

Geometry and measures – native teaching supplement

AQA · GCSE · Mathematics

Supported teaching and tier boundary

8300: Geometry and measures. Version: Version 1.0, 12 September 2014; first examination 2017.

Foundation teaching and Higher additions are labelled below. This reference packages the existing native-lesson crosswalk. It does not certify unreviewed specification rows or a whole qualification. Original diagnostics are separate and are not reproduced.

■ Angles, lengths and area · Foundation

Use angle facts with a stated reason. Similar shapes have equal corresponding angles and proportional corresponding lengths. Areas of rectangles and triangles come from their dimensions; compound shapes can be split into simpler parts.

$$A_{\text{rectangle}} = LW, \quad A_{\text{triangle}} = \frac{1}{2}bh$$

A rectangle of length 8 cm and width 5 cm has area $A=LW=40 \text{ cm}^2$. A triangle on the same base and height has area $A=bh/2=20 \text{ cm}^2$. For a pentagon, the interior-angle sum is $(5-2) \times 180 = 540^\circ$.

Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

Use a labelled sketch and appropriate units. This Foundation/Core lesson does not test area/volume scale factors or advanced circle-theorem proofs.

■ Angle reasoning, similarity and mensuration · Higher

For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 . State angle reasons explicitly. A circle's tangent is perpendicular to the radius at the contact point.

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

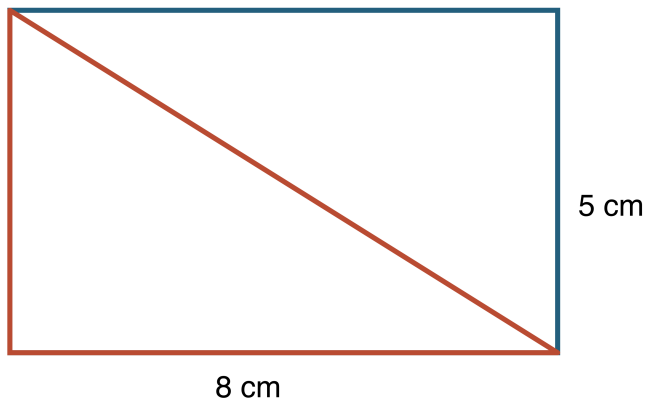
If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108 \text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216 \text{ cm}^3$. A cylinder with $r=3, h=5$ has volume $\pi r^2 h = 45\pi$.

Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion. A scale drawing is evidence only when the task permits measurement.

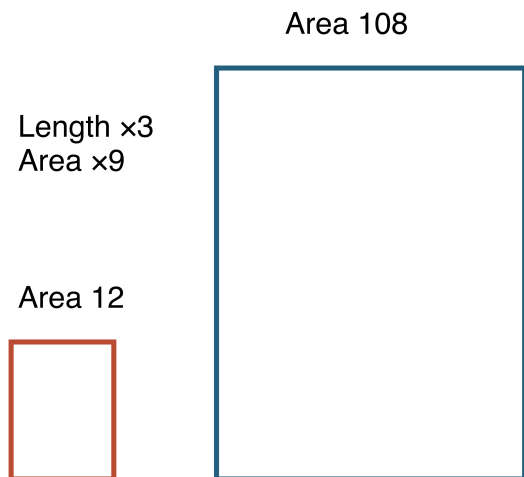
Same base and height, half the area

Rectangle: 40 cm^2 ; triangle: 20 cm^2



Original native-lesson illustration; labels belong to its worked example.

A length scale factor is squared for area



Original native-lesson illustration; labels belong to its worked example.

■ Right-angled triangles in two dimensions · Foundation

Use Pythagoras in a right triangle and use sine, cosine or tangent with the sides labelled relative to the chosen angle.

$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. A right triangle with legs 6 and 8 has area $6 \times 8 / 2 = 24$.

Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.

This Foundation/Core lesson uses right-angled triangles only. Sine and cosine rules for non-right triangles belong to the advanced-tier lesson.

■ Right triangles and non-right triangles · Higher

In a right triangle $a^2 + b^2 = c^2$; $\sin \theta = \text{opposite} / \text{hypotenuse}$, $\cos \theta = \text{adjacent} / \text{hypotenuse}$ and $\tan \theta = \text{opposite} / \text{adjacent}$. For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C / 2$.

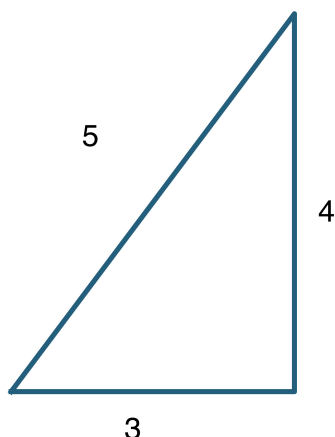
$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.

Use a plan or elevation for a three-dimensional problem before applying a triangle rule. Explain why the chosen triangle contains the required length or angle.

Label the known sides before choosing a rule



Original native-lesson illustration; labels belong to its worked example.

■ Vectors and transformation geometry · Foundation

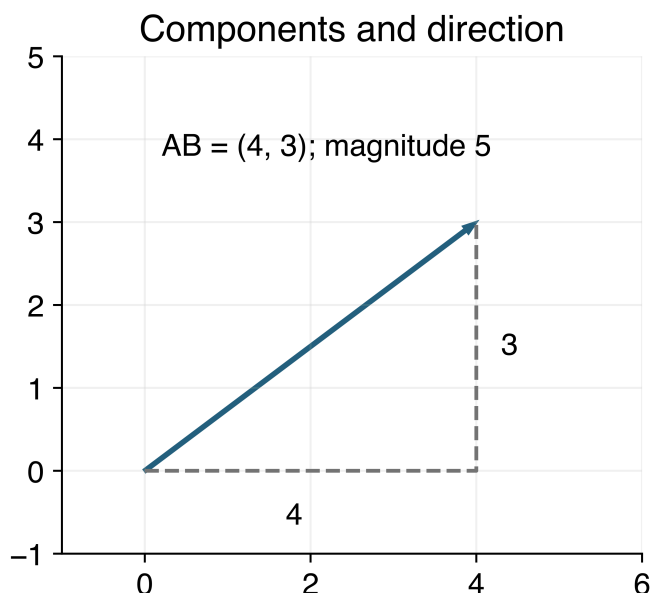
Add corresponding vector components and subtract position vectors to find a displacement. A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

With $a=(4,1)$ and $b=(1,3)$, $a+b=(5,4)$ and $2a-b=(7,-1)$. From $A=(1,2)$ to $B=(5,5)$, $AB=(4,3)$ and its length is 5. Write each 2D vector as a column with the horizontal component above the vertical component: the translation AB has top entry 4 and bottom entry 3. A negative horizontal entry moves left; a negative vertical entry moves down. Drawing b from the head of a gives the head-to-tail diagram for $a+b$. Subtracting b adds its opposite $-b$.

The order of subtraction matters: $BA=-AB$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.

Draw arrows with direction and identify the starting and ending points. Scalar products, spatial line equations and advanced angle calculations are excluded from this Foundation/Core lesson.



Original native-lesson illustration; labels belong to its worked example.

■ Reflections, rotations and enlargements · Foundation

A translation adds a vector. A reflection reverses signed perpendicular distance from a mirror line. A rotation needs a centre, angle and direction. For enlargement from C, use new $P=C+k(P-C)$.

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

For centre $C=(1,1)$, point $P=(3,2)$ and factor 2, the image is $(5,3)$. At factor $1/2$ the image is $C+(1/2)(P-C)=(2,1.5)$, halfway from C to P. Reflecting $(3,2)$ in the y-axis gives $(-3,2)$. Rotating it 90° anticlockwise about the origin gives $(-2,3)$. Translation by $(2,-1)$ gives $(5,1)$. State the centre, line or vector as appropriate.

Specify a reflection line, rotation centre/angle/direction or enlargement centre/factor. Fractional positive enlargement factors reduce a shape without reversing its position about the centre.

AQA G7 Foundation includes single reflections, rotations, translations and enlargements with positive integer/fractional factors. Negative factors and combinations of isometries belong to separate Higher teaching.

