conclusion while proving it. In contradiction, identify the impossible consequence and reject the initial contrary assumption. Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Checking the first ten integers proves a claim for every positive integer. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- AQA proof includes deduction, exhaustion and contradiction. IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction. Keep the method matched to the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A single valid case that disproves a universal claim. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
 - This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
-

Choose the mathematical structure

- Factor where possible; otherwise complete the square or use the quadratic formula. A quadratic inequality needs the sign on intervals, not only the roots. The discriminant identifies repeated or missing real roots.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

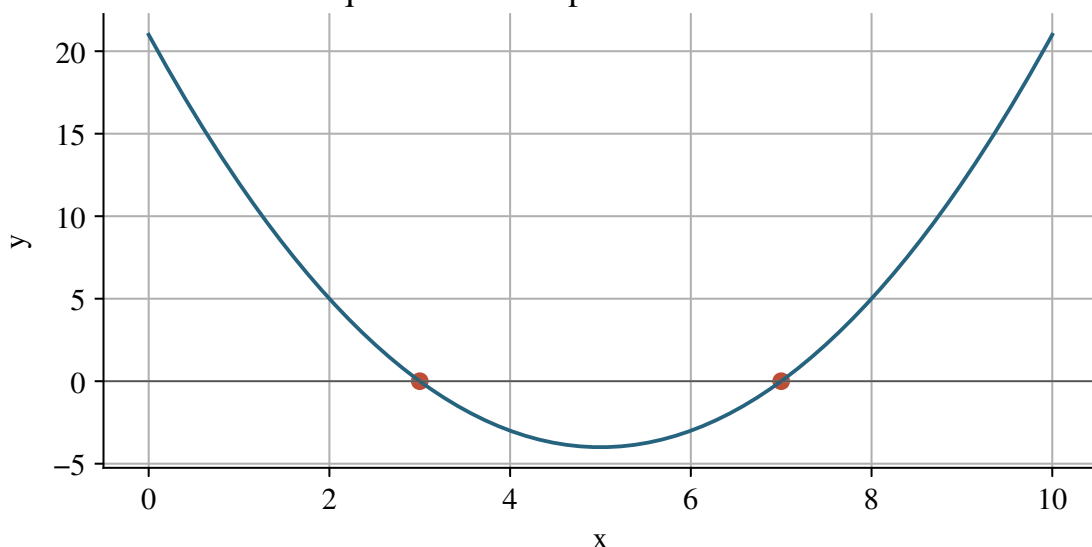
$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The condition is $x(10-x) \geq 21$, so $x^2-10x+21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.

The quadratic is nonpositive between 3 and 7



Test a tempting shortcut

- Multiplying an inequality by a negative number reverses its direction. A sketch must show which side of each root satisfies the inequality. Geometry may restrict x further.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A positive discriminant means that a quadratic has no real roots. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use the vertex to interpret an optimum. Check an endpoint and a point between the roots; the algebra and graph should tell the same story.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Series, finite differences and Taylor expansions

Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **Taylor polynomial** 泰勒多项式: A polynomial formed from a function's derivatives at a chosen centre.

Choose the mathematical structure

- About $x=a$, $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots$ when the expansion is valid. At $a=0$ it is a Maclaurin expansion. Finite sums may also simplify by cancellation or standard sum formulae.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

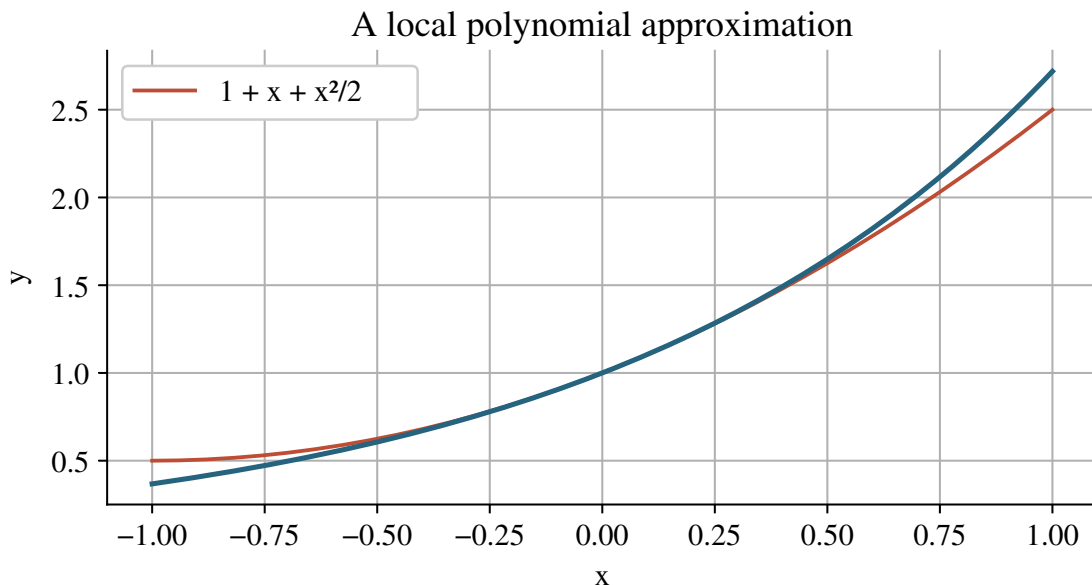
Work through a checked case

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The first three terms of e^x are $1+x+x^2/2$. At $x=0.1$ this gives 1.105, close to $e^{0.1} \approx 1.105170$. For the sum of k^2 from $k=1$ to 5, $n(n+1)(2n+1)/6$ gives $5 \times 6 \times 11/6 = 55$.



Test a tempting shortcut

- The factorial belongs in the denominator of every Taylor coefficient. The expansion centre is not always zero. An approximation close to its centre can be poor far away.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A Taylor polynomial is always exact for the original function at every input. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a telescoping series, write several terms and identify both surviving ends. For a Taylor approximation, state its order and compare with a known value or a remainder estimate where required.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A polynomial formed from a function's derivatives at a chosen centre. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Complex numbers and roots

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
 - This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.
-

Choose the mathematical structure

- Write $z=a+bi$ with $i^2=-1$. The conjugate is $a-bi$ and z times its conjugate $=a^2+b^2$. The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant. Conjugate roots occur for polynomials with real coefficients.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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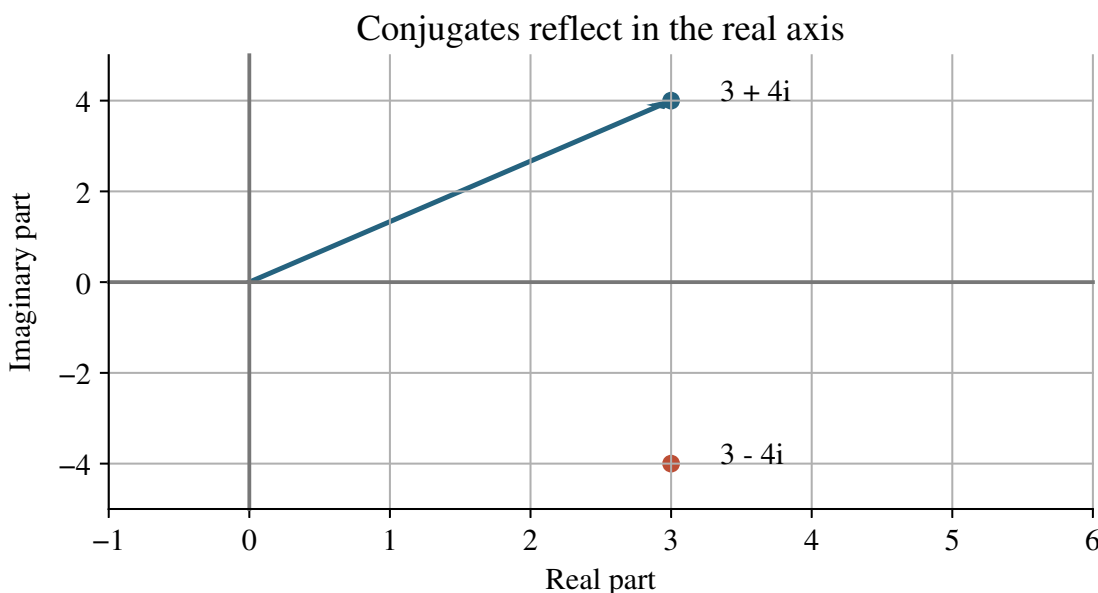
Work through a checked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



Test a tempting shortcut

- The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Draw the real axis horizontally and imaginary axis vertically. For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
 - This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.
-

Choose the mathematical structure

- For $dy/dx=ky$, separate variables: $dy/y=k dx$, giving $y=Ae^{(kx)}$. Use an initial condition to find A. Euler's method takes $y_{next}=y+h f(x,y)$, with a chosen step h .
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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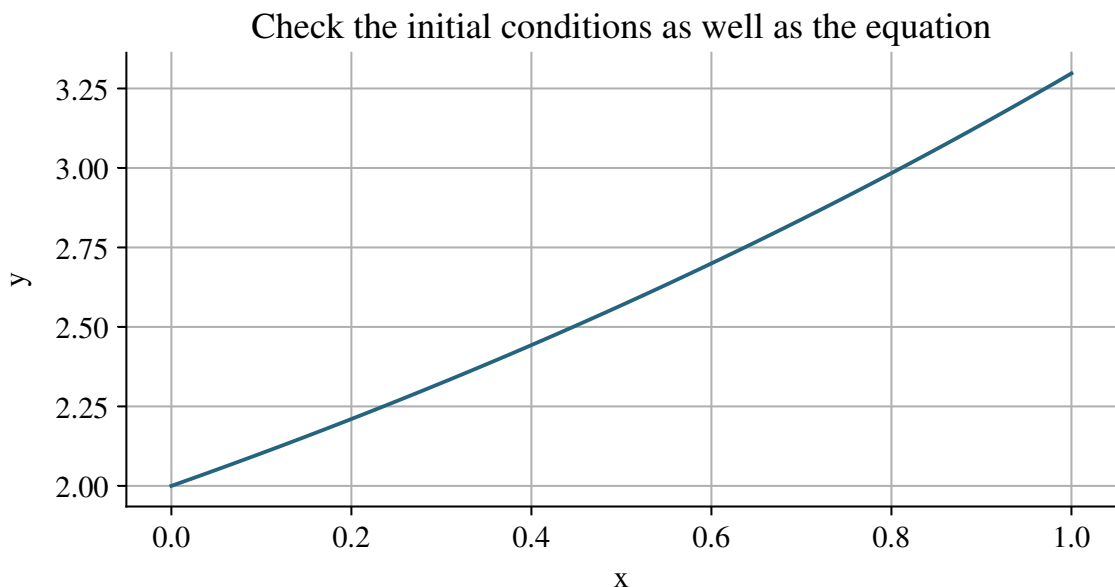
Work through a checked case

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With $\frac{dy}{dx}=0.5y$ and $y(0)=2$, $y=2e^{(0.5x)}$. Euler with $h=0.2$ gives $y(0.2)\approx 2+0.2\times 1=2.2$ and $y(0.4)\approx 2.2+0.2\times 1.1=2.42$. The exact second value is about 2.4428.



Test a tempting shortcut

- An initial condition determines the integration constant; it is not a replacement for integrating. Dividing by y can omit an equilibrium solution $y=0$. Euler accuracy depends on step size and the equation.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every step size gives exactly the same Euler approximation. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions. This is Further Pure scope; check the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A specified value that selects a particular solution from a family. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Second-order linear differential equations

Why do we need two initial conditions?

- A displacement model involves both velocity and acceleration. Solving only a first-order rate equation cannot capture both initial conditions.
 - This lesson studies **complementary function** 互补函数: The general solution of the associated homogeneous linear differential equation.
-

Choose the mathematical structure

- For $y'' + ay' + by = f(x)$, solve the auxiliary quadratic for the homogeneous part. Add a suitable particular integral for the forcing term. Repeated and complex roots require their correct forms.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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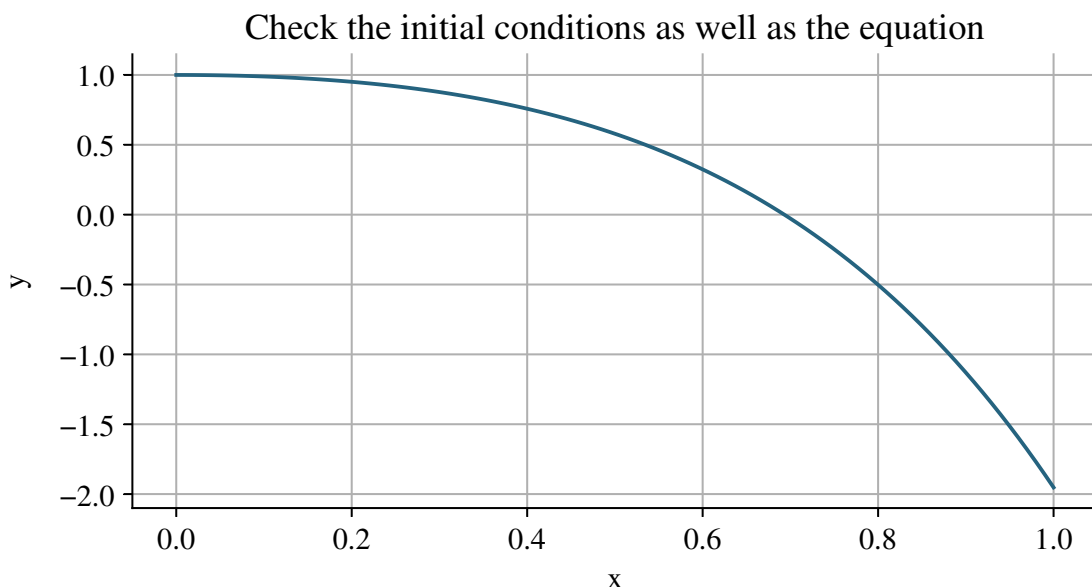
Work through a checked case

$$y'' + ay' + by = f(x), \quad m^2 + am + b = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y'' - 3y' + 2y = 0$, the auxiliary equation is $m^2 - 3m + 2 = 0$ with roots 1, 2. Hence $y = Ae^x + Be^{2x}$. With $y(0) = 1$ and $y'(0) = 0$, $A + B = 1$ and $A + 2B = 0$, so $A = 2, B = -1$.



