

4.3.3 Exact Trig

Name: _____ Class: _____ Date: _____

KEY WORDS exact value

<!-- Session 2 native-skill companion -->

Objective

Practise exact trig within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Exact trigonometric values from special triangles · Foundation

Bisect an equilateral triangle of side 2 to obtain a 30–60–90 triangle with sides 1, $\sqrt{3}$, 2. A right isosceles triangle has sides 1, 1, $\sqrt{2}$. Apply opposite/hypotenuse, adjacent/hypotenuse and opposite/adjacent to derive sine, cosine and tangent. Use degree angles and retain surds exactly; $\tan 90^\circ$ is undefined.

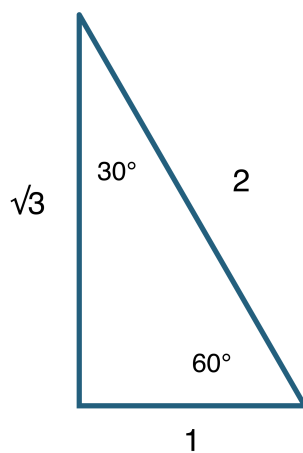
$$\sin 30^\circ = \frac{1}{2}, \quad \cos 45^\circ = \frac{\sqrt{2}}{2}, \quad \tan 60^\circ = \sqrt{3}$$

For angles $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$, sine values are 0, $1/2, \sqrt{2}/2, \sqrt{3}/2, 1$; cosine values are 1, $\sqrt{3}/2, \sqrt{2}/2, 1/2, 0$. For $0^\circ, 30^\circ, 45^\circ, 60^\circ$, tangent values are 0, $1/\sqrt{3}, 1, \sqrt{3}$. The side 1 opposite 30° in the bisected equilateral triangle gives $\sin 30^\circ = 1/2$; the adjacent $\sqrt{3}$ gives $\cos 30^\circ = \sqrt{3}/2$ and $\tan 30^\circ = 1/\sqrt{3}$. In the isosceles right triangle, $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2} = \sqrt{2}/2$. A right triangle with hypotenuse 10 and angle 30° has opposite side 5 and adjacent side $5\sqrt{3}$. Since $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$, tangent at 90° would divide by zero.

Sine and cosine interchange when the chosen acute angle changes to its complement. Do not turn $\sqrt{3}$ into a rounded decimal when an exact answer is required. Tangent at 90° is not zero.

AQA G21 is additional Foundation and includes the listed exact values. Derive them from labelled special triangles, then use them in G20 right-triangle calculations.

Bisected equilateral triangle: sides 1, $\sqrt{3}$, 2



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Foundation) Find $\sin 30^\circ$.

2.2 (Foundation) Find $\tan 45^\circ$.

2.3 (Foundation) Find the opposite length for hypotenuse 10 and angle 30° .

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: All trigonometric functions have finite values at 90° .

Solutions

2.1 Opposite 1 divided by hypotenuse 2.

2.2 Equal opposite and adjacent sides give ratio 1.

2.3 $10 \times \sin 30^\circ = 5$.

3.1 False. $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$, but $\tan 90^\circ = \sin 90^\circ / \cos 90^\circ$ is undefined.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.