■ Reflections, rotations and enlargements · Higher

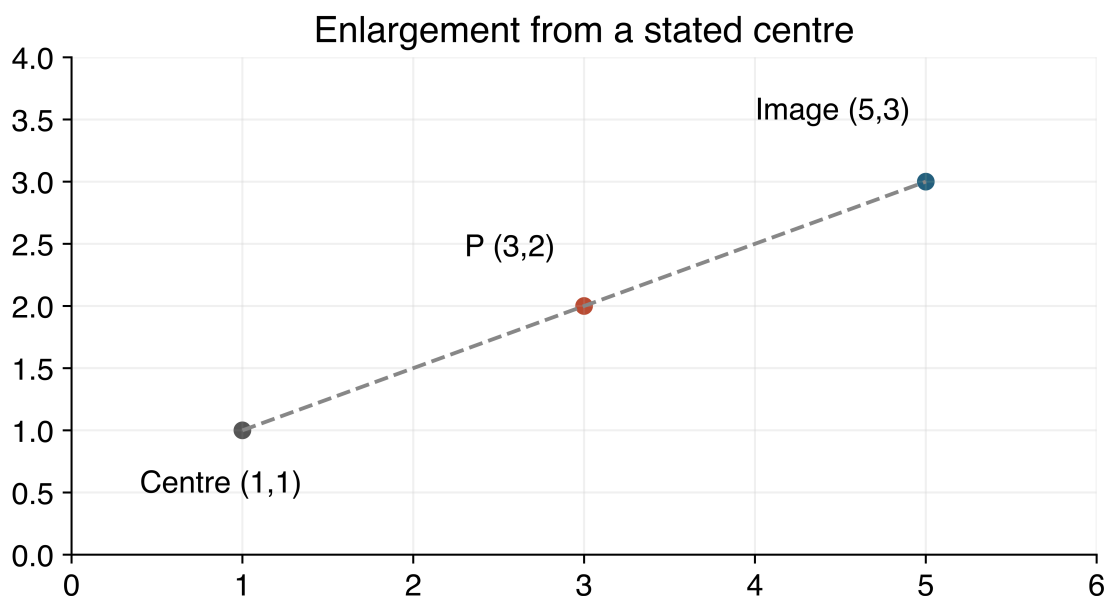
A translation adds a vector. A reflection reverses signed perpendicular distance from a mirror line. A rotation needs a centre, angle and direction. For enlargement from C, use new $P=C+k(P-C)$.

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

For $C=(1,1), P=(3,2), k=2$, the image is $C+2(P-C)=(5,3)$. With $k=1/2$ it is $(2,1.5)$; with $k=-2$ it is $(-3,-1)$, on the opposite side of the centre. Reflecting $(3,2)$ in the y -axis gives $(-3,2)$, and a 90° anticlockwise rotation about the origin gives $(-2,3)$. Identify centre, factor, line, angle and direction as applicable.

A rotation needs its centre and direction, not just an angle. A negative enlargement factor places the image on the opposite side of the centre. A translation does not change orientation or size.

Describe a transformation completely before constructing the image. Check corresponding distances and angles. For combined transformations, apply them in the stated order; they usually do not commute.



Original native-lesson illustration; labels belong to its worked example.

■ Constructions, loci and geometric conditions · Foundation

Points equally distant from A and B lie on the perpendicular bisector of AB. Points at fixed distance r from C lie on a circle. Points equally distant from two intersecting lines lie on their angle bisectors.

$$PA = PB, \quad h^2 + 3^2 = 5^2$$

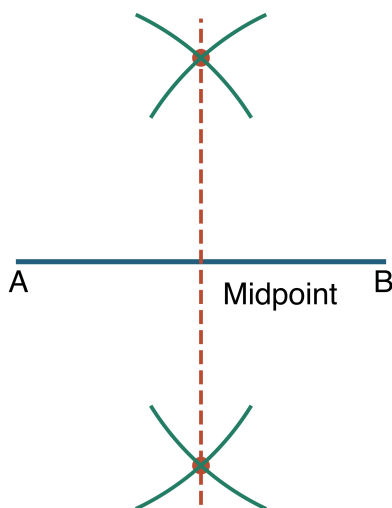
For endpoints A and B 6 cm apart, draw equal-radius arcs above and below AB, with compass opening greater than 3 cm. Join the arc intersections to obtain the perpendicular bisector, crossing AB at its midpoint 3 cm from each end. To construct

a perpendicular from P to a line, use a circle centred at P to mark two line points, then bisect their segment. For a perpendicular at P on the line, mark equal distances on each side of P and use equal arcs. For an angle bisector, draw one vertex-centred arc meeting both arms, then equal arcs from those two points; join their intersection to the vertex. Equal-radius circles centred at the endpoints of a segment construct an equilateral triangle and hence a 60° angle. For a point equally distant from A and B and 5 cm from A, intersect the bisector with a 5 cm circle centred at A. Each intersection is 4 cm perpendicular to AB by a 3–4–5 triangle. The shortest point-to-line distance follows a perpendicular, since any slanted route is a longer hypotenuse.

The perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines. A sketch is not a ruler-and-compass construction: preserve arcs as evidence of the method.

AQA G2 uses ruler-and-compass constructions, including a 60° angle, perpendiculars, bisectors and intersections of loci. Preserve construction arcs and justify the equidistance condition.

Equal compass radii: join the two arc intersections



Original native-lesson illustration; labels belong to its worked example.

■ Circle theorems and reasoned proofs · Higher

The angle at the centre is twice the angle at the circumference on the same arc. Angles in the same segment are equal. Opposite angles of a cyclic quadrilateral sum to 180° . A radius is perpendicular to a tangent at contact.

$$\theta_{\text{centre}} = 2\theta_{\text{circumference}}$$

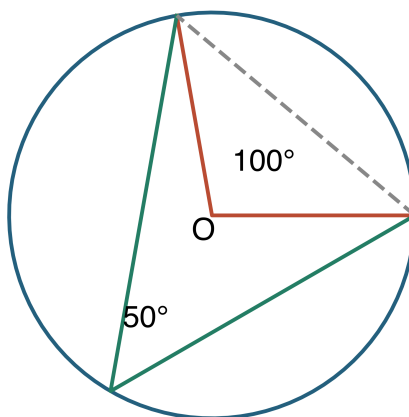
If a central angle is 100° , the corresponding angle at the circumference is 50° . In a

cyclic quadrilateral with one angle 112° , its opposite angle is $180-112=68^\circ$. A radius meeting a tangent gives 90° , even if the drawing looks oblique.

Identify the same chord and the correct arc before using a theorem. Two visible right angles do not prove a quadrilateral cyclic without a valid converse argument. A diagram need not be to scale.

Write one reason alongside each angle calculation. For the alternate-segment theorem, name the tangent and chord, then identify the angle in the opposite segment. Use auxiliary radii only when they help the proof.

The centre angle is twice the circumference angle



Original native-lesson illustration; labels belong to its worked example.

■ Shape vocabulary, properties and symmetry · Foundation

A point marks a position; a line extends in both directions and a segment has two endpoints. A plane is a flat two-dimensional surface. A vertex is a corner; an edge is a boundary segment where solid faces meet. A polygon is a closed plane shape made of straight sides; regular means all sides and all interior angles are equal. Points label vertices; AB names a side and angle ABC has vertex B. Parallel lines have the same direction; perpendicular lines meet at 90° . Reflection symmetry uses a mirror line; rotational symmetry counts matches during one full turn, including 360° .

$$\text{order} = \frac{360^\circ}{\text{smallest matching rotation}}$$

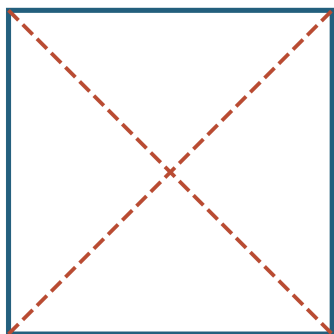
A square has four equal sides, four right angles, four reflection axes and rotational order 4. A non-square rectangle has opposite sides equal, four right angles, two axes and order 2. A non-square rhombus has four equal sides, opposite angles equal, two

diagonal axes and order 2. A parallelogram has two parallel side pairs and opposite angles equal; a general one has no reflection axis. A kite has two pairs of adjacent equal sides; a trapezium has a pair of parallel sides. An equilateral triangle has three equal sides/angles and three reflection axes; an isosceles triangle has two equal sides and equal base angles; scalene means no equal sides. Acute, right and obtuse classify triangles by their largest angle. Pentagons, hexagons, octagons and decagons have 5,6,8,10 sides. A rectangle has equal diagonals that bisect each other. A rhombus has perpendicular bisecting diagonals; a square has both properties. For a parallelogram, a diagonal splits it into triangles: alternate angles on the two parallel side pairs agree and the diagonal is shared, so ASA establishes congruence and opposite sides are equal. Opposite-angle equality also follows from parallel-line angle facts.