Test a tempting shortcut

- Two arbitrary constants need two independent conditions. If the trial particular integral duplicates a complementary-function term, multiply the trial by x as required. Do not discard a valid oscillatory solution because the auxiliary roots are complex.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

One initial value is always sufficient to determine both constants in a second-order general solution. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Differentiate the final expression and substitute into the original differential equation. Check both initial conditions separately, and interpret the permitted solution interval.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The general solution of the associated homogeneous linear differential equation. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Polar curves and area

Why describe a curve by direction?

- A petal-shaped curve is simpler when its radius depends on direction. Cartesian coordinates can hide this structure.
 - This lesson studies **polar coordinate** 极坐标: A position described by distance r and angle θ from a chosen origin and reference ray.
-

Choose the mathematical structure

- Use $x=r \cos \theta$ and $y=r \sin \theta$. A polar area is one half the integral of r^2 with respect to θ over a correctly chosen interval. Identify symmetry and repeated tracing before selecting limits.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

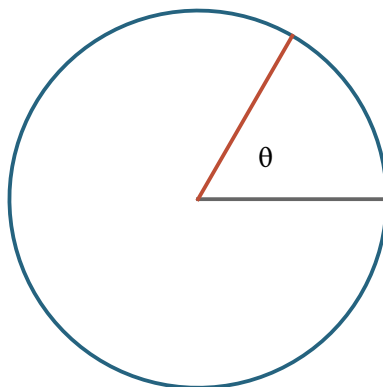
$$x = r \cos \theta, \quad y = r \sin \theta, \quad A = \frac{1}{2} \int r^2 d\theta$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $r=2$ and $\theta=\pi/3$, $x=1$ and $y=\sqrt{3}$. For the circle $r=2$ over a complete turn, $A=(1/2) \int_0^{2\pi} 4 d\theta=4\pi$. A half-turn gives 2π , exactly half the disk.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Test a tempting shortcut

- Negative r places a point in the opposite direction; it is not an ordinary negative distance along the same ray. A parametrisation may trace the same region more than once, so a full parameter interval can overcount area.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Using any full parameter interval always counts each polar region exactly once. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Sketch enough points to establish orientation and bounds. For an enclosed region between two curves, determine intersections and which radial square contributes the outer boundary on each interval.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A position described by distance r and angle θ from a chosen origin and reference ray. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Hyperbolic functions and inverse relations

How can growth and decay make a symmetric curve?

- A hanging cable has a curved profile related to exponentials. Hyperbolic functions combine growth and decay symmetrically.
 - This lesson studies **hyperbolic cosine** 双曲余弦: The function $\cosh x = (e^x + e^{-x})/2$.
-

Choose the mathematical structure

- Define $\sinh x = (e^x - e^{-x})/2$ and $\cosh x = (e^x + e^{-x})/2$. Their identity is $\cosh^2 x - \sinh^2 x = 1$. Derivatives are $\sinh' = \cosh$ and $\cosh' = \sinh$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

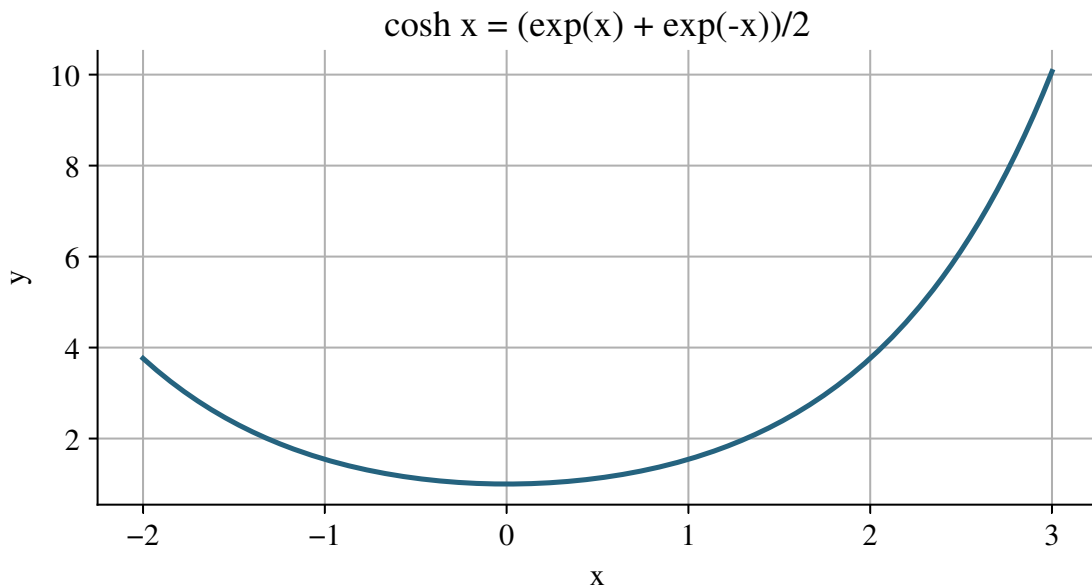
Work through a checked case

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \cosh^2 x - \sinh^2 x = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

At $x=\ln 2$, $e^x=2$ and $e^{-x}=1/2$. Thus $\cosh x=1.25$ and $\sinh x=0.75$. Their squared difference is $1.5625-0.5625=1$. To invert $y=\sinh x$, solve a quadratic in e^x and choose the positive root.



Test a tempting shortcut

- The hyperbolic identity has a minus sign. $\cosh$ is not one-to-one on all real inputs; its usual inverse uses $x \geq 0$. Ordinary circular-trigonometric identities cannot be substituted unchanged.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The identity for hyperbolic functions is $\cosh^2 x + \sinh^2 x = 1$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use exponential definitions to prove identities and solve equations. State domain restrictions for inverse functions before differentiating or integrating them.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The function $\cosh x = (e^x + e^{-x})/2$. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Conics, parametric curves and tangent reasoning

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
 - This lesson studies **parametric equation** 参数方程: A representation in which coordinates are expressed using a common parameter.
-

Choose the mathematical structure

- For $x=a \cos t, y=b \sin t$, eliminating t gives $x^2/a^2 + y^2/b^2 = 1$. For parametric curves, $dy/dx = (dy/dt)/(dx/dt)$, where $dx/dt \neq 0$. A zero denominator may signal a vertical tangent.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

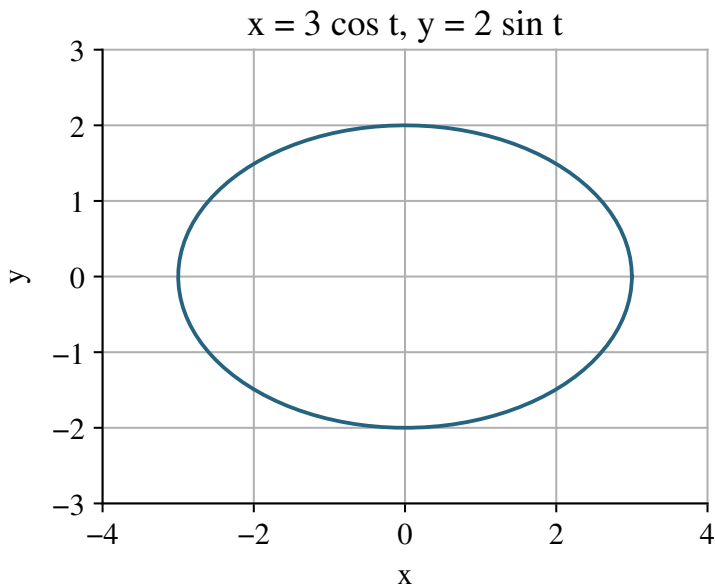
Work through a checked case

$$x = a \cos t, \quad y = b \sin t, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $x=3\cos t, y=2\sin t$ at $t=\pi/4$, the gradient is $(2\cos t)/(-3\sin t)=-2/3$. The point is $(3/\sqrt{2}, \sqrt{2})$. The area inside the ellipse is $\pi ab=6\pi$; its semiaxes are 3 and 2.



Test a tempting shortcut

- Eliminating a parameter may lose a domain restriction or direction of travel. A vertical tangent cannot be assigned a finite dy/dx . Distinguish semiaxes from full widths.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Eliminating a parameter always preserves every restriction and direction automatically. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For advanced coordinate geometry, use the defining equation and a consistent parameter. For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A representation in which coordinates are expressed using a common parameter. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.
-

Choose the mathematical structure

- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

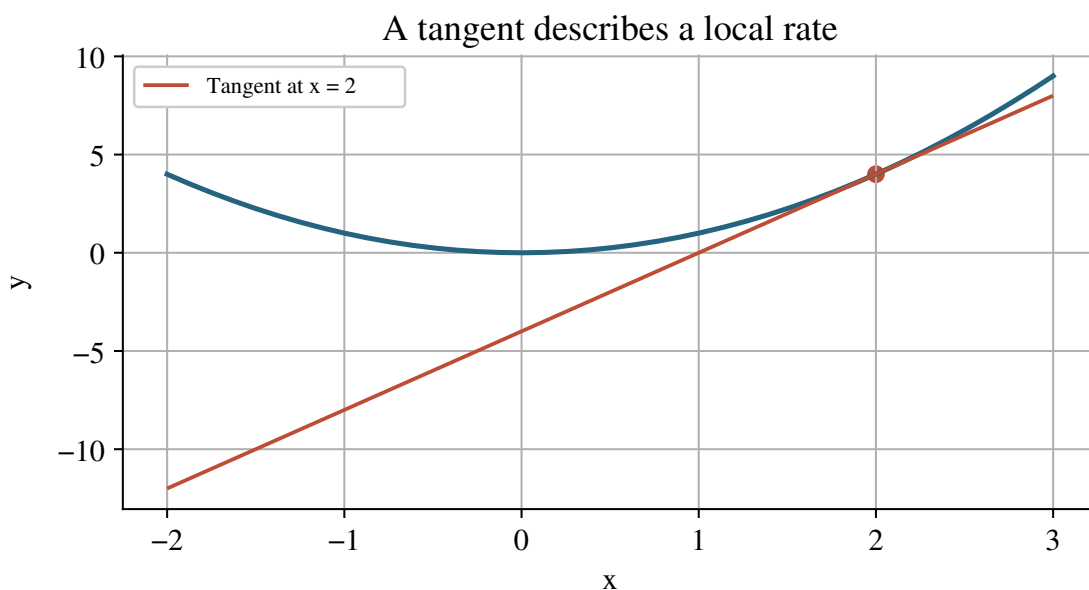
Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Test a tempting shortcut

- A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The derivative of $u(x)v(x)$ is always u prime times v prime. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as $x e^x$ cannot be integrated by separately integrating each factor.
 - This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.
-

Choose the mathematical structure

- Use substitution when an inner derivative appears as a factor. By parts, $\int u v' = uv - \int u' v$. For a rational function, divide first if needed, then decompose into partial fractions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

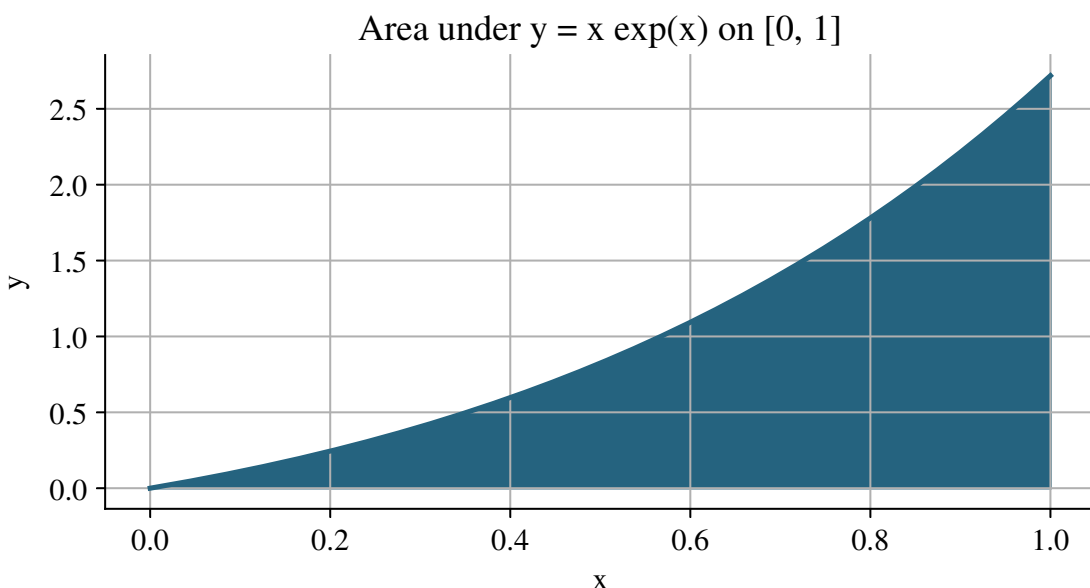
Work through a checked case

$$\int u v' dx = uv - \int u' v dx$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For integral $\int x e^x dx$, take $u=x$ and $v' = e^x$. Then the integral is $x e^x - e^x + C$.
For integral $\int \frac{2x}{x^2+1} dx$, substitute $w=x^2+1$, $dw=2x dx$; the answer is $\ln(x^2+1) + C$.



