

4.2.6 Solid Measure

Name: _____ Class: _____ Date: _____

KEY WORDS frustum

<!-- Session 2 native-skill companion -->

Objective

Practise solid measure within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Spheres, cones, pyramids and frustums · Foundation

Pyramid and cone volumes are one third of base area times perpendicular height. A sphere has volume $4\pi r^3/3$ and area $4\pi r^2$. A cone's curved area is πrl using slant height l ; its volume uses perpendicular height h . For a frustum subtract the removed similar solid. Composite surface area counts only exposed faces; joined faces are hidden.

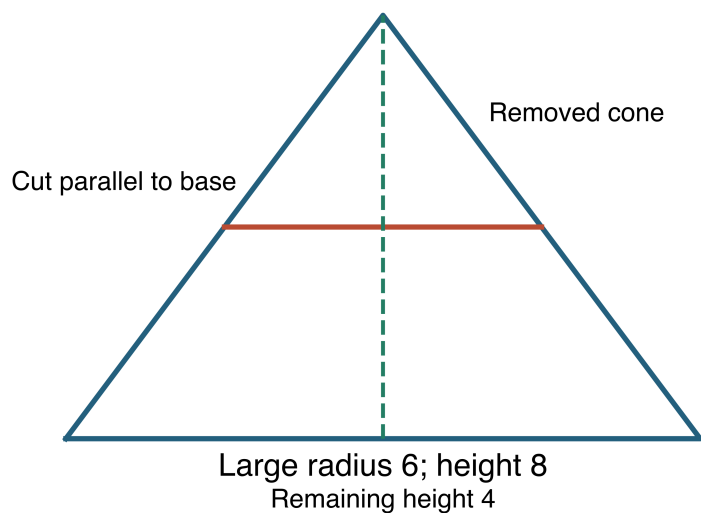
$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h, \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

A cone with radius 3 cm and perpendicular height 4 cm has slant height 5 cm. Volume is $12\pi \text{ cm}^3$, curved area $15\pi \text{ cm}^2$ and total closed area $24\pi \text{ cm}^2$. A sphere of radius 3 has volume $36\pi \text{ cm}^3$ and area $36\pi \text{ cm}^2$, with different units. A square pyramid of base side 6 and height 4 has volume $6^2 \times 4/3 = 48 \text{ cm}^3$; each triangular face has slant height $\sqrt{(4^2 + 3^2)} = 5$, so lateral area is $4 \times (6 \times 5/2) = 60 \text{ cm}^2$ and total area 96 cm^2 . A large cone $r=6, h=8$ loses a similar top cone $r=3, h=4$: frustum volume is $96\pi - 12\pi = 84\pi \text{ cm}^3$. Its slant height is $10 - 5 = 5$; curved area is $60\pi - 15\pi = 45\pi$, and two circular ends add $36\pi + 9\pi$ for total $90\pi \text{ cm}^2$.

Perpendicular height and slant height are not interchangeable. A frustum is not a full cone of the leftover height. Shared composite faces do not contribute exposed area.

AQA G17 additional Foundation includes spheres, pyramids, cones, composite solids and frustums. Use similarity to find missing removed dimensions, then subtract volumes or exposed areas with matching units.

Frustum volume = whole cone – removed similar cone



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Foundation) Find the coefficient of pi in cone volume for $r=3, h=4$.

2.2 (Foundation) Find square-pyramid volume for base side 6, height 4.

2.3 (Foundation) Find the coefficient of pi in frustum volume for large cone $r=6, h=8$ minus small $r=3, h=4$.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: A cone's slant height can always replace its perpendicular height in the volume formula.

Solutions

2.1 $(1/3) \times 3^2 \times 4 = 12$.

2.2 $(1/3) \times 36 \times 4 = 48$.

2.3 $96 - 12 = 84$.

3.1 False. Radius 3, perpendicular height 4 and slant height 5 give volume $(1/3)\pi \times 9 \times 4 = 12\pi$. Using 5 instead incorrectly gives 15π .

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.