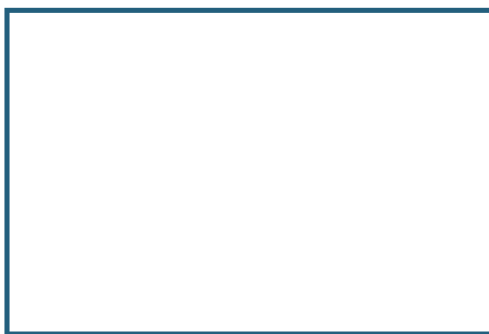
Equal-looking lengths need stated equal-length marks or a deduction. Regular does not mean equal sides alone. An axis of symmetry is a full line, not just an internal diagonal.

AQA G1/G4 requires conventional names, notation and derived shape properties. State the property that justifies a classification and allow overlapping classes, such as square and rectangle.

Square diagonals are symmetry axes



Square



Rectangle

Original native-lesson illustration; labels belong to its worked example.

■ Angle facts and polygon reasoning · Foundation

Angles around one point sum to 360° ; angles on a straight line sum to 180° ; vertically opposite angles are equal. When the crossed lines are parallel, corresponding and alternate angles are equal and co-interior angles sum to 180° . A triangle has angle sum 180° . Split an n-sided polygon into n-2 triangles to derive its interior sum (n-

2) $\times 180^\circ$.

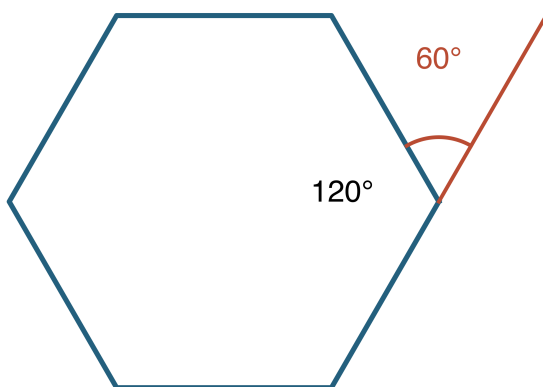
$$S_n = (n - 2)180^\circ, \quad E_{\text{regular}} = \frac{360^\circ}{n}$$

If a straight-line angle is 68° , its neighbour is 112° ; its vertically opposite angle is 68° . A parallel-line corresponding angle is also 68° , with the reason named. Drawing a line through a triangle vertex parallel to the opposite side transfers its two base angles by alternate-angle equality; the three angles then form a straight line and sum to 180° . A pentagon splits into three triangles, giving 540° . A regular hexagon has total 720° , each interior angle 120° and each exterior turn 60° . Exterior turns of a convex polygon sum to one full turn, 360° ; a regular polygon with turn 45° has eight sides.

Corresponding/alternate equalities require parallel lines. Name the theorem rather than using informal letter-shape labels. An interior angle is not the same as an exterior turning angle.

AQA G3/G6 expects reasons and derivations. Mark the given parallelism, identify the angle positions and build a chain of justified equalities rather than reading angles from a sketch.

Regular hexagon: interior 120° , exterior 60°



Original native-lesson illustration; labels belong to its worked example.

■ Triangle congruence and geometric proof · Foundation

Use SSS (three sides), SAS (two sides and their included angle), ASA (two angles and the corresponding side), or RHS (right angle, hypotenuse and one other side). Match vertices in the same order. AAA establishes similarity, not congruence. SSA generally permits more than one triangle. A proof needs a given fact, a valid criterion and a matching-part conclusion.

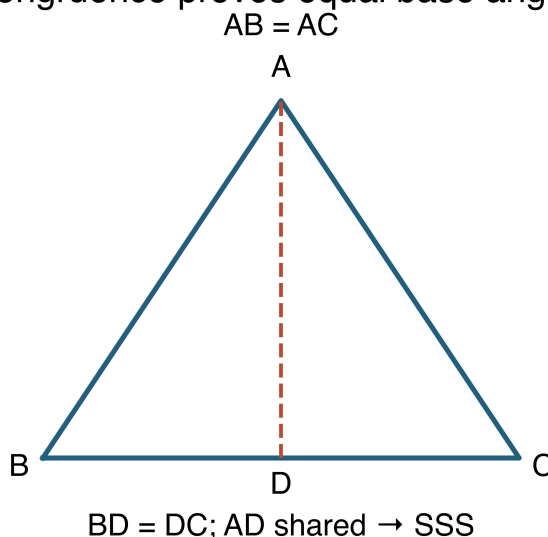
$$(a + b)^2 = 4\frac{ab}{2} + c^2 \implies a^2 + b^2 = c^2$$

For an isosceles triangle ABC with $AB=AC$, let D be the midpoint of BC. Triangles ABD and ACD have $AB=AC$, $BD=DC$ and shared AD, so SSS gives congruence. Matching base angles ABC and BCA are therefore equal; the two angles at D are equal and form 180° , so each is 90° . To derive Pythagoras, arrange four congruent right triangles with legs a,b around a tilted square of side c inside a square of side a+b. Area gives $(a+b)^2=4(ab/2)+c^2$, hence $a^2+b^2=c^2$. For $a=3, b=4$, $c^2=9+16=25$, so $c=5$. These arguments establish results independently of a scale drawing.

The SAS angle must lie between the named sides. RHS uses the hypotenuse, not two arbitrary sides. A proof diagram supports the argument; it cannot establish equality just by appearance.

AQA G5/G6 uses basic congruence criteria and simple geometric proofs, including isosceles base angles and Pythagoras. State every matching pair and explain which criterion applies.

Congruence proves equal base angles



Original native-lesson illustration; labels belong to its worked example.

■ Circle parts and geometric definitions · Foundation

A circle consists of points a fixed radius from its centre. A diameter is a chord through the centre and has length $2r$. A chord joins two circumference points; an arc is part of the circumference. A sector lies between two radii and an arc. A segment lies between a chord and an arc. A tangent touches at one point and is perpendicular to the radius there.

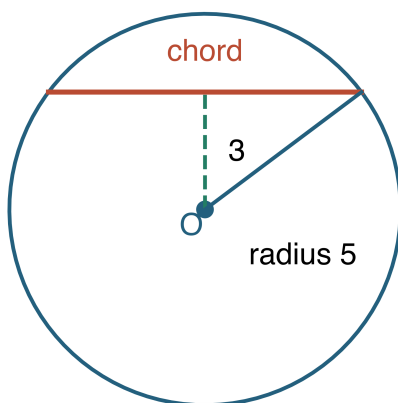
$$d = 2r, \quad \left(\frac{c}{2}\right)^2 + h^2 = r^2$$

For centre O and radius 5 cm, every circumference point is 5 cm from O and the diameter is 10 cm. A chord 3 cm from O has half-length $\sqrt{(5^2-3^2)}=4$ cm, so its whole length is 8 cm. The perpendicular from O meets the chord at its midpoint. Joining the two chord endpoints to O creates a sector; the smaller region between chord and arc is a segment. At the rightmost circumference point, the vertical touching line is tangent and the horizontal radius is perpendicular to it. The circumference is a length, $2\pi r=10\pi$ cm, rather than an area.

A chord need not pass through the centre; only a diameter must. Sector and segment have different straight boundaries. Do not confuse circumference length with the shaded area inside a circle.

AQA G9 includes all named circle parts at Foundation. Use the given radius and position labels to identify the part; Higher circle-theorem proofs are developed separately.

Circle of radius 5: distances use one consistent scale



Perpendicular bisects chord: half-length 4

Original native-lesson illustration; labels belong to its worked example.

■ Geometric reasoning on coordinate axes · Foundation

Find horizontal or vertical length by subtracting coordinates; use Pythagoras for a diagonal. Midpoint coordinates are averages of the endpoints. Equal coordinate changes identify translations. To prove a quadrilateral property, justify side directions and lengths, rather than naming a shape from its appearance.

