

5.2.2 Conditional Counts

Name: _____ Class: _____ Date: _____

KEY WORDS conditional probability

<!-- Session 2 native-skill companion -->

Objective

Practise conditional counts within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Conditional probability with tables and expected frequencies · Higher

Conditioning restricts the denominator to the given group. In a table, divide the intersection count by the condition’s row/column total. $P(A \text{ given } B) = P(A \text{ and } B) / P(B)$ when $P(B) > 0$. Reversing the condition usually changes the denominator. Expected-frequency trees help interpret percentages without treating the two conditions as interchangeable.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Using a class of 30 with 18 cyclists, 12 swimmers and 7 doing both, $P(\text{swim given cycle}) = 7/18$, while $P(\text{cycle given swim}) = 7/12$. The unconditional swim probability is $12/30 = 0.4$. For an expected cohort of 1000, suppose 20% have a condition; a test is positive for 90% with it and 10% without it. Expected positive counts are 180 from 200 with the condition and 80 from 800 without it. Of 260 positive tests, the conditional proportion with the condition is $180/260 = 9/13 \approx 0.6923$, not 90%. These are stated illustrative model rates, not claims about a real diagnostic test.

Given positive and positive given condition are different questions. Use the conditioned group total, not the grand total. A rare starting category can make false-positive counts significant even when the detection rate is high.

AQA P9 Higher requires conditional calculations and interpretation using two-way tables, trees and Venn diagrams. Name the restricted group and retain expected counts until the final ratio.

Given positive: use 180 out of 260, not 180 out of 200

	Positive	Negative	Total
Condition	180	20	200
No condition	80	720	800
Total	260	740	1000

Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Higher) From cyclists 18 and cyclist swimmers 7, find $P(\text{swim given cycle})$.

2.2 (Higher) Find total expected positives in the illustrative 1000-person model.

2.3 (Higher) Find the numerator of $180/260$ in lowest terms.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: $P(A \text{ given } B)$ always equals $P(B \text{ given } A)$.

Solutions

2.1 Restrict to 18 cyclists; 7 swim.

2.2 180 true positives plus 80 false positives.

2.3 Divide numerator and denominator by 20 to get $9/13$.

3.1 False. In a group of 100, let $|A|=20$, $|B|=50$ and $|A \cap B|=10$. Then $P(A|B)=10/50=1/5$, but $P(B|A)=10/20=1/2$.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.