

4.4.6 Vector Geometry Proofs

Name: _____ Class: _____ Date: _____

KEY WORDS collinear

<!-- Session 2 native-skill companion -->

Objective

Practise vector geometry proofs within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Vector geometry and midpoint arguments · Higher

Add/subtract components and multiply a vector by a scalar. Position vectors locate points from one origin; displacement $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ joins two points. A scalar multiple gives parallel directions; to establish collinearity, also connect the displacements to a common point. Midpoints average position vectors. Give a chain of vector equalities with clear start and end points.

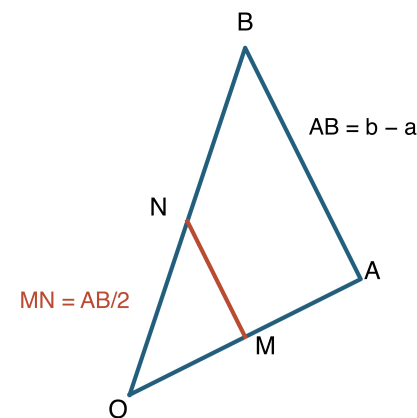
$$\overrightarrow{MN} = \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\overrightarrow{AB}$$

Let $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$, with M midpoint of OA and N midpoint of OB. Then $\mathbf{OM} = \mathbf{a}/2$ and $\mathbf{ON} = \mathbf{b}/2$, so $\mathbf{MN} = \mathbf{b}/2 - \mathbf{a}/2 = (\mathbf{b} - \mathbf{a})/2 = \mathbf{AB}/2$. Thus MN is parallel to AB and half as long. For $\mathbf{a} = (4, 2), \mathbf{b} = (2, 6)$, $\mathbf{M} = (2, 1), \mathbf{N} = (1, 3)$, $\mathbf{AB} = (-2, 4)$ and $\mathbf{MN} = (-1, 2)$, verifying the general result. In parallelogram OACB with $\mathbf{OC} = \mathbf{a} + \mathbf{b}$, the midpoint of OC is $(\mathbf{a} + \mathbf{b})/2$; the midpoint of AB is the same, so the diagonals bisect each other. If $\mathbf{AP} = 3\mathbf{AB}/2$, P lies on line AB beyond B; if $\mathbf{AP} = \mathbf{AB}/2$, P is its midpoint. A parallel vector at another location alone does not prove three specified points collinear.

Keep $\mathbf{AB} = \mathbf{b} - \mathbf{a}$, not $\mathbf{a} - \mathbf{b}$. Parallelism alone does not locate a line. A numerical example can check the algebra but cannot replace the general midpoint proof.

AQA G25 Higher uses vectors for geometric arguments/proofs, while G24 and the basic G25 operations also belong to Foundation. This proof lesson avoids scalar products and spatial line equations.

Midpoint segment: half length, parallel direction



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Higher) For $\mathbf{a} = (4, 2), \mathbf{b} = (2, 6)$, find the x-component of AB.

2.2 (Higher) Find the y-component of midpoint M of OA.

2.3 (Higher) Find the y-component of MN for the two midpoint vectors.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: If two vectors are parallel, any three points used in their diagrams must be collinear.

Solutions

2.1 $b-a$ gives $2-4=-2$.

2.2 Half of $a_y=2$ is 1.

2.3 Half of $b_y-a_y=(6-2)/2=2$.

3.1 False. AB from $(0,0)$ to $(1,0)$ and CD from $(0,1)$ to $(2,1)$ are parallel, but A,B,C are not collinear. Collinearity requires a shared line, not merely parallel directions.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.