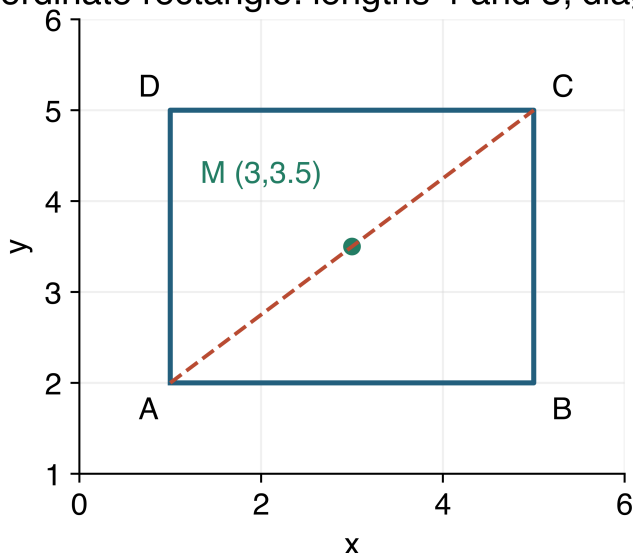
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

For $A=(1,2)$, $B=(5,2)$, $C=(5,5)$, $D=(1,5)$, $AB=4$ and $BC=3$. AB is horizontal, BC vertical, so they are perpendicular; opposite sides have matching directions and lengths, establishing a rectangle. Diagonal AC has length $\sqrt{4^2+3^2}=5$. Its midpoint is $((1+5)/2, (2+5)/2)=(3,3.5)$. Diagonal BD has the same midpoint, confirming that the diagonals bisect each other. A translation by $(2,-1)$ sends A to $(3,1)$ and C to $(7,4)$, preserving the diagonal length. A square would additionally need adjacent side lengths equal; these lengths 4 and 3 exclude it.

A coordinate difference can be negative even though a length is nonnegative. The diagonal is not the sum of the two edge lengths. One pair of equal sides alone does not establish a rectangle.

AQA G11 solves geometric problems on axes. Keep point labels, coordinate arithmetic and geometric reasons linked in the written argument; use the diagram to choose a method, then verify it numerically.

Coordinate rectangle: lengths 4 and 3; diagonal 5



Original native-lesson illustration; labels belong to its worked example.

■ Solid properties, plans and elevations · Foundation

A face is a flat boundary polygon; curved surfaces must be named separately. Edges join faces and vertices are corners. A prism has two congruent parallel end faces and a constant cross-section. A pyramid joins one polygon base to an apex. Plan is the view from above; front and side elevations are direct views without perspective. Give the viewing direction and align corresponding widths, depths and heights.

$$A_{\text{plan}} = wd, \quad A_{\text{front}} = wh$$

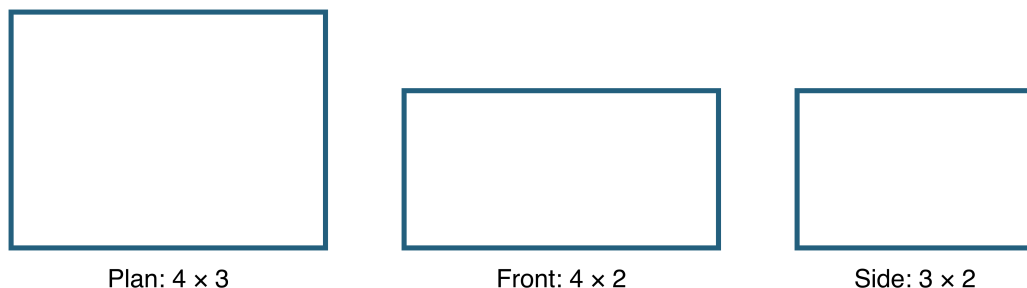
A cube or cuboid has 6 faces, 12 edges and 8 vertices. A triangular prism has 5 faces,

9 edges and 6 vertices; a square pyramid has 5 faces, 8 edges and 5 vertices. A cylinder has two circular flat faces and one curved surface, with no vertices. A cone has one flat circular face, one curved surface and one apex; a sphere has only a curved surface. For a cuboid of width 4, depth 3 and height 2 units, the plan is 4 by 3, front elevation 4 by 2 and side elevation 3 by 2. A stack with front-row column heights 1 and 3, and back-row heights 2 and 1, occupies four cells. The front elevation has column maxima 2 and 3; the side elevation has depth-row maxima 3 and 2. These views do not determine every hidden cube uniquely.

Do not draw perspective diagonals in an orthographic elevation. A plan alone cannot give height. Different hidden arrangements can share the same plan and elevations; do not claim a unique reconstruction without enough information.

AQA G12/G13 covers the named solids and construction/interpretation of views. Label dimensions and directions, preserve alignment between views and explain any hidden-space ambiguity.

Cuboid width 4, depth 3, height 2



Original native-lesson illustration; labels belong to its worked example.

■ Measuring angles and three-figure bearings · Foundation

Draw a north line at the starting point, then measure clockwise to the route. Write three digits: east 090°, south 180°, west 270°, north 000°. NE, SE, SW and NW are 045°, 135°, 225°, 315°. For the reverse bearing add 180° and reduce modulo 360°. Use a ruler for a stated-scale length and the correct protractor scale for a stated angle.

$$b_{\text{reverse}} = (b + 180^\circ) \bmod 360^\circ$$

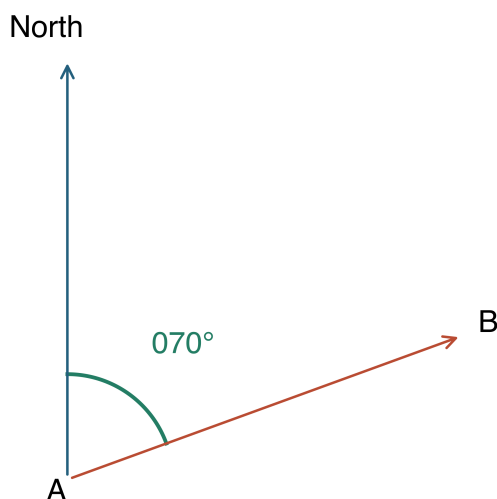
From A to B the bearing 070° is 70° clockwise from north. From B to A it is

$070+180=250^\circ$. A bearing of 320° reverses to $500-360=140^\circ$. East is 090° , not 90 without its three-digit form. A route bearing 120° points southeast of the starting point, making 30° below east. On a 1:10000 map, a 3 cm route represents 300 m; to construct bearing 120° , place the protractor centre at the route start, align its zero with north and measure clockwise. Keep the ruler scale and angular direction as separate decisions.

A bearing is measured at the departure point, not the destination. Read clockwise from north rather than the acute angle to the nearest compass axis. The back bearing differs by 180° , even when the diagram is oblique.

AQA G15 includes measured lengths/angles, maps, the eight compass directions and three-figure bearings. State which point supplies north and distinguish a numerical bearing from a measured scale distance.

Clockwise from north at A; reverse bearing at B is 250°



Original native-lesson illustration; labels belong to its worked example.

■ Combined isometries and invariants · Higher

Apply each transformation to the current image in the stated order. Rotations, reflections and translations preserve lengths and angles, so their combinations also preserve them. Reflections reverse orientation; rotations and translations preserve it. Two reflections in parallel lines give a translation; in intersecting lines they give a rotation through twice the directed angle between the mirrors.