Test a tempting shortcut

- Choosing u and v prime well matters: the remaining integral should become simpler. In a definite substitution, either change the limits or return to x before applying the original limits.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The integral of a product equals the product of its separate integrals. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check by differentiating. For volumes of revolution around the x -axis use $V = \pi \int y^2 dx$; do not confuse the square of a function with the integral of the function.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An integration method based on the derivative of a product. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Spatial vectors, lines and angles

Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
- This lesson studies **direction vector** 方向向量: A nonzero vector giving the direction of a line.

Choose the mathematical structure

- A line has $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ with position vector $\mathbf{a}$ and nonzero direction $\mathbf{b}$. To find an intersection, equate all components and solve for both line parameters. For an angle use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where scalar products are in the named syllabus.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

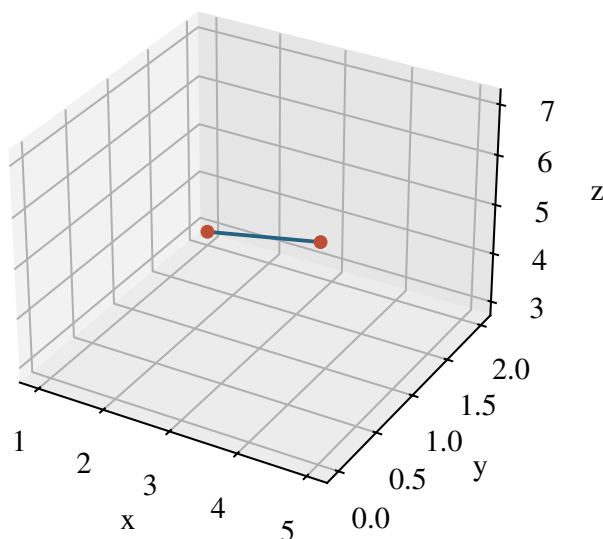
$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}, \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $a=(1,2,3), b=(2,-1,2)$, $\lambda=2$ gives $r=(5,0,7)$. The magnitude of b is $\sqrt{(4+1+4)}=3$. With $c=(1,0,-1)$, $b \cdot c=2-2=0$, so b and c are perpendicular.

$$r = (1, 2, 3) + \lambda(2, -1, 2)$$



Test a tempting shortcut

- Satisfying two component equations does not guarantee the third. Nonparallel lines may be skew. A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Any two nonparallel lines in three dimensions must intersect. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State whether a requested angle is acute, directed or an angle between lines. Use the syllabus-specific plane or distance method only when it is actually required for that unit.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A nonzero vector giving the direction of a line. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Matrix transformations, inverses and eigenvalues

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
 - This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
-

Choose the mathematical structure

- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero. Transform column vectors by left multiplication. The rightmost matrix acts first in a product.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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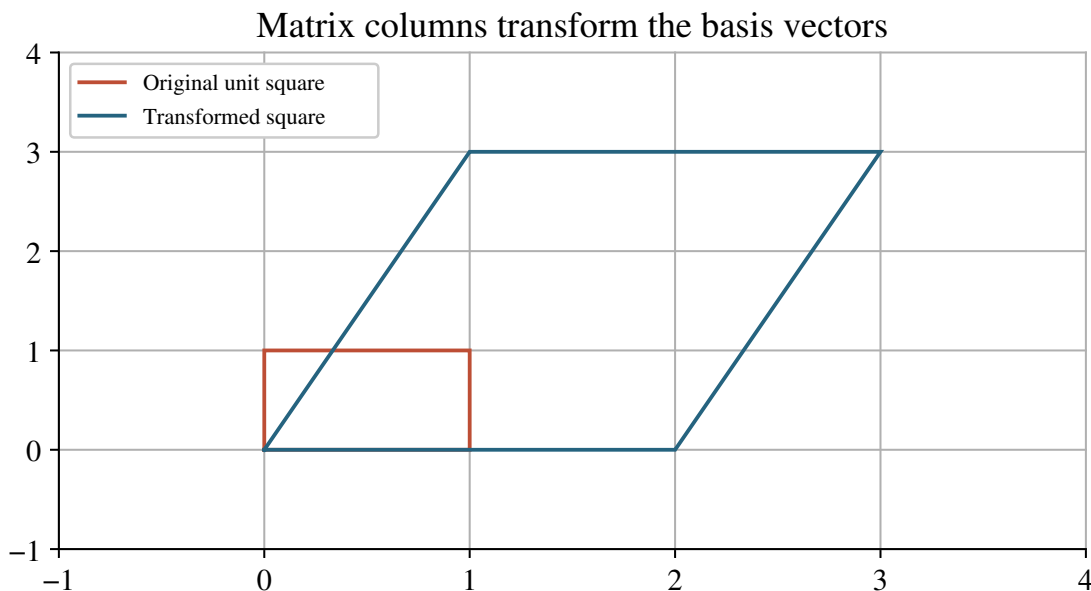
Work through a checked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. The characteristic equation is $(2-\lambda)(3-\lambda)=0$, so the eigenvalues are 2 and 3.



Test a tempting shortcut

- Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every square matrix has an inverse. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Find an eigenvector by solving $(A - \lambda I)v = 0$ with $v \neq 0$. Explain the geometrical meaning: this vector keeps its line direction under the transformation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor. Choose the relationship, show the method, check its assumptions and interpret the result.

M1 · Kinematics and Newton's laws

Which force accelerates the trolley?

- A trolley accelerates while friction opposes its motion. The engine's force alone does not equal mass times acceleration.
 - This lesson studies **resultant force** 合力: The vector sum of all external forces acting on the modelled object.
-

Choose the mathematical structure

- Choose a positive direction and draw a force diagram. Use resultant $F = ma$. For constant acceleration, $v = u + at$ and $s = ut + at^2/2$. A particle model ignores size; a smooth surface ignores friction.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

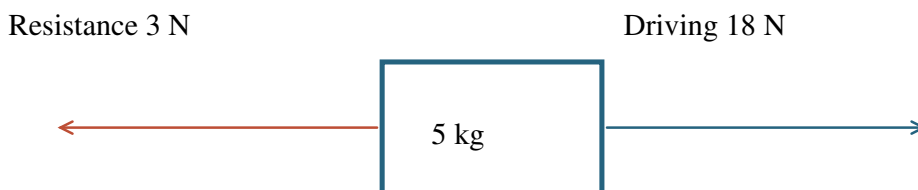
$$F_{\text{resultant}} = ma, \quad s = ut + \frac{1}{2}at^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A 5 kg trolley has driving force 18 N and resistance 3 N. The resultant is 15 N, so $a = 15/5 = 3 \text{ m/s}^2$. Starting from rest for 4 s gives $v = 0 + 3 \times 4 = 12 \text{ m/s}$ and $s = 0 \times 4 + 3 \times 4^2 / 2 = 24 \text{ m}$.

Identify the resultant before using $F = ma$



Test a tempting shortcut

- The normal reaction need not equal weight on a slope. Connected-particle models need one equation per particle and a consistent acceleration relation. The constant-acceleration formulae cannot be used for arbitrary variable acceleration.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The constant-acceleration formulae apply unchanged when acceleration varies with time. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Explain what each idealisation permits and what it leaves out. For a velocity-time graph, gradient represents acceleration and signed area represents displacement.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The vector sum of all external forces acting on the modelled object. Choose the relationship, show the method, check its assumptions and interpret the result.

M1 · Vectors and transformation geometry

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
 - This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.
-

Choose the mathematical structure

- Add corresponding vector components and subtract position vectors to find a displacement. A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

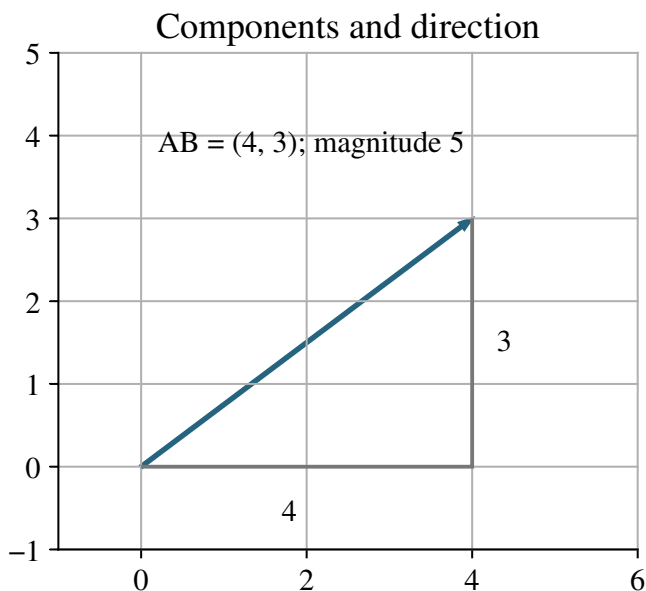
Work through a checked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With $\mathbf{a}=(4,1)$ and $\mathbf{b}=(1,3)$, $\mathbf{a}+\mathbf{b}=(5,4)$. From $A=(1,2)$ to $B=(5,5)$, displacement $\overrightarrow{AB}=(4,3)$. Its magnitude is $\sqrt{4^2+3^2}=5$.



Test a tempting shortcut

- The order of subtraction matters: $\mathbf{BA} = -\mathbf{AB}$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

$\mathbf{AB}$ and $\mathbf{BA}$ always have the same components. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use vector components, scalar multiples and magnitudes for displacements and forces. This unit does not require the scalar product.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The vector sum representing the combined displacement or force. Choose the relationship, show the method, check its assumptions and interpret the result.

M1 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.

- This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.

Choose the mathematical structure

- For equilibrium, both the resultant force and resultant moment are zero. Take moments about a useful pivot to remove unknown forces through that point. A combined centre of mass is a mass-weighted mean position.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

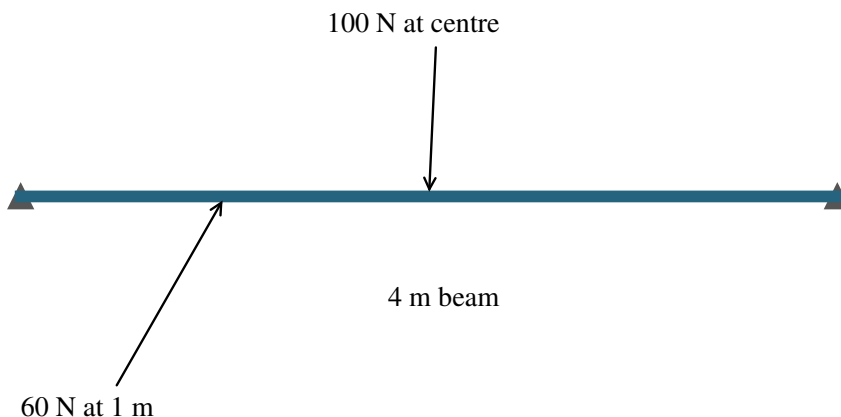
Work through a checked case

$$M = Fd_{\perp}, \quad \sum M = 0, \quad \sum \mathbf{F} = \mathbf{0}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give $4R_{\text{right}} = 100 \times 2 + 60 \times 1 = 260$, so $R_{\text{right}} = 65$ N. Vertical equilibrium gives $R_{\text{left}} = 95$ N.



Test a tempting shortcut

- Use the perpendicular lever arm, not a diagonal distance to the point of application. A zero resultant force alone does not prevent rotation. A support that would need a negative normal reaction may lose contact.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Zero resultant force by itself is enough for a rigid body's equilibrium. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution. A removed region has negative mass in that calculation, not a new physical negative material.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Force multiplied by the perpendicular distance from its line of action to a pivot. Choose the relationship, show the method, check its assumptions and interpret the result.

