

2.3.1 Sequences

Name: _____ Class: _____ Date: _____

KEY WORDS common difference

<!-- Session 2 native-skill companion -->

Objective

Practise sequences within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Arithmetic sequences and nth terms · Foundation

Find a constant difference for an arithmetic sequence. Its n th term is $a+(n-1)d$. A term-to-term rule describes how to reach the next term; a position-to-term rule gives a term directly.

$$u_n = a + (n - 1)d$$

For 5,8,11,14,... the common difference is 3. The n th term is $5+3(n-1)=3n+2$. At $n=8$, $u_8=26$. To find the position of 62, solve $3n+2=62$, giving $n=20$.

The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.

Generate several terms and check a proposed n th-term rule. Infinite geometric series and advanced sum formulae are excluded from this Foundation/Core lesson.

■ Arithmetic and finite geometric sequences · Higher

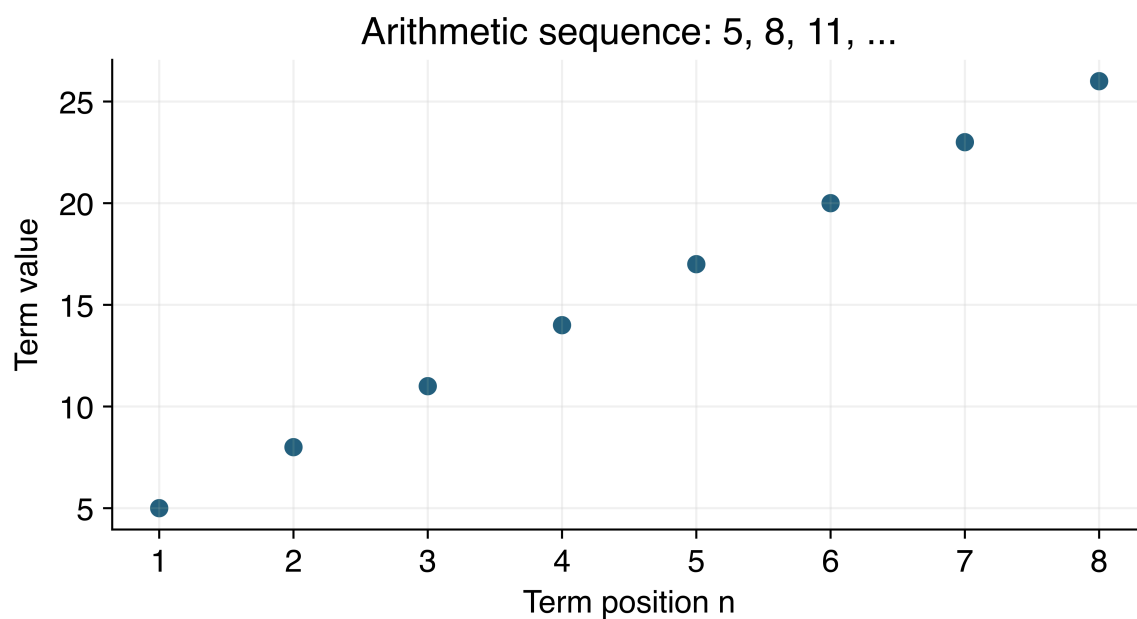
An arithmetic sequence adds a constant difference and has n th term $a+(n-1)d$. A geometric sequence multiplies by a constant ratio and has n th term $ar^{(n-1)}$. Check the starting position and keep a surd ratio exact when one is supplied.

$$u_n = a + (n - 1)d, \quad v_n = ar^{n-1}$$

For 5,8,11,... the n th term is $3n+2$, so $u_8=26$. For 2,6,18,... the common ratio is 3 and $u_n=2 \times 3^{(n-1)}$, giving $u_4=54$. The geometric sequence $1, \sqrt{2}, 2, 2\sqrt{2}, \dots$ has ratio $\sqrt{2}$. These are finite-term calculations; no infinite-series sum is used.

A geometric ratio is not a common difference. The exponent is $n-1$ when a is the first term at position 1. A finite pattern does not itself justify an infinite-sum formula.

AQA A23–A25 Higher uses arithmetic rules, geometric progression patterns and quadratic n th terms. Arithmetic-series formulae and infinite-series sums are excluded from this GCSE lesson.



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 (Foundation) For $a=5, d=3$, find u_8 .

2.2 (Foundation) At what position does 62 occur in $u_n=3n+2$?

2.3 (Foundation) Find u_4 for $u_n=3n+2$.

2.4 (Higher) Find the common ratio of 2,6,18.

2.5 (Higher) Find the fourth term of 2,6,18,... .

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: The first term of a sequence always has index zero.

Solutions

2.1 $u_8 = 5 + (8-1) \times 3 = 26$.

2.2 Solve $3n+2=62$: $3n=60$, hence $n=20$.

2.3 $u_4 = 3 \times 4 + 2 = 14$.

2.4 For 2,6,18,... the ratio is $6/2=3$.

2.5 The fourth term is $2 \times 3^3 = 54$.

3.1 False. The convention $a_1=5$, $a_2=8$, $a_3=11$ indexes its first term by one. An initial value a_0 is possible only when explicitly specified.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.