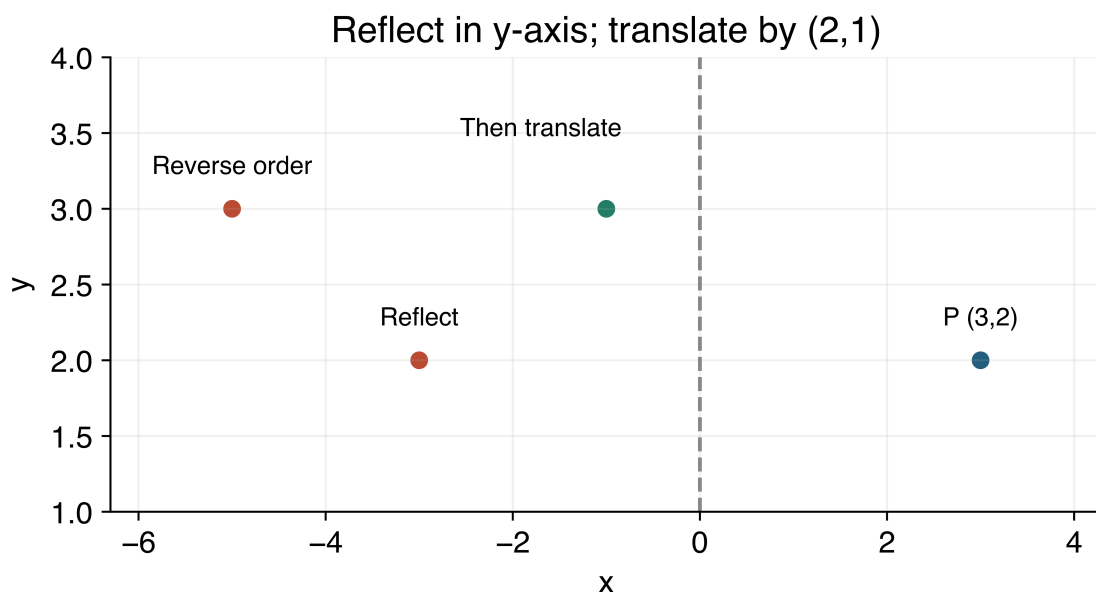
$$T \circ R \neq R \circ T \quad \text{in general}$$

Start with $P=(3,2)$. Reflect in the y-axis to get $(-3,2)$, then translate by $(2,1)$ to get $(-1,3)$. Reversing the order gives $(5,3)$ then $(-5,3)$, a different point. Reflecting in $x=0$

followed by $x=2$ maps $(3,2)$ to $(-3,2)$ then $(7,2)$, equivalent to translation by $(4,0)$. Two reflections reverse orientation twice, restoring it. Reflecting in the x -axis then y -axis sends $(3,2)$ to $(-3,-2)$, a 180° rotation about the origin. Every pairwise length and angle is unchanged, but position generally changes.

Preserved length does not mean every point stays fixed. Do not commute transformations unless a checked argument permits it. A single reflection reverses orientation, while two reflections restore it.

AQA G8 Higher requires changes and invariance under combinations of rigid transformations. Describe the resulting map completely and test it on more than one point before making a whole-shape claim.



Original native-lesson illustration; labels belong to its worked example.

■ Circle angles and proof chains · Higher

The central angle on an arc is twice a circumference angle standing on that same arc. A diameter therefore gives a 90° circumference angle. Angles in the same segment are equal. Opposite angles of a cyclic quadrilateral sum to 180° . State the chord, arc and segment before applying a theorem.

$$\angle AOB = 2\angle ACB, \quad \angle A + \angle C = 180^\circ$$

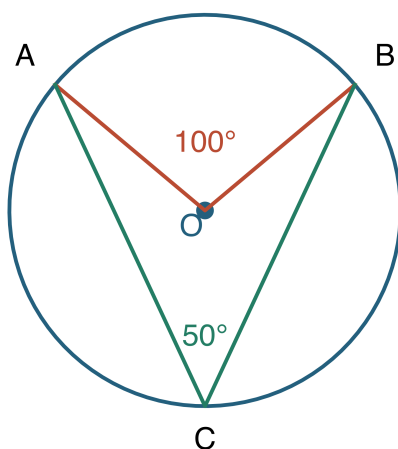
Let A, B, C lie on the circle and let O be the centre inside angle ACB . Put $\alpha = \text{angle } ACO$ and $\beta = \text{angle } OCB$. Equal radii make triangles AOC and BOC isosceles. Thus angle $AOC = 180 - 2\alpha$ and angle $COB = 180 - 2\beta$. Angles around O give the remaining angle $AOB = 360 - (180 - 2\alpha) - (180 - 2\beta) = 2(\alpha + \beta) = 2 \text{ angle } ACB$. Other centre positions need the corresponding subtraction of isosceles angles, with the same result for the chosen arc.

If AB is a diameter, angle AOB=180°, so angle ACB=90°. Two circumference angles on the same chord in the same segment each equal half the same central angle, so they agree. For opposite cyclic angles, their arcs together make 360°; half-arc angles therefore sum to 180°. A central angle 100° gives 50° at the circumference; an angle opposite 112° in a cyclic quadrilateral is 68°.

Distinguish the reflex central angle from the smaller one and identify which arc does not contain the circumference vertex. Opposite segments can give supplementary rather than equal angles. A theorem must be tied to named points.

AQA G10 Higher requires application and proof. Draw auxiliary radii, use isosceles base angles and point sums, then extend the proof to the intended configuration rather than inferring equality from the drawing.

Angles on the same upper arc: centre = twice circumference



Original native-lesson illustration; labels belong to its worked example.

■ Tangents, chords and the alternate segment · Higher

A tangent is perpendicular to its contact radius. Tangents from one external point are equal. The perpendicular from the centre to a chord bisects it. The tangent–chord angle equals the angle on that chord in the alternate segment. Use equal radii, right triangles and the central-angle theorem to prove these relationships.

$$PA = PB, \quad OM \perp AB \implies AM = MB$$

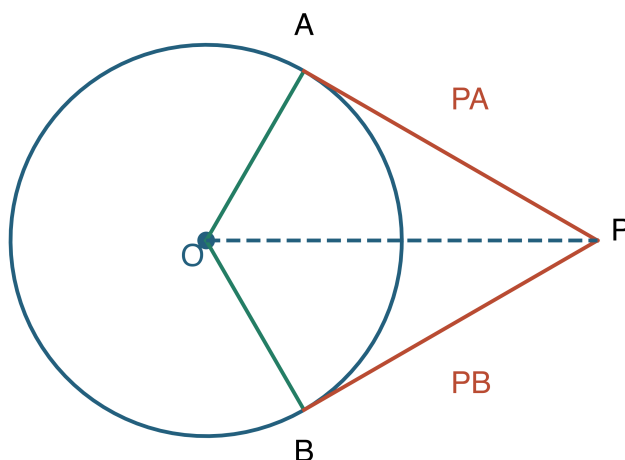
At contact T, the radius OT is perpendicular to the tangent: a non-perpendicular line through T would have a smaller centre-to-line distance than the radius and cut the circle twice. For tangents PA and PB, OA=OB, OP is shared and both contact angles are 90°. RHS makes OAP and OBP congruent, so PA=PB. For a centre perpendicular OM

to chord AB, $OA=OB$ and OM is shared; RHS gives $AM=MB$. If $OA=5$ and $OM=3$, $AM=4$, hence $AB=8$. For a chord AB with minor central angle φ , triangle OAB has base angle $(180-\varphi)/2$. The adjacent tangent–chord angle is $90-(180-\varphi)/2=\varphi/2$, equal to the circumference angle in the alternate segment. Thus a tangent–chord angle of 35° gives 35° in that segment. Reflex/supplementary configurations require the matching arc and angle.

A tangent–chord angle and a radius–chord angle are different. Equal tangent lengths refer to one external point. The chord is bisected by a perpendicular from the centre, not by any line that happens to cross it.

AQA G10 Higher includes these theorem proofs and related results. Give the congruence criterion or angle chain explicitly and identify the relevant chord, contact point and alternate segment.

Shared OP and equal radii $\rightarrow$ RHS $\rightarrow PA = PB$



Original native-lesson illustration; labels belong to its worked example.