M1 · Inclined planes, friction and connected particles

Is friction already at its maximum?

- A crate on a slope is at rest. Friction can adjust to balance the force down the slope; it need not already be at its maximum.
 - This lesson studies **limiting friction** 极限摩擦力: The maximum static friction before slipping, equal to μ times the normal reaction in the model.
-

Choose the mathematical structure

- Resolve parallel and perpendicular to the plane. Weight components are $mg \sin \theta$ and $mg \cos \theta$. Static friction satisfies $F \leq \mu R$ and equals μR only at the limiting case. Write separate equations for connected particles.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

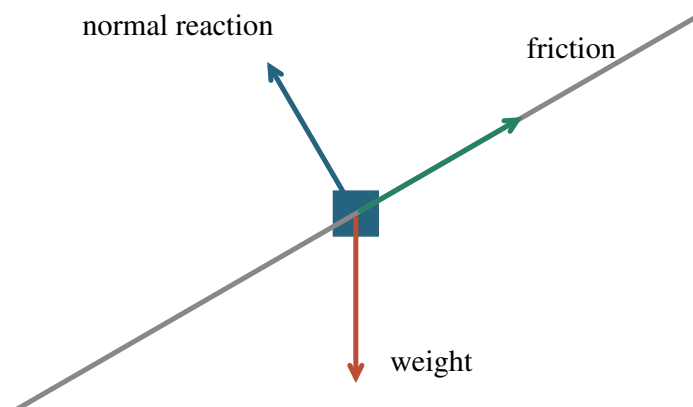
$$R = mg \cos \theta, \quad F \leq \mu R$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A 5 kg crate on a 30° slope has $R = 5 \times 9.8 \times \cos 30^\circ \approx 42.44$ N. The downhill weight component is 24.5 N. With $\mu = 0.6$, maximum friction ≈ 25.46 N, so equilibrium is possible with friction 24.5 N.

Resolve weight along and normal to the slope



Test a tempting shortcut

- Friction opposes motion or the tendency to move, not always the coordinate direction. A taut light inextensible string over a smooth pulley gives equal tension and a common acceleration magnitude; each assumption has a job.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Static friction is always exactly μR even when the body is not about to slip. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Start with force diagrams and a proposed direction of motion. If the calculated direction conflicts with the friction assumption, revisit the model instead of keeping inconsistent signs.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL further mathematics; official unit M1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The maximum static friction before slipping, equal to μ times the normal reaction in the model. Choose the relationship, show the method, check its assumptions and interpret the result.

M2 · Kinematics and Newton's laws

Which force accelerates the trolley?

- A trolley accelerates while friction opposes its motion. The engine's force alone does not equal mass times acceleration.
- This lesson studies **resultant force** 合力: The vector sum of all external forces acting on the modelled object.

Choose the mathematical structure

- Choose a positive direction and draw a force diagram. Use resultant $F=ma$. For constant acceleration, $v=u+at$ and $s=ut+\frac{1}{2}at^2$. A particle model ignores size; a smooth surface ignores friction.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

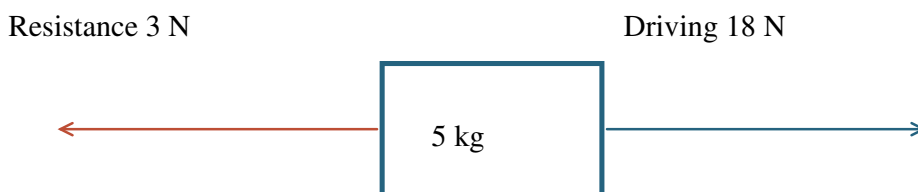
$$F_{\text{resultant}} = ma, \quad s = ut + \frac{1}{2}at^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A 5 kg trolley has driving force 18 N and resistance 3 N. The resultant is 15 N, so $a = 15/5 = 3 \text{ m/s}^2$. Starting from rest for 4 s gives $v = 0 + 3 \times 4 = 12 \text{ m/s}$ and $s = 0 \times 4 + 3 \times 4^2 / 2 = 24 \text{ m}$.

Identify the resultant before using $F = ma$



Test a tempting shortcut

- The normal reaction need not equal weight on a slope. Connected-particle models need one equation per particle and a consistent acceleration relation. The constant-acceleration formulae cannot be used for arbitrary variable acceleration.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The constant-acceleration formulae apply unchanged when acceleration varies with time. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Explain what each idealisation permits and what it leaves out. For a velocity-time graph, gradient represents acceleration and signed area represents displacement.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The vector sum of all external forces acting on the modelled object. Choose the relationship, show the method, check its assumptions and interpret the result.

M2 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
 - This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.
-

Choose the mathematical structure

- For equilibrium, both the resultant force and resultant moment are zero. Take moments about a useful pivot to remove unknown forces through that point. A combined centre of mass is a mass-weighted mean position.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

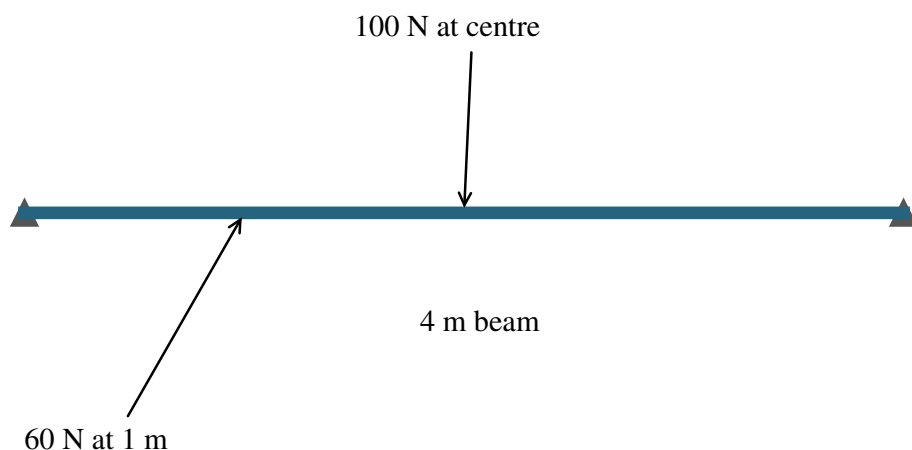
Work through a checked case

$$M = Fd_{\perp}, \quad \sum M = 0, \quad \sum \mathbf{F} = \mathbf{0}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give $4R_{\text{right}} = 100 \times 2 + 60 \times 1 = 260$, so $R_{\text{right}} = 65$ N. Vertical equilibrium gives $R_{\text{left}} = 95$ N.



Test a tempting shortcut

- Use the perpendicular lever arm, not a diagonal distance to the point of application. A zero resultant force alone does not prevent rotation. A support that would need a negative normal reaction may lose contact.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Zero resultant force by itself is enough for a rigid body's equilibrium. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution. A removed region has negative mass in that calculation, not a new physical negative material.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Force multiplied by the perpendicular distance from its line of action to a pivot. Choose the relationship, show the method, check its assumptions and interpret the result.

M2 · Work, energy, power and collisions

What does a slower rebound tell us?

- A ball rebounds slower after hitting a wall. Momentum and kinetic energy answer different parts of the collision story.
 - This lesson studies **coefficient of restitution** 恢复系数: The ratio of relative separation speed to relative approach speed along the line of impact.
-

Choose the mathematical structure

- Work is force times displacement in its direction. Kinetic energy is $mv^2/2$ and gravitational potential energy is mgh . Momentum is conserved for an isolated collision; restitution gives the relative speed relation, not conservation of kinetic energy.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

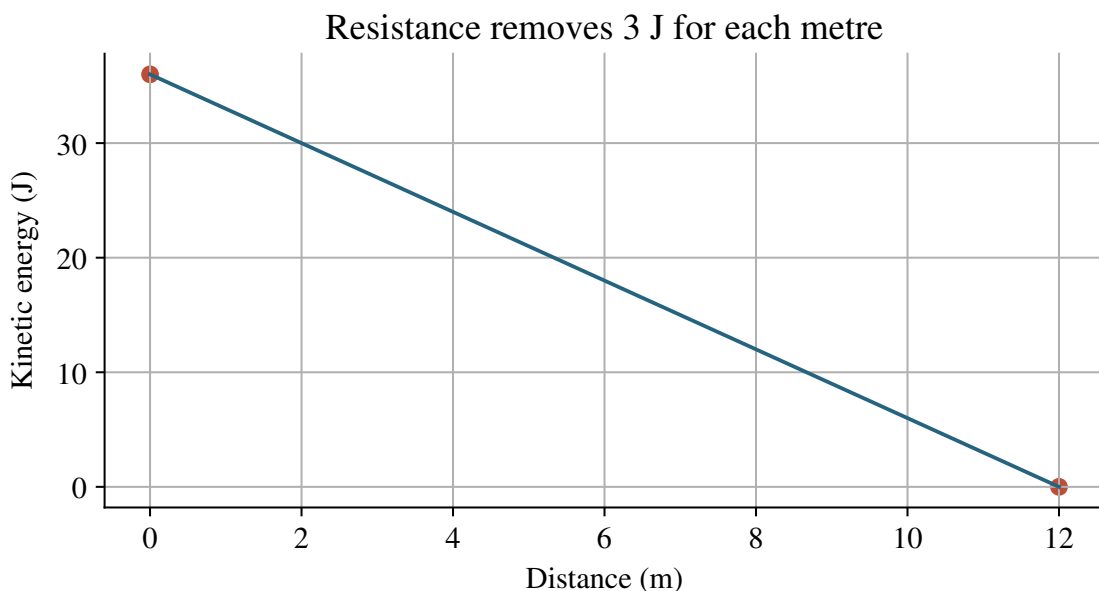
Work through a checked case

$$E_k = \frac{1}{2}mv^2, \quad W = Fs, \quad P = \frac{dE}{dt}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A 2 kg particle moving at 6 m/s has kinetic energy $2 \times 6^2 / 2 = 36$ J. If a constant opposing force of 3 N stops it, $\text{work} = 3s = 36$, giving $s = 12$ m. A wall bounce with approach speed 5 and $e = 0.6$ has rebound speed 3 m/s in the opposite direction.



Test a tempting shortcut

- Conserve momentum only when the external impulse is negligible. Kinetic energy is generally lost in an inelastic collision. Power is a rate of energy transfer, not an amount of energy.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every collision conserves kinetic energy. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State the direction signs before writing collision equations. For a connected motion problem, choose the whole system for energy when internal tensions cancel; account for frictional work explicitly.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The ratio of relative separation speed to relative approach speed along the line of impact. Choose the relationship, show the method, check its assumptions and interpret the result.

M2 · Projectile motion and variable acceleration

Why does horizontal motion share vertical time?

- A ball is launched at an angle. Its horizontal and vertical motion share time but follow different equations.
 - This lesson studies **projectile** 抛体: A particle moving under gravity after launch in a model that neglects air resistance.
-

Choose the mathematical structure

- Resolve initial velocity into horizontal $u \cos\theta$ and vertical $u \sin\theta$. With no air resistance, horizontal acceleration is zero and vertical acceleration is $-g$. Use the same time in both components.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

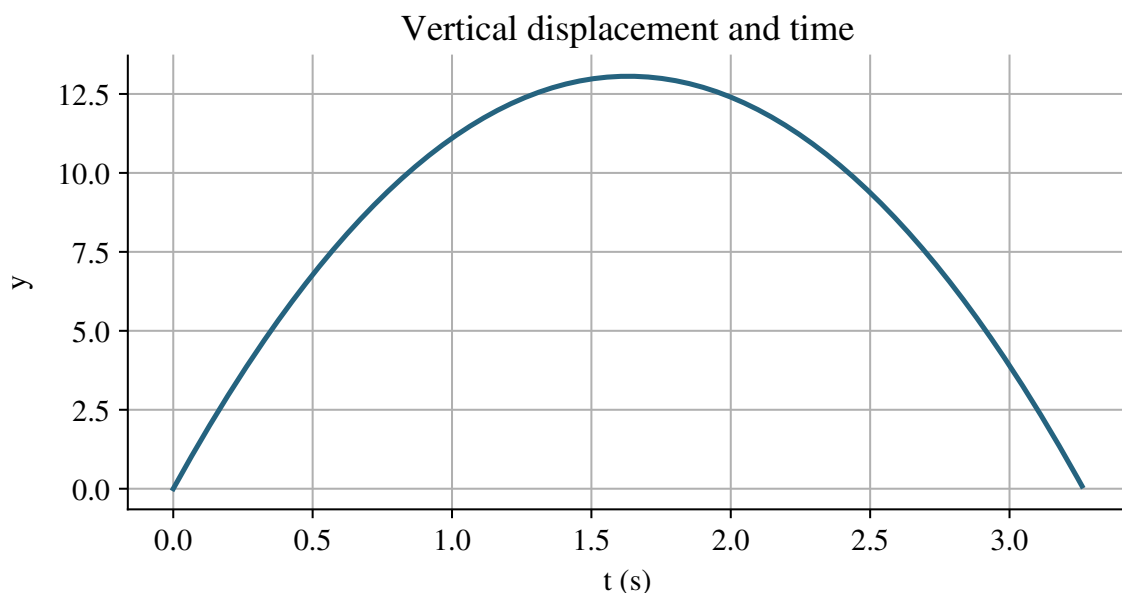
Work through a checked case

$$x = u_x t, \quad y = u_y t - \frac{1}{2} g t^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For components $u_x=12$, $u_y=16$ and $g=9.8$, at $t=2$ s the horizontal displacement is $x=12 \times 2=24$ m and vertical displacement is $y=16 \times 2 - 9.8 \times 2^2 / 2 = 12.4$ m. At the highest point, $v_y=0$ but v_x remains 12.



