

3.1.3 Proportion Models

Name: _____ Class: _____ Date: _____

KEY WORDS constant of proportionality

<!-- Session 2 native-skill companion -->

Objective

Practise proportion models within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Direct and inverse proportional relationships · Foundation

Direct proportion $y=kx$ preserves y/x for nonzero x and gives a straight line through the origin. Inverse proportion $y=k/x$ preserves xy and gives a reciprocal curve, with $x=0$ excluded. State the fixed assumptions: a start-up charge breaks direct proportion, while changing productivity can break inverse proportion. Higher constructs the equation from a known pair; Foundation interprets and uses a given equation.

$$y = kx \quad \text{or} \quad y = \frac{k}{x}, \quad x \neq 0$$

For a given fabric cost $C=3L$, lengths 2,4,6 m cost 6,12,18 yuan: $C/L=3$. A 5-yuan fixed fee instead gives $C=5+3L$, which is linear but not directly proportional. For a given fixed-job model $t=24/w$, workers 2,4,6 need 12,6,4 hours, and $wt=24$ worker-hours. Doubling workers halves time. The inverse model assumes equally productive workers on one fixed job.

An increasing graph need not be direct proportion. An inverse relationship is proportional to $1/x$, not to $-x$. Keep a product constant for inverse proportion and a quotient constant for direct proportion.

AQA R10/R13/R14 covers numerical, algebraic and graphical direct/inverse proportion. Interpret the units of k and the gradient. In Foundation use the supplied equations; Higher also constructs and interprets them from the context.

■ Direct and inverse proportional relationships · Higher

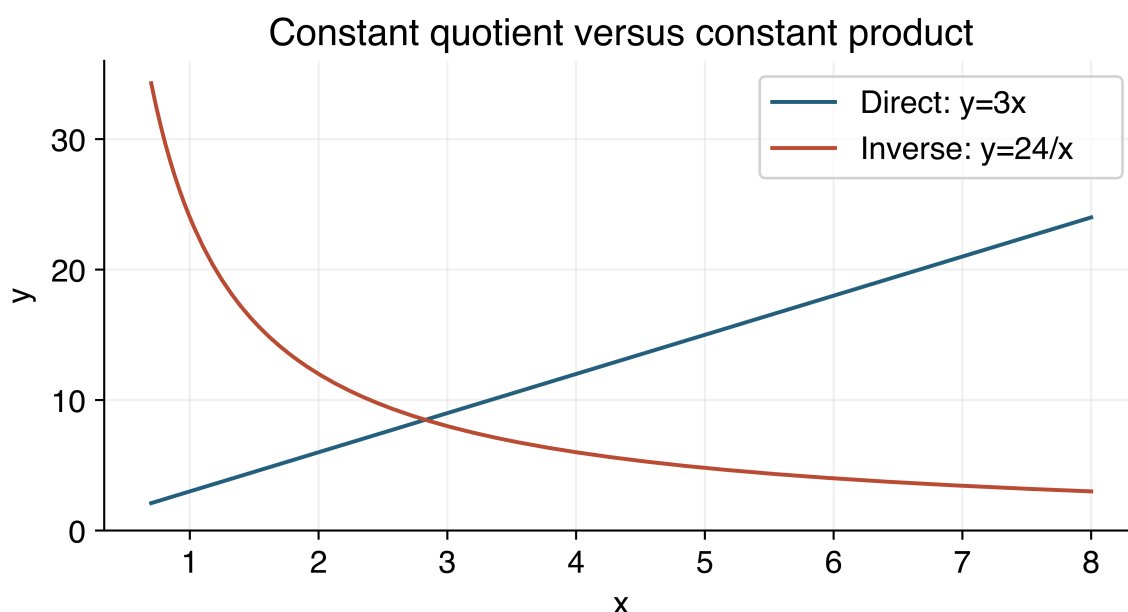
Direct proportion $y=kx$ preserves y/x for nonzero x and gives a straight line through the origin. Inverse proportion $y=k/x$ preserves xy and gives a reciprocal curve, with $x=0$ excluded. State the fixed assumptions: a start-up charge breaks direct proportion, while changing productivity can break inverse proportion. Higher constructs the equation from a known pair; Foundation interprets and uses a given equation.

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For a given fabric cost $C=3L$, lengths 2,4,6 m cost 6,12,18 yuan: $C/L=3$. A 5-yuan fixed fee instead gives $C=5+3L$, which is linear but not directly proportional. For a given fixed-job model $t=24/w$, workers 2,4,6 need 12,6,4 hours, and $wt=24$ worker-hours. Doubling workers halves time. Higher construction: if y is directly proportional to x and $y=18$ at $x=6$, $k=18/6=3$, hence $y=3x$. If t is inversely proportional to w and $t=6$ at $w=4$, $k=tw=24$, hence $t=24/w$. These models assume a constant rate and exclude impossible negative worker counts.

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Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Foundation) Use $C=3L$ to find C for $L=6$.

2.2 (Foundation) Use $t=24/w$ to find t for $w=4$.

2.3 (Foundation) Find the constant product wt for this inverse model.

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Every straight line represents direct proportion.

Solutions

2.1 Substitute $L=6$: $C=18$.

2.2 Substitute $w=4$: $t=24/4=6$ hours.

2.3 Multiply: $w \times (24/w)=24$ for $w \neq 0$.

3.1 False. $y=2x+3$ is straight but does not pass through the origin: $y(0)=3$. Direct proportion requires $y=kx$ and a constant quotient for nonzero x .

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.