■ Areas, perimeters and composite plane shapes · Foundation

Triangle area is $bh/2$, parallelogram area bh and trapezium area $(a+b)h/2$ for parallel sides a, b . Heights are perpendicular to the selected base. Perimeter adds only the outside boundary. Split a composite shape into non-overlapping parts, or subtract a missing region from a containing shape.

$$A_{\triangle} = \frac{1}{2}bh, \quad A_{\text{trap}} = \frac{1}{2}(a+b)h$$

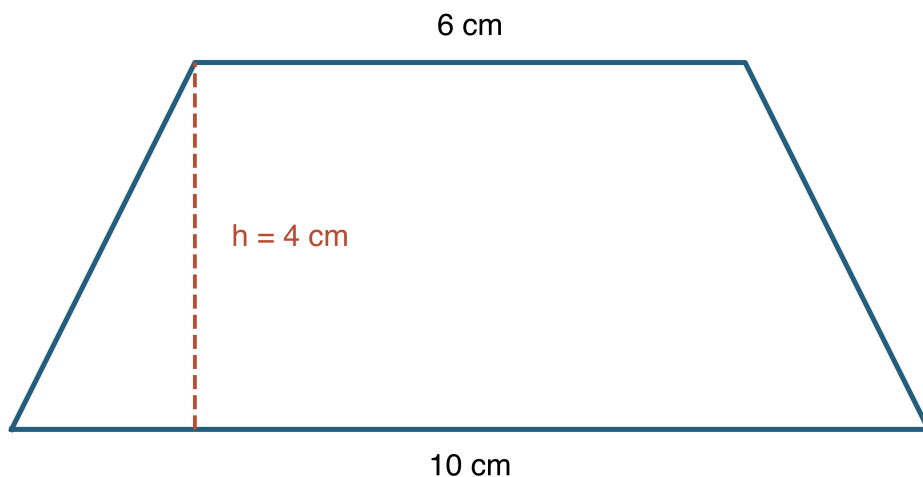
A triangle with base 8 cm and height 5 cm has area 20 cm^2 ; a parallelogram with the same base/height has area 40 cm^2 . A trapezium with parallel sides 6 and 10 cm and

height 4 cm has area $(6+10) \times 4/2 = 32 \text{ cm}^2$. A rectangular 8 by 6 cm sheet with a 3 by 2 cm corner removed has area $48-6=42 \text{ cm}^2$. Its perimeter is still 28 cm: the two removed outside segments total 5 cm and the two new notch edges also total 5 cm. This perimeter equality depends on a corner rectangular cut; an internal hole adds a separate boundary. Rearranging triangle area gives $h=2A/b$.

Use perpendicular height rather than a sloping side. A shared internal division line is not part of the perimeter. An area answer has squared units; a perimeter answer has length units.

AQA G16/G17 includes triangle/parallelogram/trapezium area and composite perimeter/area. Mark the parallel bases, perpendicular height and counted outside edges before calculating.

Trapezium area = average parallel sides $\times$ height



Original native-lesson illustration; labels belong to its worked example.

■ Prism volume and cylinder measurement · Foundation

For a right prism, volume is constant cross-sectional area times perpendicular length. Cuboid volume is lwh . A cylinder is a circular prism: $V = \pi r^2 h$. Total closed-cylinder surface area includes two circular ends and the curved surface $2\pi rh$. Open containers omit the specified faces.

$$V = A_{\text{cross}} L, \quad V_{\text{cylinder}} = \pi r^2 h$$

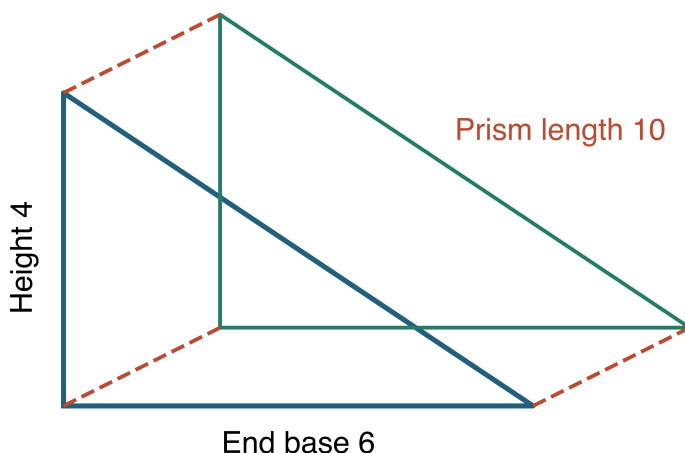
A triangular prism with end base 6 cm, end height 4 cm and length 10 cm has cross-section 12 cm^2 and volume 120 cm^3 . A cylinder of radius 3 cm and height 5 cm has volume $45\pi \text{ cm}^3$. Its curved surface unwraps to a rectangle of width $2\pi r = 6\pi$ and height 5, so curved area is $30\pi \text{ cm}^2$. Adding two ends gives total area $30\pi + 18\pi = 48\pi$

cm². An open-top tank of those dimensions has surface area 39π cm² because it keeps only one end. If a prism has volume 180 cm³ and cross-section 15 cm², its length is 12 cm. Convert 2000 cm³ to 2 litres, keeping volume conversion separate from surface area.

Use cross-sectional area, not perimeter, in the volume formula. Distinguish cylinder radius from diameter and identify whether end faces are present. A length times an area gives cubic units.

AQA G16/G17 includes cuboids, right prisms and cylinders. Draw the constant end shape, calculate it first and label the extrusion length. Preserve exact multiples of pi when asked.

Constant triangular end area 12 → volume 120



Original native-lesson illustration; labels belong to its worked example.

■ Circle lengths, areas and composite boundaries · Foundation

Circle circumference is $2\pi r = \pi d$ and area is πr^2 . A semicircle has half the circle area, but its complete perimeter includes the diameter as well as half the circumference. For composite shapes count every exposed boundary once, and separate straight edges from arcs.

$$C = 2\pi r, \quad A = \pi r^2, \quad P_{\text{semi}} = \pi r + 2r$$

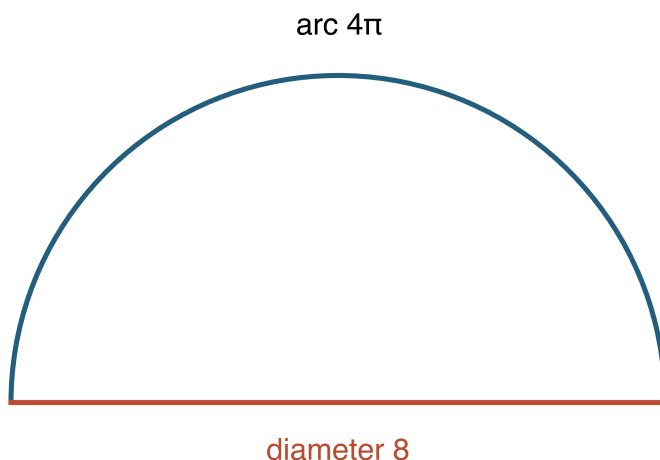
For radius 4 cm, circumference is 8π cm and area 16π cm². A semicircle of that radius has area 8π cm² and perimeter $4\pi + 8$ cm. A rectangular 8 by 3 cm window topped by this semicircle has area $24 + 8\pi$ cm². Its perimeter is the bottom 8, two vertical sides totalling 6 and the curved top 4π : $14 + 4\pi$ cm. The diameter across the join is internal and is not counted. An annulus with outer radius 5 and inner radius 3 has

area $\pi(25-9)=16\pi \text{ cm}^2$. Its two circular boundaries together have length $10\pi+6\pi=16\pi \text{ cm}$, despite this accidental equality of coefficients; length and area still have different units.

Radius must be halved from a given diameter before squaring. Half a circle's circumference is only its arc, not the complete semicircle perimeter. An internal join is not exposed boundary.

AQA G17 includes exact pi answers and composite circle perimeters/areas. Give both the exact expression and the required final rounded value, keeping the meaning and units clear.

Semicircle perimeter: curved arc + straight diameter



Original native-lesson illustration; labels belong to its worked example.

■ Spheres, cones, pyramids and frustums · Foundation

Pyramid and cone volumes are one third of base area times perpendicular height. A sphere has volume $4\pi r^3/3$ and area $4\pi r^2$. A cone's curved area is πrl using slant height l ; its volume uses perpendicular height h . For a frustum subtract the removed similar solid. Composite surface area counts only exposed faces; joined faces are hidden.

