

5.1.99 Independent transfer: Foundation then Higher

Name: _____ Class: _____ Date: _____

KEY WORDS conditional probability • conditional probability

<!-- Session 2 independent transfer -->

Objective

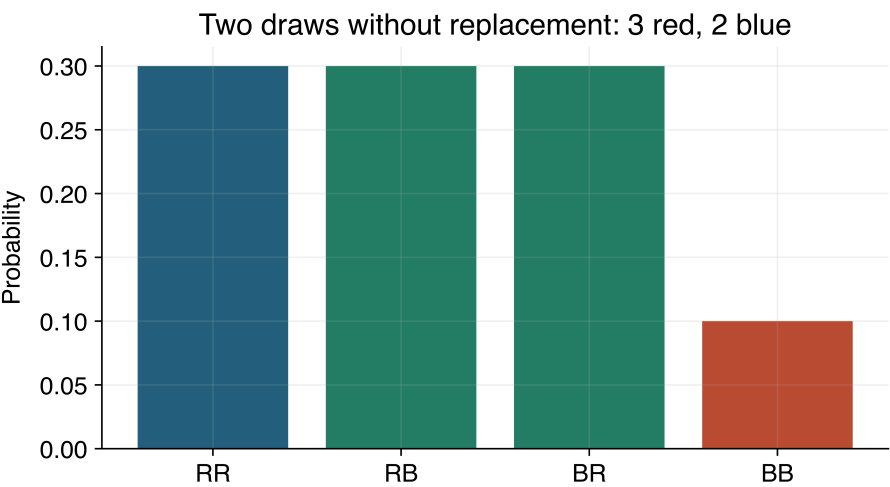
Apply this topic to unfamiliar Foundation and Higher problems; keep the two scopes distinct. Original questions and local marking guidance.

1 Key knowledge and worked examples

After a draw without replacement, the population decreases by one and the colour counts depend on the first result. Multiply conditional branch probabilities along a path and add disjoint paths. Independence requires the intersection probability to equal the product of marginal probabilities; reversed conditions are different questions.

A bag contains 3 red and 2 blue counters. With replacement, $P(\text{two red}) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25} = 0.36$. Without replacement, the red-red branch is $\frac{3}{5} \times \frac{2}{4} = 0.3$. Label each branch before multiplying.

Using a class of 30 with 18 cyclists, 12 swimmers and 7 doing both, $P(\text{swim given cycle}) = \frac{7}{18}$, while $P(\text{cycle given swim}) = \frac{7}{12}$. The unconditional swim probability is $\frac{12}{30} = 0.4$. For an expected cohort of 1000, suppose 20% have a condition; a test is positive for 90% with it and 10% without it. Expected positive counts are 180 from 200 with the condition and 80 from 800 without it. Of 260 positive tests, the conditional proportion with the condition is $\frac{180}{260} = \frac{9}{13} \approx 0.6923$, not 90%. These are stated illustrative model rates, not claims about a real diagnostic test.



Prerequisite example; use the transfer task's own data.

Given positive: use 180 out of 260, not 180 out of 200

	Positive	Negative	Total
Condition	180	20	200
No condition	80	720	800
Total	260	740	1000

Prerequisite example; use the transfer task's own data.

2 Practice

2.1 Foundation: A bag has three red and two blue counters. Two are drawn without replacement. Make a complete possibility tree and find the probability of different colours. Explain how the second-stage denominators change.

3 Exam-style questions

3.1 Higher: In 50 students, 30 study art, 25 study music and 15 study both. Find music given art and art given music. Are the events independent? Reconcile the counts using a two-way table.

Solutions

2.1 First probabilities are R:3/5, B:2/5. After R, the second probabilities are R:2/4, B:2/4. After B they are R:3/4, B:1/4. Terminal probabilities RR, RB, BR, BB are 6/20, 6/20, 6/20, 2/20 and sum to one. Different colours are the disjoint RB and BR paths, so probability is $6/20 + 6/20 = 3/5$. Multiplying stages follows a path; adding combines mutually exclusive paths. Replacement would change the model.

3.1 Both=15, art only=15, music only=10 and neither=10; their sum is 50. $P(M | A) = 15/30 = 1/2$ while $P(A | M) = 15/25 = 3/5$. The intersection probability is $15/50 = 3/10$, equal to $(30/50)(25/50) = 3/10$, so these events are independent in this finite model. Their unequal conditional probabilities do not contradict independence, since independence compares each condition with the matching unconditional probability: $P(M) = 1/2$ and $P(A) = 3/5$.

Local guidance: select and explain the relationship; calculate every requested result; check units, scope, conditions and completeness. These are authored classroom criteria.