

4.5.4 Circle Tangent Proofs

Name: _____ Class: _____ Date: _____

KEY WORDS alternate segment theorem

<!-- Session 2 native-skill companion -->

Objective

Practise circle tangent proofs within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Tangents, chords and the alternate segment · Higher

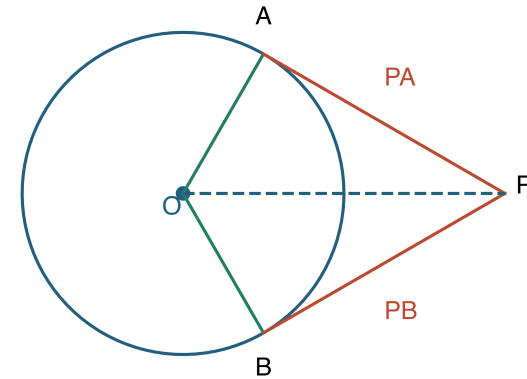
A tangent is perpendicular to its contact radius. Tangents from one external point are equal. The perpendicular from the centre to a chord bisects it. The tangent–chord angle equals the angle on that chord in the alternate segment. Use equal radii, right triangles and the central-angle theorem to prove these relationships.

$$PA = PB, \quad OM \perp AB \implies AM = MB$$

At contact T, the radius OT is perpendicular to the tangent: a non-perpendicular line through T would have a smaller centre-to-line distance than the radius and cut the circle twice. For tangents PA and PB, $OA=OB$, OP is shared and both contact angles are 90° . RHS makes OAP and OBP congruent, so $PA=PB$. For a centre perpendicular OM to chord AB, $OA=OB$ and OM is shared; RHS gives $AM=MB$. If $OA=5$ and $OM=3$, $AM=4$, hence $AB=8$. For a chord AB with minor central angle φ , triangle OAB has base angle $(180-\varphi)/2$. The adjacent tangent–chord angle is $90-(180-\varphi)/2=\varphi/2$, equal to the circumference angle in the alternate segment. Thus a tangent–chord angle of 35° gives 35° in that segment. Reflex/supplementary configurations require the matching arc and angle.

A tangent–chord angle and a radius–chord angle are different. Equal tangent lengths refer to one external point. The chord is bisected by a perpendicular from the centre, not by any line that happens to cross it.

AQA G10 Higher includes these theorem proofs and related results. Give the congruence criterion or angle chain explicitly and identify the relevant chord, contact point and alternate segment.

Shared OP and equal radii $\rightarrow$ RHS $\rightarrow PA = PB$ 

Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Higher) Find the radius–tangent angle.

2.2 (Higher) One tangent from P has length 7. Find the other tangent length.

2.3 (Higher) Find the alternate-segment angle for a 35° tangent–chord angle.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Any line through a circle centre bisects every chord it meets.

Solutions

2.1 A tangent is perpendicular to the radius at contact.

2.2 Tangents from the same external point have equal length.

2.3 Apply the alternate segment theorem to the same chord.

3.1 False. In a unit circle, the horizontal diameter meets the chord from $(0,1)$ to $(1,0)$ at its endpoint $(1,0)$, not its midpoint $(1/2,1/2)$. A perpendicular from the centre bisects a chord.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.