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h, \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

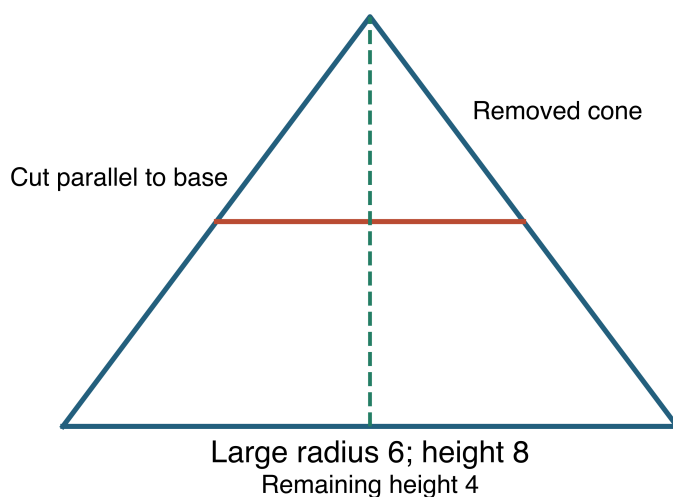
A cone with radius 3 cm and perpendicular height 4 cm has slant height 5 cm. Volume is $12\pi \text{ cm}^3$, curved area $15\pi \text{ cm}^2$ and total closed area $24\pi \text{ cm}^2$. A sphere of radius 3 has volume $36\pi \text{ cm}^3$ and area $36\pi \text{ cm}^2$, with different units. A square pyramid of base side 6 and height 4 has volume $6^2 \times 4/3 = 48 \text{ cm}^3$; each triangular face has slant height $\sqrt{4^2+3^2}=5$, so lateral area is $4 \times (6 \times 5/2) = 60 \text{ cm}^2$ and total area 96 cm^2 . A

large cone $r=6, h=8$ loses a similar top cone $r=3, h=4$: frustum volume is $96\pi - 12\pi = 84\pi \text{ cm}^3$. Its slant height is $10 - 5 = 5$; curved area is $60\pi - 15\pi = 45\pi$, and two circular ends add $36\pi + 9\pi$ for total $90\pi \text{ cm}^2$.

Perpendicular height and slant height are not interchangeable. A frustum is not a full cone of the leftover height. Shared composite faces do not contribute exposed area.

AQA G17 additional Foundation includes spheres, pyramids, cones, composite solids and frustums. Use similarity to find missing removed dimensions, then subtract volumes or exposed areas with matching units.

Frustum volume = whole cone – removed similar cone



Original native-lesson illustration; labels belong to its worked example.

■ Arcs, sectors and reverse angle calculations · Foundation

For an angle θ in degrees, fraction of a turn is $\theta/360$. Arc length is this fraction of $2\pi r$ and sector area is this fraction of πr^2 . Sector perimeter adds the two radii. Rearrange the same fraction to recover an angle from an arc or an area. Keep degree and length units separate.

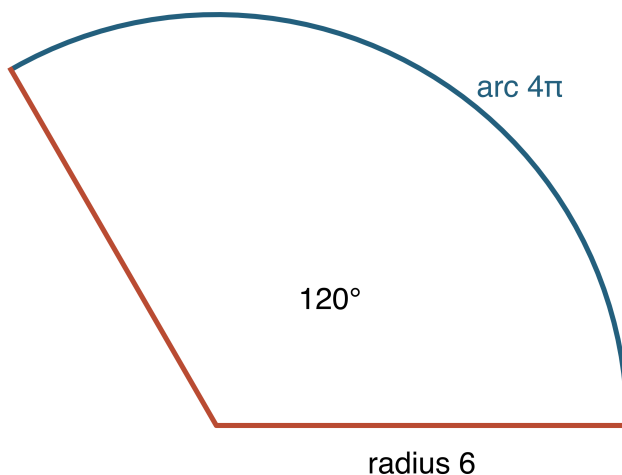
$$s = \frac{\theta}{360^\circ} 2\pi r, \quad A = \frac{\theta}{360^\circ} \pi r^2$$

At $r=6 \text{ cm}$ and $\theta=120^\circ$, the fraction is $1/3$: arc is $4\pi \text{ cm}$, area is $12\pi \text{ cm}^2$ and perimeter is $4\pi + 12 \text{ cm}$. If another $r=6$ sector has area $9\pi \text{ cm}^2$, its fraction is $9\pi/36\pi = 1/4$ and angle is 90° . If its arc instead measures $3\pi \text{ cm}$, its fraction is $3\pi/12\pi = 1/4$, giving the same angle. A full 360° sector has the whole circle area, while the circle boundary has no extra radii. A 60° sector of radius 3 has area $(1/6) \times 9\pi = 1.5\pi \text{ cm}^2$.

An arc length is not a sector perimeter. Use the angle as a fraction of 360° , not 180° . Radius is squared only for area, not arc length.

AQA G18 includes arc lengths, sector angles and areas in degree-based geometry. Show the full-circle quantity and fraction before multiplying, then check that the result fits the angle.

One third of a turn: sector area 12π



Original native-lesson illustration; labels belong to its worked example.

■ Exact trigonometric values from special triangles · Foundation

Bisect an equilateral triangle of side 2 to obtain a 30° – 60° – 90° triangle with sides $1, \sqrt{3}, 2$. A right isosceles triangle has sides $1, 1, \sqrt{2}$. Apply opposite/hypotenuse, adjacent/hypotenuse and opposite/adjacent to derive sine, cosine and tangent. Use degree angles and retain surds exactly; $\tan 90^\circ$ is undefined.

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 45^\circ = \frac{\sqrt{2}}{2}, \quad \tan 60^\circ = \sqrt{3}$$

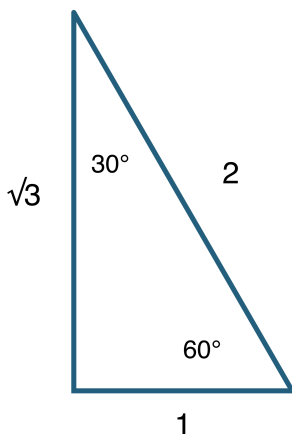
For angles $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$, sine values are $0, 1/2, \sqrt{2}/2, \sqrt{3}/2, 1$; cosine values are $1, \sqrt{3}/2, \sqrt{2}/2, 1/2, 0$. For $0^\circ, 30^\circ, 45^\circ, 60^\circ$, tangent values are $0, 1/\sqrt{3}, 1, \sqrt{3}$. The side 1 opposite 30° in the bisected equilateral triangle gives $\sin 30^\circ = 1/2$; the adjacent $\sqrt{3}$ gives $\cos 30^\circ = \sqrt{3}/2$ and $\tan 30^\circ = 1/\sqrt{3}$. In the isosceles right triangle, $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2} = \sqrt{2}/2$. A right triangle with hypotenuse 10 and angle 30° has opposite side 5 and adjacent side $5\sqrt{3}$. Since $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$, tangent at 90° would divide by zero.

Sine and cosine interchange when the chosen acute angle changes to its complement. Do not turn $\sqrt{3}$ into a rounded decimal when an exact answer is required. Tangent at

90° is not zero.

AQA G21 is additional Foundation and includes the listed exact values. Derive them from labelled special triangles, then use them in G20 right-triangle calculations.

Bisected equilateral triangle: sides 1, $\sqrt{3}$, 2



Original native-lesson illustration; labels belong to its worked example.

■ Sine rule, cosine rule and triangle area · Higher

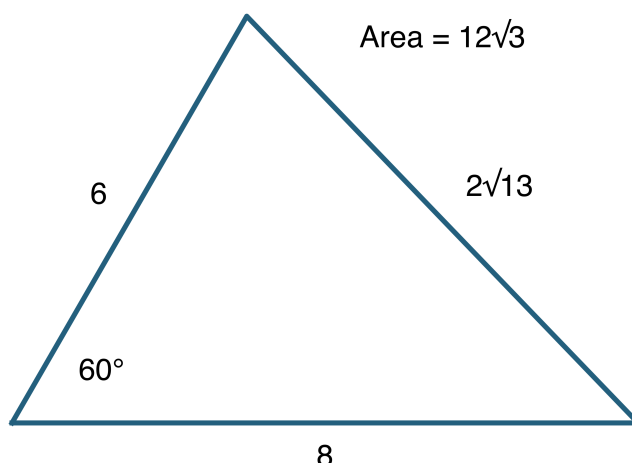
Sine rule pairs opposite sides/angles: $a/\sin A = b/\sin B = c/\sin C$. Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ uses the angle opposite a . Area is $ab \sin C / 2$ when C lies between a and b . Choose the rule from the known information. An inverse sine may give an acute angle and an obtuse supplement; check the angle sum and supplied sides before accepting either.

$$a^2 = b^2 + c^2 - 2bc \cos A, \quad A_{\triangle} = \frac{1}{2} ab \sin C$$

With sides 6 and 8 enclosing 60° , $c^2 = 36 + 64 - 96 \times (1/2) = 52$, hence $c = 2\sqrt{13}$. Its area is $(1/2) \times 6 \times 8 \times \sin 60^\circ = 12\sqrt{3}$. For $a = 4$ opposite $A = 30^\circ$ and $B = 45^\circ$, $b = 4 \sin 45^\circ / \sin 30^\circ = 4\sqrt{2}$. If sides $a = 7, b = 5, c = 6$, $\cos A = (25 + 36 - 49) / (2 \times 5 \times 6) = 1/5$, so $A \approx 78.5^\circ$. To find an angle from area 12 with enclosing sides 6 and 8, $\sin C = 24/48 = 1/2$; C could be 30° or 150° until the remaining data selects a shape. Label opposite pairs and check triangle inequalities to reject impossible side combinations.