Test a tempting shortcut

- Zero vertical velocity at the highest point does not mean zero total speed. Launch and landing heights need not be equal. Do not use a range formula that assumes equal heights without checking them.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The total speed of a projectile is zero at its highest point. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For variable acceleration, integrate $a(t)$ to get $v(t)$ and use the initial velocity to find the constant, then integrate for displacement. Confirm units and the physical time interval.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A particle moving under gravity after launch in a model that neglects air resistance. Choose the relationship, show the method, check its assumptions and interpret the result.

M3 · Elasticity, circular motion and variable dynamics

Which length stretches the spring?

- A stretched spring stores energy. A particle moving in a circle needs a resultant force towards the centre, even at constant speed.
 - This lesson studies **extension** 伸长量: The increase in an elastic string's or spring's length from its natural length.
-

Choose the mathematical structure

- For an elastic string, tension = $\lambda x/l$ when stretched, with natural length l and modulus λ . Elastic energy = $\lambda x^2/(2l)$. For circular motion, radial acceleration = v^2/r ; resolve actual forces in the radial direction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$T = \frac{\lambda x}{l}, \quad E = \frac{\lambda x^2}{2l}, \quad F_r = \frac{mv^2}{r}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With $\lambda=40$ N, $l=2$ m, $x=0.3$ m, tension = $40 \times 0.3/2 = 6$ N and elastic energy = $40 \times 0.3^2/(2 \times 2) = 0.9$ J. A 2 kg particle at speed 3 m/s on radius 1.5 m needs resultant inward force = $2 \times 3^2/1.5 = 12$ N.

Tension depends on extension, not total length

natural length $l = 2 \text{ m}$



extension $x = 0.3 \text{ m}$

Test a tempting shortcut

- Centripetal force is the name of the inward resultant, not an extra force to add to the diagram. A slack string has no tension. Distinguish extension from total stretched length.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Centripetal force is an additional force that must be added to all the real forces. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For variable acceleration, use $a = dv/dt$ or $a = v dv/dx$ when appropriate. Combine the chosen motion relation with the force or energy model, then check the permitted range of motion.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The increase in an elastic string's or spring's length from its natural length. Choose the relationship, show the method, check its assumptions and interpret the result.

M3 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

Choose the mathematical structure

- For $dy/dx=ky$, separate variables: $dy/y=k dx$, giving $y=Ae^{kx}$. Use an initial condition to find A. Euler's method takes $y_{next}=y+h f(x,y)$, with a chosen step h .
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

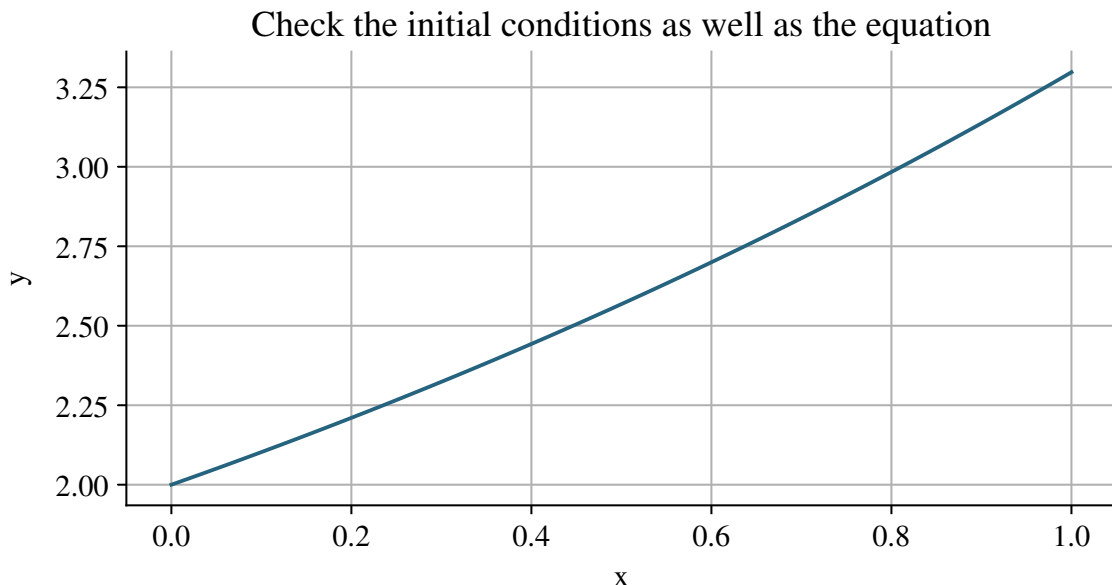
Work through a checked case

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With $\frac{dy}{dx}=0.5y$ and $y(0)=2$, $y=2e^{(0.5x)}$. Euler with $h=0.2$ gives $y(0.2)\approx 2+0.2\times 1=2.2$ and $y(0.4)\approx 2.2+0.2\times 1.1=2.42$. The exact second value is about 2.4428.



Test a tempting shortcut

- An initial condition determines the integration constant; it is not a replacement for integrating. Dividing by y can omit an equilibrium solution $y=0$. Euler accuracy depends on step size and the equation.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every step size gives exactly the same Euler approximation. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions. This is Further Pure scope; check the named unit.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A specified value that selects a particular solution from a family. Choose the relationship, show the method, check its assumptions and interpret the result.

M3 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
 - This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.
-

Choose the mathematical structure

- For equilibrium, both the resultant force and resultant moment are zero. Take moments about a useful pivot to remove unknown forces through that point. A combined centre of mass is a mass-weighted mean position.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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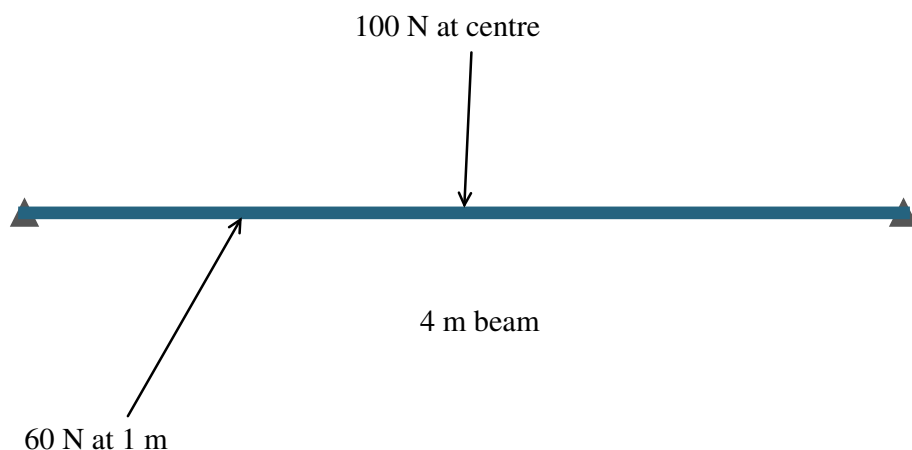
Work through a checked case

$$M = Fd_{\perp}, \quad \sum M = 0, \quad \sum \mathbf{F} = \mathbf{0}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give $4R_{\text{right}} = 100 \times 2 + 60 \times 1 = 260$, so $R_{\text{right}} = 65$ N. Vertical equilibrium gives $R_{\text{left}} = 95$ N.



Test a tempting shortcut

- Use the perpendicular lever arm, not a diagonal distance to the point of application. A zero resultant force alone does not prevent rotation. A support that would need a negative normal reaction may lose contact.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Zero resultant force by itself is enough for a rigid body's equilibrium. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution. A removed region has negative mass in that calculation, not a new physical negative material.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit M3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Force multiplied by the perpendicular distance from its line of action to a pivot. Choose the relationship, show the method, check its assumptions and interpret the result.

S1 · Data summaries, histograms and interpretation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
 - This lesson studies **frequency density** 频率密度: Frequency divided by class width, used as histogram height.
-

Choose the mathematical structure

- Compare an appropriate average and spread in context. A histogram uses area for frequency, so $\text{height} = \text{frequency} / \text{class width}$. Grouped estimates assume representative values within intervals.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

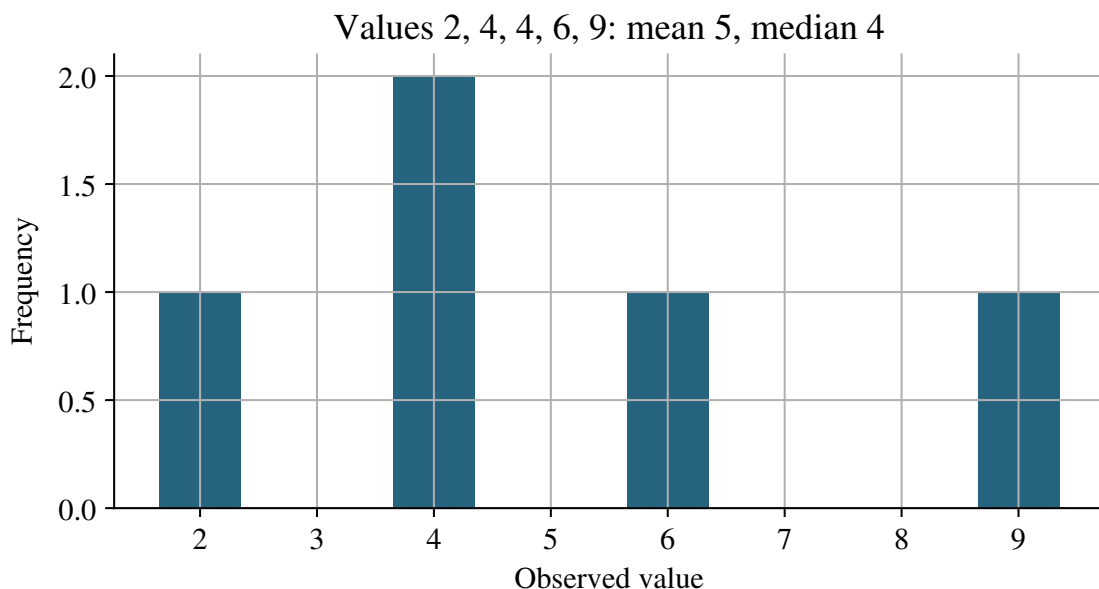
Work through a checked case

$$\bar{x} = \frac{\sum x_i}{n}, \quad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A class from 10 to 20 with frequency 30 has density $30/10=3$. A class from 20 to 40 with frequency 20 has density $20/20=1$. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.



Test a tempting shortcut

- The tallest histogram bar need not contain the most observations. A grouped mean is an estimate. Correlation does not prove causation, and extrapolation extends beyond the observed range.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A histogram bar's height always equals its frequency, even with unequal class widths. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a display that fits the data type. Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Frequency divided by class width, used as histogram height. Choose the relationship, show the method, check its assumptions and interpret the result.

S1 · Probability, trees and conditional reasoning

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
 - This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.
-

Choose the mathematical structure

- Multiply along a tree branch and add disjoint branches. With replacement, the composition stays fixed. Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

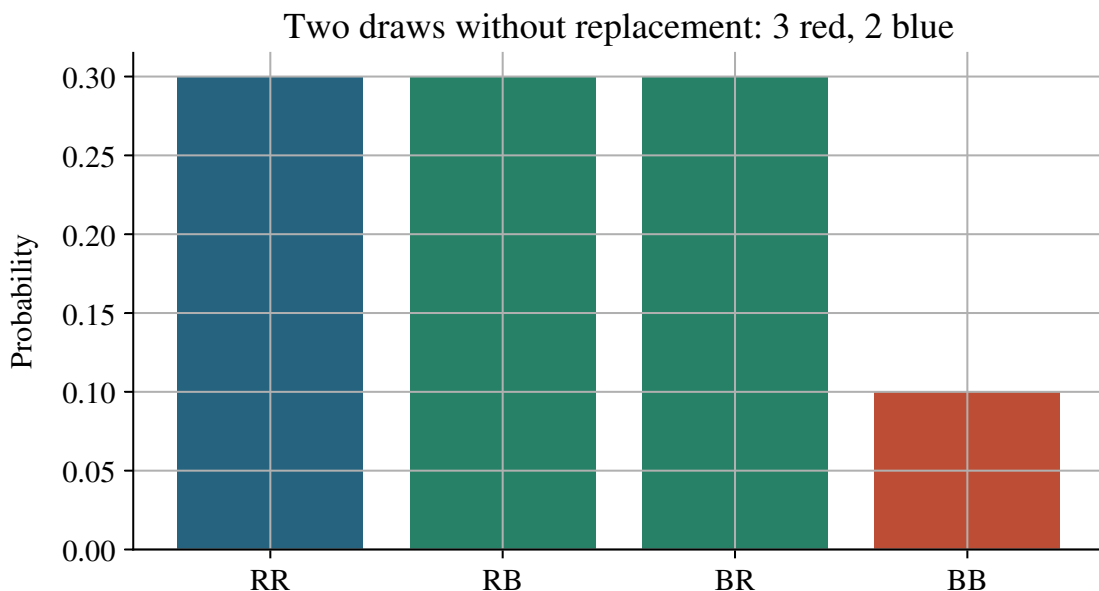
Work through a checked case

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



Test a tempting shortcut

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Mutually exclusive events with positive probabilities must be independent. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- A two-way table makes the restricted denominator visible. Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The probability of an event after restricting the sample space to a stated condition. Choose the relationship, show the method, check its assumptions and interpret the result.