Do not pair a side with its adjacent angle in the sine rule. The area angle must be included. A calculator's first inverse-sine answer need not be the only possible triangle. AQA G22/G23 Higher includes unknown sides/angles and areas of general triangles. Write the chosen rule before substitution and state any second possible configuration.

Two sides and their included angle



Original native-lesson illustration; labels belong to its worked example.

■ Right triangles in three-dimensional shapes · Higher

Identify a plane containing the wanted length or angle. Calculate a base diagonal first, then combine it with the perpendicular height. For an angle between a line and a plane, use the angle between the line and its orthogonal projection onto that plane. Mark right angles; general-triangle rules are used only when the selected triangle is not right-angled.

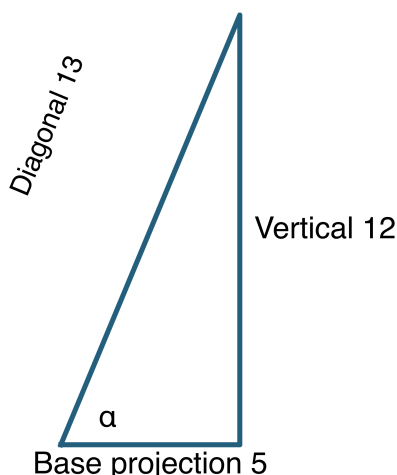
$$d = \sqrt{l^2 + w^2 + h^2}, \quad \tan \alpha = \frac{h}{\sqrt{l^2 + w^2}}$$

A cuboid of width 3, depth 4 and height 12 has base diagonal $\sqrt{9+16}=5$ and space diagonal $\sqrt{25+144}=13$. The angle α of the space diagonal to the horizontal base satisfies $\tan \alpha = 12/5$, giving about 67.4° . Its sine is $12/13$; its cosine is $5/13$. In a square-based pyramid of side 6 and vertical height 4, the base centre-to-side-midpoint distance is 3, giving face slant height 5. The centre-to-corner distance is $3\sqrt{2}$, giving edge length $\sqrt{18+16}=\sqrt{34}$. The face slant and edge lengths differ because their base projections differ.

Do not combine unrelated lengths as if they met at a right angle. The angle to a plane uses the base projection, not an arbitrary base edge. A pyramid's face slant is different from its sloping edge.

AQA G20 Higher extends right-triangle and, where possible, general-triangle reasoning into 3D. Draw the relevant section triangle separately and name which spatial points it represents.

Cuboid section: first $\sqrt{3^2+4^2}=5$, then $\sqrt{5^2+12^2}=13$



Original native-lesson illustration; labels belong to its worked example.

■ Vector geometry and midpoint arguments · Higher

Add/subtract components and multiply a vector by a scalar. Position vectors locate points from one origin; displacement $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ joins two points. A scalar multiple gives parallel directions; to establish collinearity, also connect the displacements to a common point. Midpoints average position vectors. Give a chain of vector equalities with clear start and end points.

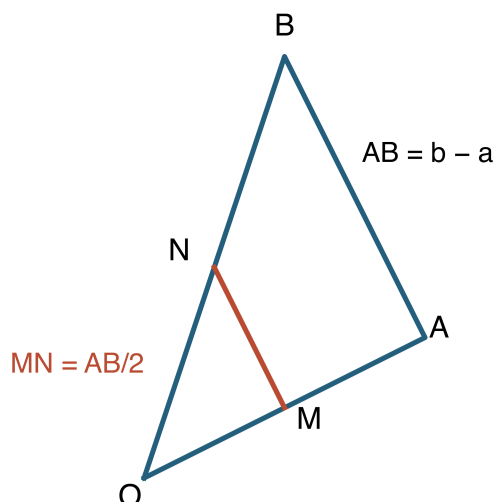
$$\overrightarrow{MN} = \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\overrightarrow{AB}$$

Let $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$, with M midpoint of OA and N midpoint of OB. Then $\mathbf{OM} = \mathbf{a}/2$ and $\mathbf{ON} = \mathbf{b}/2$, so $\mathbf{MN} = \mathbf{b}/2 - \mathbf{a}/2 = (\mathbf{b} - \mathbf{a})/2 = \overrightarrow{AB}/2$. Thus MN is parallel to AB and half as long. For $\mathbf{a} = (4, 2)$, $\mathbf{b} = (2, 6)$, $\mathbf{M} = (2, 1)$, $\mathbf{N} = (1, 3)$, $\overrightarrow{AB} = (-2, 4)$ and $\overrightarrow{MN} = (-1, 2)$, verifying the general result. In parallelogram OACB with $\mathbf{OC} = \mathbf{a} + \mathbf{b}$, the midpoint of OC is $(\mathbf{a} + \mathbf{b})/2$; the midpoint of AB is the same, so the diagonals bisect each other. If $\mathbf{AP} = 3\overrightarrow{AB}/2$, P lies on line AB beyond B; if $\mathbf{AP} = \overrightarrow{AB}/2$, P is its midpoint. A parallel vector at another location alone does not prove three specified points collinear.

Keep $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, not $\mathbf{a} - \mathbf{b}$. Parallelism alone does not locate a line. A numerical example can check the algebra but cannot replace the general midpoint proof.

AQA G25 Higher uses vectors for geometric arguments/proofs, while G24 and the basic G25 operations also belong to Foundation. This proof lesson avoids scalar products and spatial line equations.

Midpoint segment: half length, parallel direction



Original native-lesson illustration; labels belong to its worked example.

Original independent transfer

■ Foundation

A cylindrical container has internal radius 3 cm and height 10 cm. Find its capacity in cubic centimetres and litres. A rectangular face is 6 cm by 8 cm; find its diagonal, naming the condition for your method.

■ Foundation worked solution

Cylinder volume $V = \pi r^2 h = \pi(3 \text{ cm})^2(10 \text{ cm}) = 90\pi \text{ cm}^3$. Since 1000 cubic centimetres is one litre, capacity is $0.09\pi \text{ L}$, about 0.283 L. A rectangular face has perpendicular sides. Pythagoras gives $d = \sqrt{a^2 + b^2} = \sqrt{(6 \text{ cm})^2 + (8 \text{ cm})^2} = 10 \text{ cm}$. A diagonal is not an extra edge of the rectangle's perimeter.

■ Higher

In a circle, AB is a diameter and C lies elsewhere on the circumference. Angle CAB is 32 degrees. Find angles ACB and ABC. Find the acute angle between the tangent at A and AC, explaining the circle theorem and which side of the tangent you mean.

■ Higher worked solution

Angle ACB is 90 degrees because an angle in a semicircle is a right angle. Triangle angles give $\angle ABC = 180 - 90 - 32 = 58$ degrees. By the alternate segment theorem, the angle between chord AC and the appropriate tangent ray equals angle ABC, 58 degrees. The other angle on the tangent's straight line is 122 degrees. Naming the

acute angle selects 58; the drawing's apparent scale is unnecessary. The diameter and tangent facts are required hypotheses.

Terms

scale factor .
hypotenuse .
resultant .
centre of enlargement .
locus .
cyclic quadrilateral .
line of symmetry .
corresponding angles .
congruence .
segment .
midpoint .
elevation .
bearing .
invariant .
angle at the centre .
alternate segment theorem .
perpendicular height .
cross-section .
circumference .
frustum .
sector .
exact value .
included angle .
projection .
collinear .