S1 · Regression, financial models and residuals

Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.

- This lesson studies **residual** 残差: The observed value minus the value predicted by a fitted model.

Choose the mathematical structure

- A linear model $y=a+bx$ has intercept a and slope b with contextual units. Inspect residuals and the data range. For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

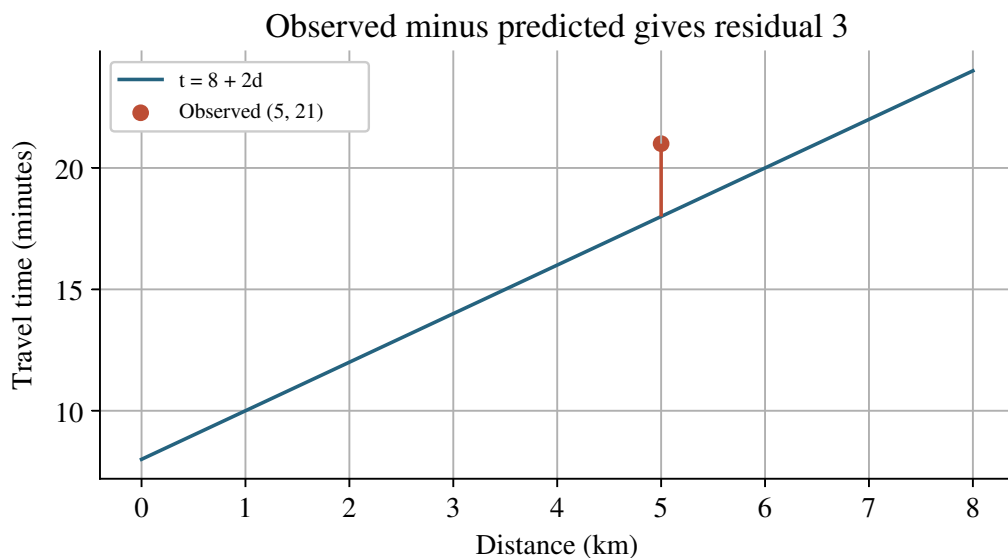
Work through a checked case

$$\hat{t} = 8 + 2d, \quad e = t - \hat{t}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The fitted model $t=8+2d$ predicts 18 minutes at $d=5$ km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes $1000 \times 1.05^3 = 1157.625$ after 3 years.



Test a tempting shortcut

- A good fit does not prove a causal mechanism. A correlation coefficient measures linear association, not the gradient. Calculator output must be translated into a model, checked and interpreted.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A high correlation permits reliable extrapolation to any distance. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- In IB AI, record the data source, domain, assumptions and calculator method. Compare an alternative model and judge predictions against residuals. A financial answer must state payment timing and whether interest is compounded.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The observed value minus the value predicted by a fitted model. Choose the relationship, show the method, check its assumptions and interpret the result.

S1 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
 - This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.
-

Choose the mathematical structure

- For a discrete random variable, sum the probabilities to 1 and calculate $E(X) = \sum xP(X=x)$. For the normal model use $z = (x - \mu)/\sigma$ and calculate an appropriate area.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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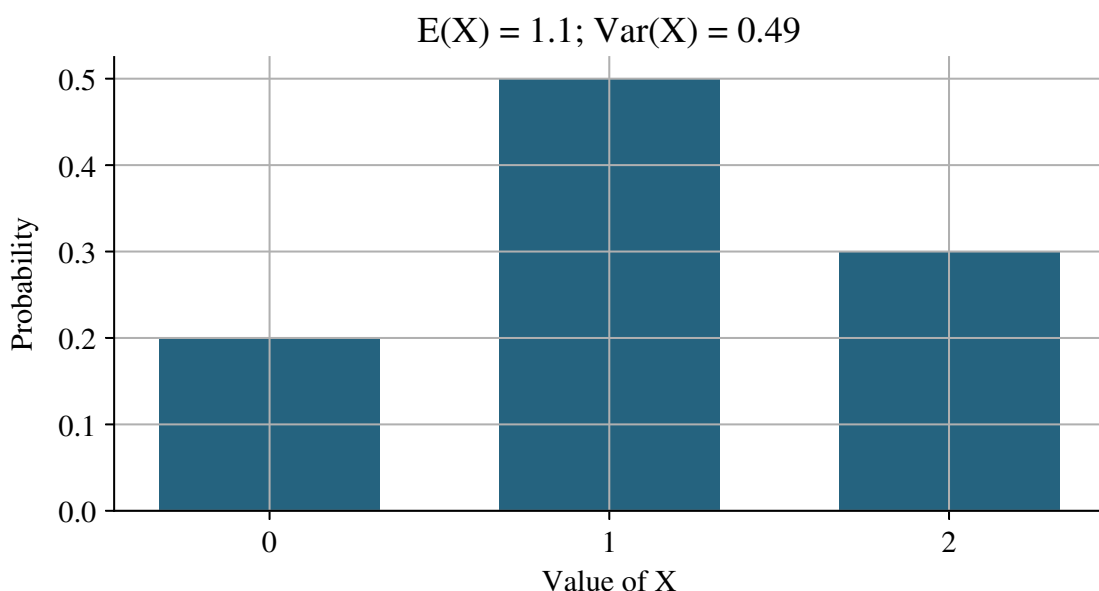
Work through a checked case

$$E(X) = \sum xP(X = x), \quad z = \frac{x - \mu}{\sigma}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If X takes 0, 1, 2 with probabilities 0.2, 0.5, 0.3, $E(X) = 0 \times 0.2 + 1 \times 0.5 + 2 \times 0.3 = 1.1$. Also $E(X^2) = 0 + 0.5 + 1.2 = 1.7$, so $\text{Var}(X) = 1.7 - 1.1^2 = 0.49$. For $\mu = 100, \sigma = 15$, a value of 130 has $z = 2$.



Test a tempting shortcut

- Not every count is binomial: changing p or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A random variable with three possible values must be binomial. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- S1 includes discrete random variables and the normal distribution. Named binomial and Poisson models are S2 content.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The probability-weighted mean of a random variable. Choose the relationship, show the method, check its assumptions and interpret the result.

S2 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.

Choose the mathematical structure

- A binomial model needs fixed n , independent trials, two outcomes and constant p . $E(X)=np$ and $\text{Var}(X)=np(1-p)$. For a normal model use $z=(x-\mu)/\sigma$. A Poisson model describes counts with a constant rate and appropriate independence assumptions.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

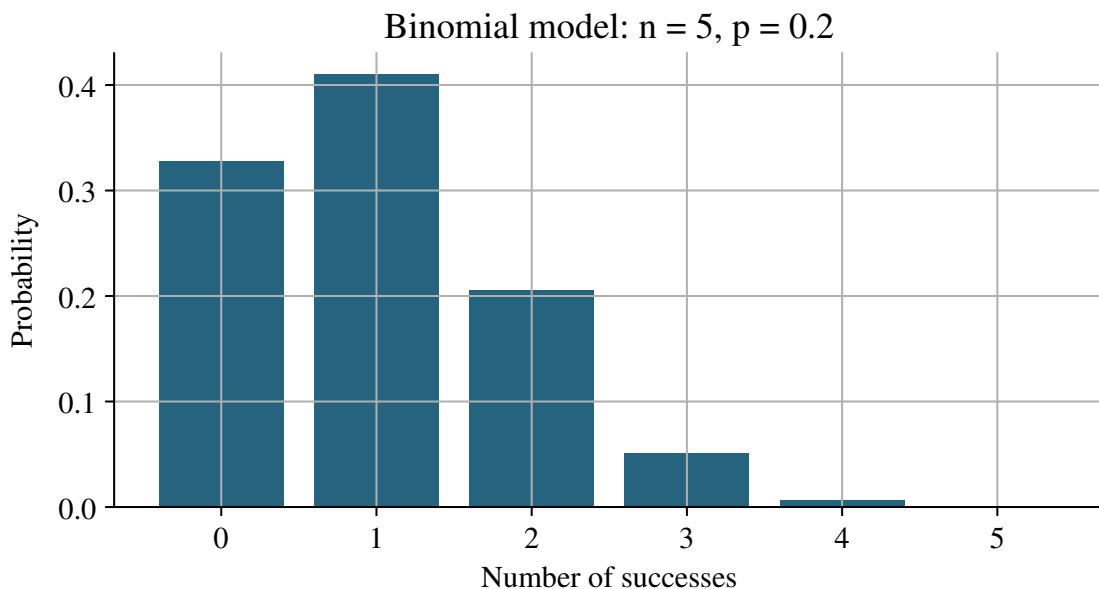
Work through a checked case

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad X \sim B(n, p)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For X binomial(5,0.2), $P(X=0)=0.8^5=0.32768$, $E(X)=1$ and $\text{Var}(X)=0.8$. For a normal quantity with $\mu=100, \sigma=15$, the value 130 has $z=2$. A Poisson mean of 3 per hour gives mean 6 over two hours.



Test a tempting shortcut

- Not every count is binomial: changing p or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every count of successes has a binomial distribution regardless of dependence. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Write the event as an inequality before using calculator distribution functions. Distinguish $P(X < k)$, $P(X \leq k)$ and a tail complement. State assumptions in context.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The probability-weighted mean of a random variable. Choose the relationship, show the method, check its assumptions and interpret the result.

S2 · Sampling and a large data set

Who is missing from the data?

- A weather database has a missing entry and several stations. Treating each row as identical can distort a comparison.
 - This lesson studies **sampling frame** 抽样框: The list or population definition from which a sample is selected.
-

Choose the mathematical structure

- Identify the population, sampling unit and frame. Distinguish random, systematic, stratified, quota and opportunity sampling. Missing data is not zero; verify units, dates and variable definitions before comparing samples.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

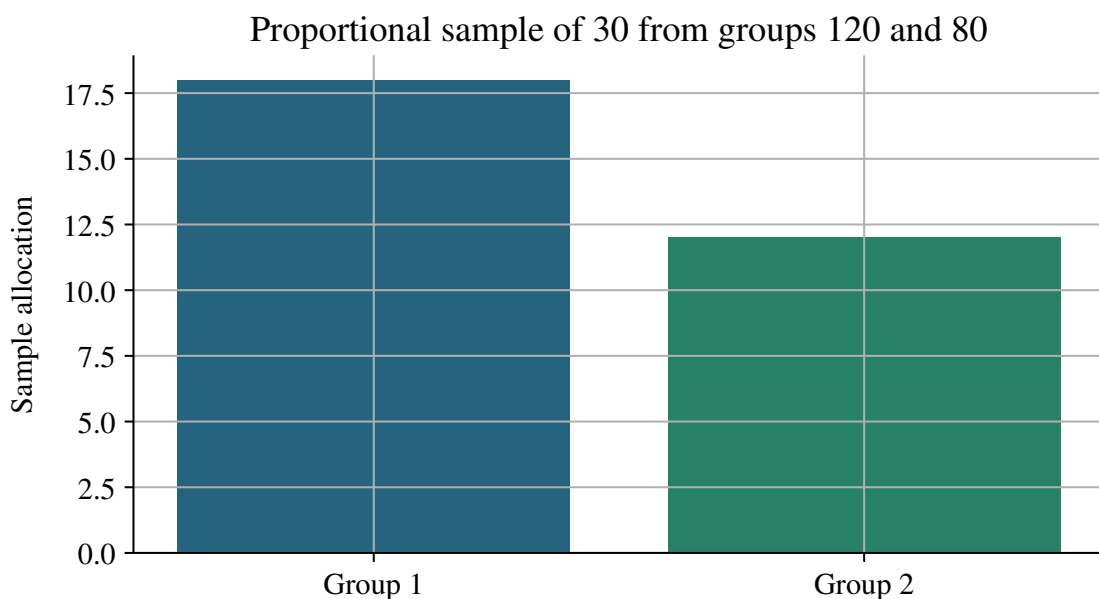
Work through a checked case

$$n_i = n \frac{N_i}{N}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A population has 120 students in one group and 80 in another. A proportional stratified sample of 30 needs $30 \times 120 / 200 = 18$ from the first and 12 from the second. Random selection is then needed within each group.



Test a tempting shortcut

- A large biased sample remains biased. Stratification is not the same as selecting whoever is available from each group. AQA large-data-set familiarity requires the actual supplied data and metadata, not invented weather values.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Doubling a biased sample automatically removes its selection bias. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For AQA, use the official large data set in a supervised spreadsheet task: identify a variable, justify a comparison, inspect missing values, create a display and explain a limitation. Save the decisions with the analysis.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The list or population definition from which a sample is selected. Choose the relationship, show the method, check its assumptions and interpret the result.

S2 · Hypothesis testing and contextual conclusions

Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
 - This lesson studies **significance level** 显著性水平: The chosen probability threshold for rejecting a null hypothesis.
-

Choose the mathematical structure

- State H_0 and H_1 in population parameters before inspecting the outcome. Calculate the appropriate tail probability under H_0 . Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

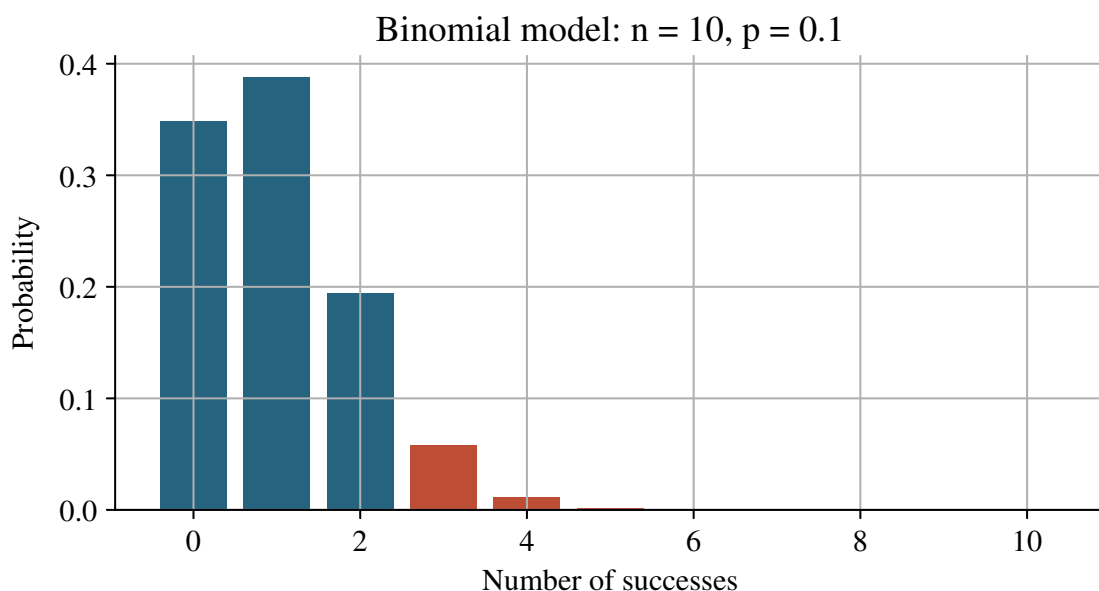
Work through a checked case

$$H_0 : p = p_0, \quad H_1 : p > p_0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $H_0: p=0.1$ and $H_1: p>0.1$ with $n=10$, observing at least 3 defects has probability $1-(0.9^{10}+10 \times 0.1 \times 0.9^9+45 \times 0.1^2 \times 0.9^8) \approx 0.070191$. At 5%, this is insufficient evidence that the defect rate exceeds 10%.



Test a tempting shortcut

- Failing to reject H_0 is not proof that H_0 is true. Choose the tail from H_1 , not from whichever tail gives a small result. Statistical significance does not measure the practical size of an effect.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Failing to reject a null hypothesis proves it is true. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Finish with a sentence about the population and the original claim. State the model's assumptions and consider whether the sampling procedure supports them.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The chosen probability threshold for rejecting a null hypothesis. Choose the relationship, show the method, check its assumptions and interpret the result.

S2 · Continuous distributions and density functions

Is a height the same as a probability?

- A waiting time can take any value in an interval. Its probability comes from area, not the graph's height at one instant.
 - This lesson studies **probability density** 概率密度: A nonnegative function whose integral over an interval gives its probability.
-

Choose the mathematical structure

- A density f must be nonnegative and integrate to 1 over its support. The cumulative distribution $F(x)$ is the integral up to x . For a continuous variable $P(X=x)=0$ and $E(X)$ is the integral of $xf(x)$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

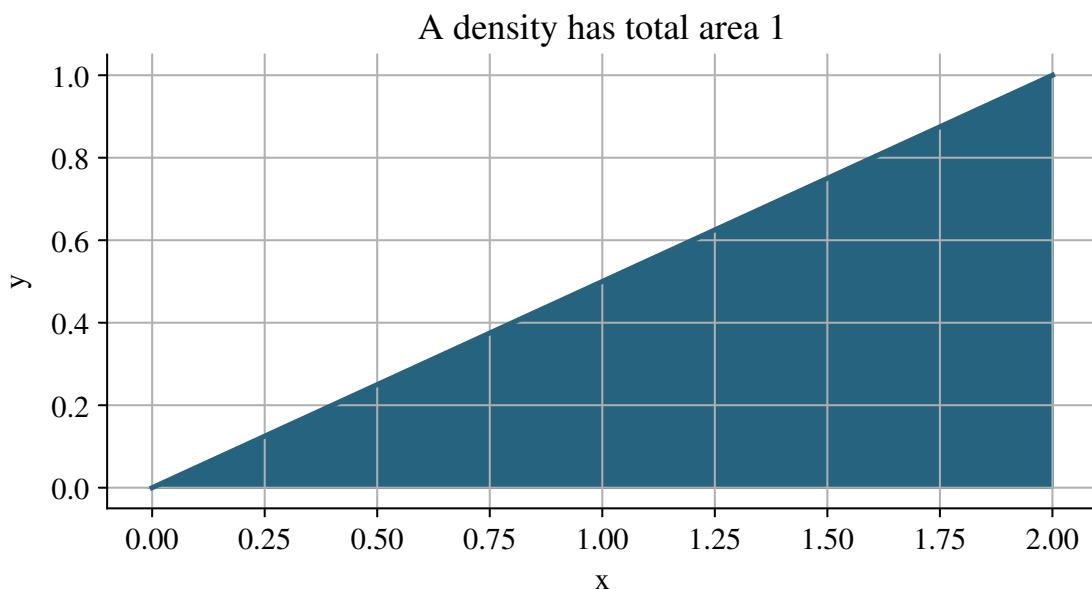
Work through a checked case

$$\int f(x) dx = 1, \quad E(X) = \int xf(x) dx$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Let $f(x)=kx$ on $0 \leq x \leq 2$ and zero elsewhere. Normalisation gives $\int_0^2 kx dx = 2k = 1$, so $k = 1/2$. Then $P(X \leq 1) = \int_0^1 x/2 dx = 1/4$ and $E(X) = \int_0^2 x^2/2 dx = 4/3$.



Test a tempting shortcut

- Density can exceed 1 without being invalid; total area must equal 1. A cumulative distribution cannot decrease. Include the zero-density region outside the support when defining a complete model.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A continuous variable has $P(X=x)$ equal to the density height $f(x)$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a uniform distribution, density is reciprocal interval length. Find medians or percentiles using the cumulative probability, and use a support-aware integral for moments.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A nonnegative function whose integral over an interval gives its probability. Choose the relationship, show the method, check its assumptions and interpret the result.

S3 · Confidence intervals, chi-square and regression

How uncertain is the sample mean?

- Two samples give different means. We need a statement about sampling uncertainty, not a promise that one sample equals the population.

- This lesson studies **confidence interval** 置信区间: An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate.
-

Choose the mathematical structure

- For a normal mean with known σ , a 95% interval is sample mean $\pm 1.96\sigma/\sqrt{n}$. For chi-square, sum (observed-expected)²/expected with the correct degrees of freedom. Choose a test that fits the data and assumptions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

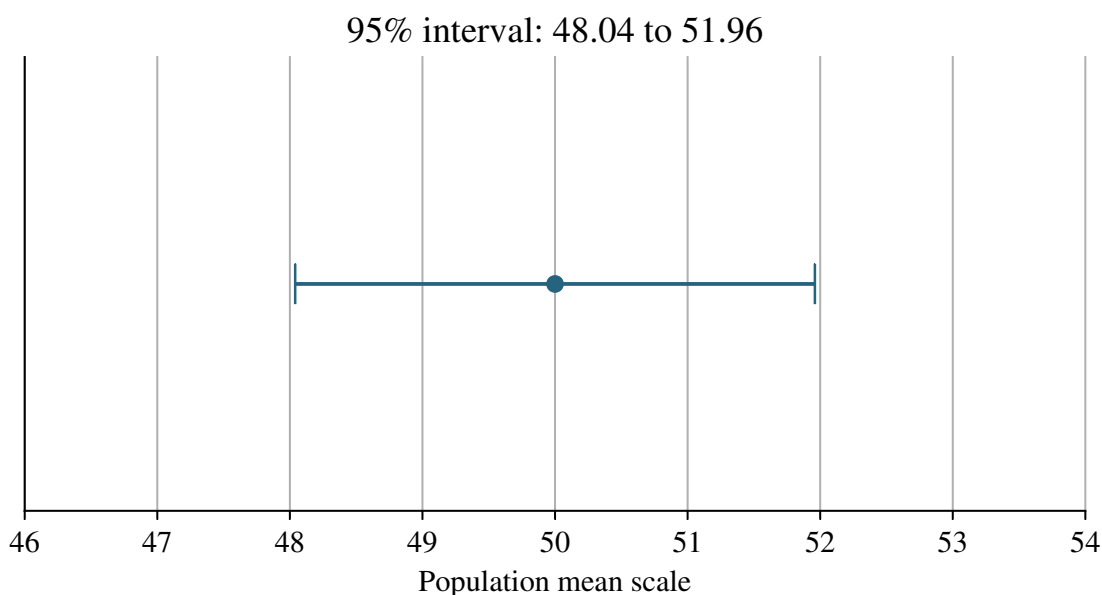
Work through a checked case

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}, \quad \chi^2 = \sum \frac{(O - E)^2}{E}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With mean=50, $\sigma=10$, $n=100$, standard error=1 and the 95% interval is [48.04, 51.96]. For observed counts 30 and 20 against expected 25 and 25, chi-square = $25/25 + 25/25 = 2$, before deciding degrees of freedom and the rejection threshold.



Test a tempting shortcut

- A 95% confidence procedure does not give a 95% probability that a fixed population mean moves within this one interval. Expected counts and estimated parameters affect the validity and degrees of freedom of chi-square tests.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A strong correlation alone establishes a causal relationship. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Report uncertainty in the context. In regression, inspect residuals, avoid unjustified extrapolation, and distinguish a fitted relationship from a causal explanation. S3 and IB AI HL require different verified scope boundaries.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate. Choose the relationship, show the method, check its assumptions and interpret the result.

S3 · Sampling and a large data set

Who is missing from the data?

- A weather database has a missing entry and several stations. Treating each row as identical can distort a comparison.
- This lesson studies **sampling frame** 抽样框: The list or population definition from which a sample is selected.

Choose the mathematical structure

- Identify the population, sampling unit and frame. Distinguish random, systematic, stratified, quota and opportunity sampling. Missing data is not zero; verify units, dates and variable definitions before comparing samples.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

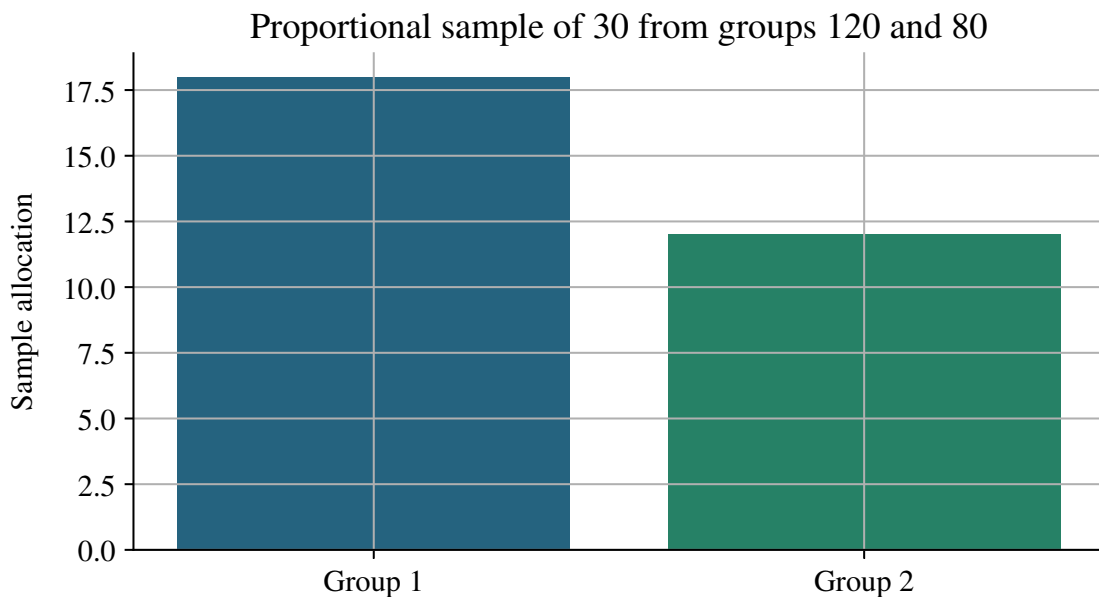
Work through a checked case

$$n_i = n \frac{N_i}{N}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A population has 120 students in one group and 80 in another. A proportional stratified sample of 30 needs $30 \times 120 / 200 = 18$ from the first and 12 from the second. Random selection is then needed within each group.



Test a tempting shortcut

- A large biased sample remains biased. Stratification is not the same as selecting whoever is available from each group. AQA large-data-set familiarity requires the actual supplied data and metadata, not invented weather values.
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Watch out

Doubling a biased sample automatically removes its selection bias. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For AQA, use the official large data set in a supervised spreadsheet task: identify a variable, justify a comparison, inspect missing values, create a display and explain a limitation. Save the decisions with the analysis.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The list or population definition from which a sample is selected. Choose the relationship, show the method, check its assumptions and interpret the result.

S3 · Regression, financial models and residuals

Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
 - This lesson studies **residual** 残差: The observed value minus the value predicted by a fitted model.
-

Choose the mathematical structure

- A linear model $y=a+bx$ has intercept a and slope b with contextual units. Inspect residuals and the data range. For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

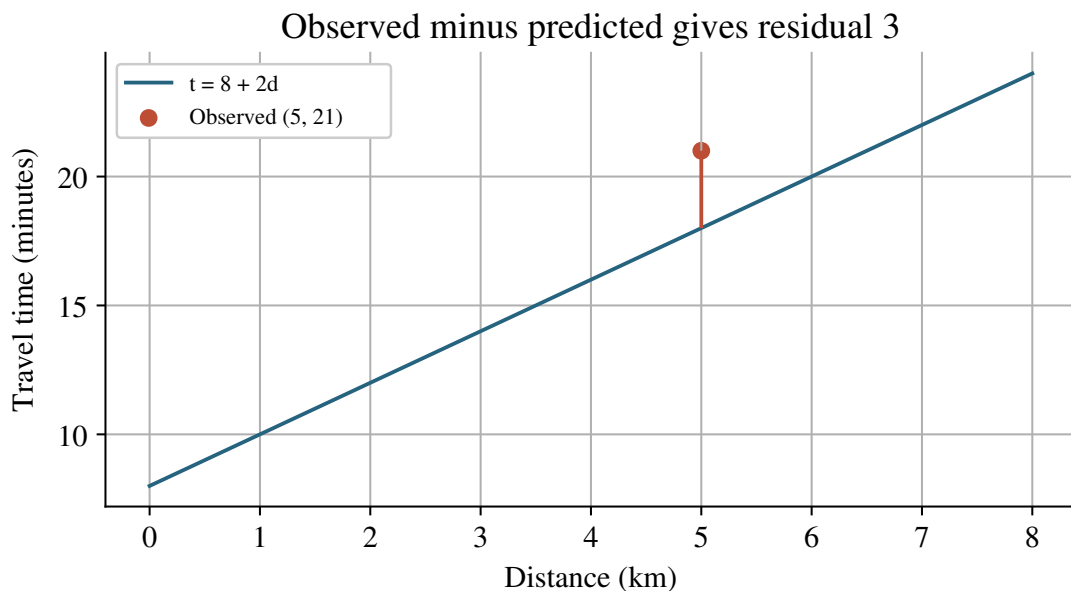
Work through a checked case

$$\hat{t} = 8 + 2d, \quad e = t - \hat{t}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The fitted model $t=8+2d$ predicts 18 minutes at $d=5$ km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes $1000 \times 1.05^3 = 1157.625$ after 3 years.



Test a tempting shortcut

- A good fit does not prove a causal mechanism. A correlation coefficient measures linear association, not the gradient. Calculator output must be translated into a model, checked and interpreted.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A high correlation permits reliable extrapolation to any distance. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- In IB AI, record the data source, domain, assumptions and calculator method. Compare an alternative model and judge predictions against residuals. A financial answer must state payment timing and whether interest is compounded.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit S3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The observed value minus the value predicted by a fitted model. Choose the relationship, show the method, check its assumptions and interpret the result.

D1 · Algorithms, networks and linear programming

Cheapest network or shortest route?

- A school must connect buildings with cable. The shortest route between two buildings and the cheapest whole network are different problems.
 - This lesson studies **minimum spanning tree** 最小生成树: A connected cycle-free network joining every vertex with the smallest possible total edge weight.
-

Choose the mathematical structure

- Kruskal selects edges in increasing weight while avoiding cycles. Dijkstra updates shortest tentative distances from a start. Linear programming optimizes a linear objective over a feasible region; inspect vertices and integer restrictions when required.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

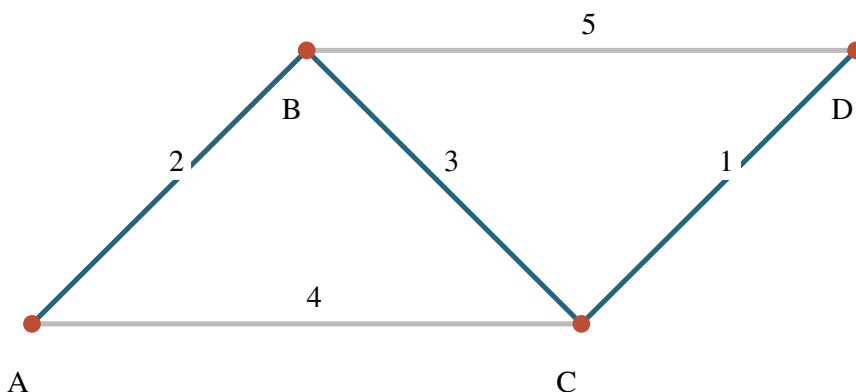
$$Z = 3x + 2y, \quad x + y \leq 4, \quad x \leq 2, \quad x, y \geq 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For vertices A,B,C,D and edges AB=2,BC=3,AC=4,CD=1,BD=5, Kruskal selects CD,AB,BC for total 6. The shortest A-to-D route is A-B-C-D, also 6, but that equality is incidental. Maximize $3x+2y$ with $x+y \leq 4, x \leq 2, x, y \geq 0$: the best vertex is (2,2), value 10.

Minimum spanning tree: weight 6



Test a tempting shortcut

- A spanning tree has no cycles and joins all vertices. The largest single edge is not automatically excluded from every optimal solution. A shortest-path algorithm cannot replace a minimum-spanning-tree algorithm.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A shortest path between two vertices must also be a minimum spanning tree of the whole network. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For critical paths, calculate earliest and latest event times and identify zero-float activities. State units and interpret the optimum. The existing archive has no D1 pairs, so these original tasks do not establish a reviewed D1 past-paper bank.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit D1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A connected cycle-free network joining every vertex with the smallest possible total edge weight. Choose the relationship, show the method, check its assumptions and interpret the result.

D1 · Critical paths and scheduling

Must every activity wait for every other?

- A school event has activities that can run in parallel. Adding every duration overestimates the minimum completion time.
 - This lesson studies **float** 浮动时间: The time an activity can be delayed without delaying completion under the network model.
-

Choose the mathematical structure

- For an activity network, calculate earliest event times forward and latest event times backward. Total float for activity i to j is $L_j - E_i - \text{duration}$. A critical activity has zero total float; more than one critical path may exist.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

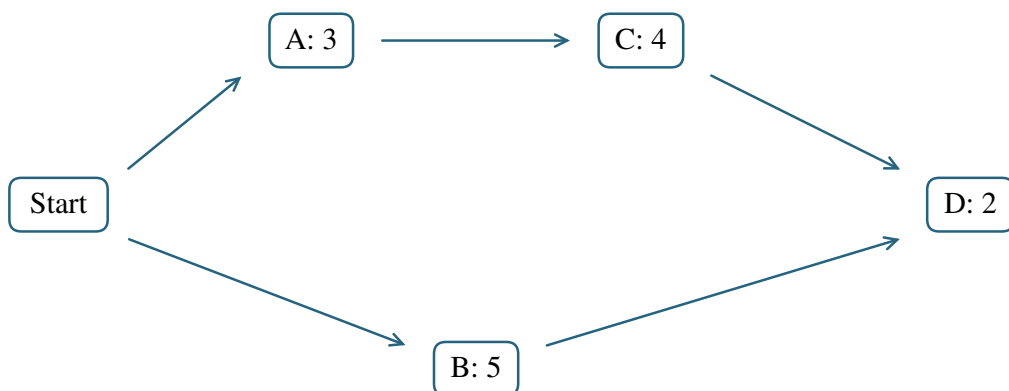
$$F_{ij} = L_j - E_i - d_{ij}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Activities A=3 and B=5 start together. C=4 follows A, and D=2 follows both B and C. The earliest completion of C is 7; D must wait until $\max(5,7)=7$ and finishes at 9. Path A-C-D is critical; B has float 2.

Parallel activities meet before D starts



Test a tempting shortcut

- Do not add durations of independent parallel activities. A zero-float activity belongs to a critical path, but there may be several such paths. Resource limits can require a schedule longer than the network's theoretical minimum.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The minimum project duration is always the sum of every activity duration. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Draw precedence relationships before assigning times. Distinguish activity duration from event time. Explain what a delay does to the completion date, and record any resource assumptions.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit D1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The time an activity can be delayed without delaying completion under the network model. Choose the relationship, show the method, check its assumptions and interpret the result.

D1 · Sorting, packing and network algorithms

Is a fast packing method always optimal?

- A packing method quickly fills boxes, but a fast valid arrangement need not use the smallest number of boxes.
 - This lesson studies **algorithm** 算法: A finite set of ordered instructions that solves a defined class of problems.
-

Choose the mathematical structure

- Trace the named algorithm exactly, including its tie rules. In first-fit packing, place each item in the first available bin that can hold it. First-fit decreasing sorts before applying first-fit. A heuristic may be valid without being optimal.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

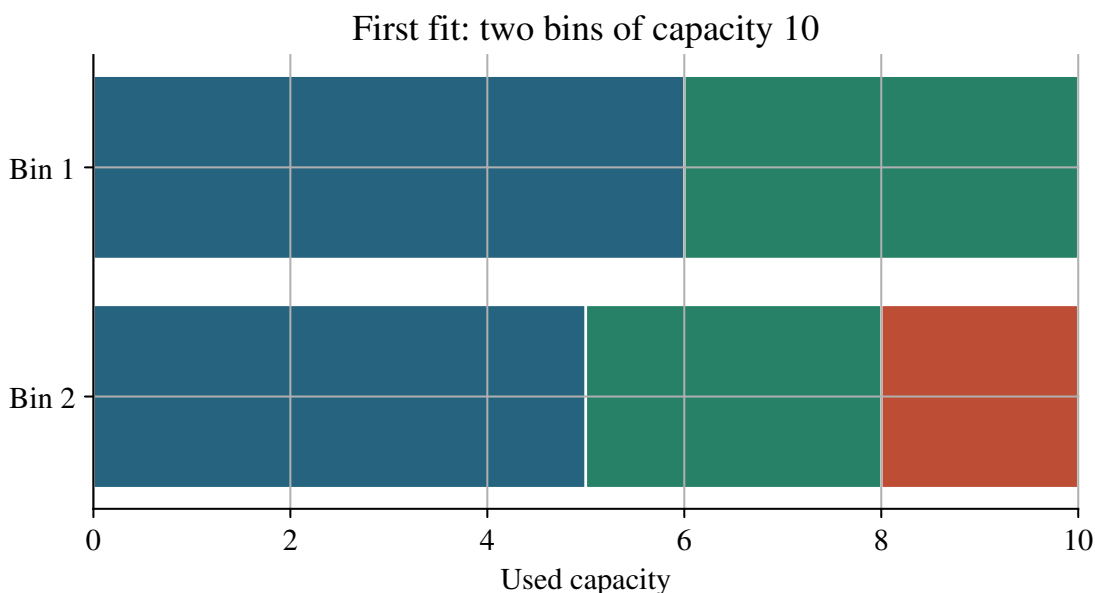
Work through a checked case

$$B_{\min} \geq \left\lceil \frac{\sum s_i}{C} \right\rceil$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With bin capacity 10 and items 6,5,4,3,2 in that order, first-fit places 6 and 4 in bin 1, then 5,3,2 in bin 2. It uses 2 bins. The total size is 20, so the lower bound is $\text{ceil}(20/10)=2$; this arrangement is optimal for this instance.



Test a tempting shortcut

- An example of success does not prove a heuristic is always optimal. Keep intermediate lists in a sorting trace; do not jump from input to a sorted final list. A shortest-path update must retain predecessor information if a route is requested.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A packing heuristic that works well on one example must always be optimal. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For Dijkstra, choose the smallest unsettled tentative label and update its neighbours. For route-inspection problems, distinguish a closed route from an open one and identify odd vertices before pairing them.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL further mathematics; official unit D1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A finite set of ordered instructions that solves a defined class of problems. Choose the relationship, show the method, check its assumptions and interpret the result.