

Exercise walk-through

4.3.4

General Triangles

AQA · GCSE · Mathematics

74. Triangle congruence and geometric proof · Foundation

Triangle congruence and geometric proof · Foundation

For an isosceles triangle ABC with $AB=AC$, let D be the midpoint of BC . Triangles ABD and ACD have $AB=AC$, $BD=DC$ and shared AD , so SSS gives congruence. Matching base angles ABC and BCA are therefore equal; the two angles at D are equal and form 180° , so each is 90° . To derive Pythagoras, arrange four congruent right triangles with legs a, b around a tilted square of side c inside a square of side $a+b$. Area gives $(a+b)^2 = 4(ab/2) + c^2$, hence $a^2 + b^2 = c^2$. For $a=3, b=4$, $c^2 = 9 + 16 = 25$, so $c=5$. These arguments establish results independently of a scale drawing.

168. Right triangles in three-dimensional shapes · Higher

Right triangles in three-dimensional shapes · Higher

Identify a plane containing the wanted length or angle. Calculate a base diagonal first, then combine it with the perpendicular height. For an angle between a line and a plane, use the angle between the line and its orthogonal projection onto that plane. Mark right angles; general-triangle rules are used only when the selected triangle is not right-angled.

176. Vector geometry and midpoint arguments · Higher

Vector geometry and midpoint arguments · Higher

Let $OA=a$ and $OB=b$, with M midpoint of OA and N midpoint of OB . Then $OM=a/2$ and $ON=b/2$, so $MN=b/2-a/2=(b-a)/2=AB/2$. Thus MN is parallel to AB and half as long. For $a=(4,2), b=(2,6)$, $M=(2,1), N=(1,3)$, $AB=(-2,4)$ and $MN=(-1,2)$, verifying the general result. In parallelogram $OACB$ with $OC=a+b$, the midpoint of OC is $(a+b)/2$; the midpoint of AB is the same, so the diagonals bisect each other. If $AP=3AB/2$, P lies on line AB beyond B ; if $AP=AB/2$, P is its midpoint. A parallel vector at another location alone does not prove three specified points collinear.

Exercises 2.1·2.3

2.1 (Higher) For sides 6,8 and included 60° , find the square of the third side.

2.2 (Higher) For sides 7,5,6, find cosine of the angle opposite 7.

2.3 (Higher) For area 12 and enclosing sides 6,8, find sine of the included angle.

3 Exam-style questions

Exercises 2.1·2.3

2.1 (Higher) For sides 6,8 and included 60° , find the square of the third side.

$$\mathbf{2.1} \quad 36+64-96\times\cos60^\circ=52.$$

2.2 (Higher) For sides 7,5,6, find cosine of the angle opposite 7.

$$\mathbf{2.2} \quad (25+36-49)/(2\times5\times6)=12/60.$$

2.3 (Higher) For area 12 and enclosing sides 6,8, find sine of the included angle.

3 Exam-style questions

$$\mathbf{2.3} \quad \sin C=2\times12/(6\times8)=1/2.$$

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: The sine rule pairs each side with any angle in the triangle.

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3.1 False. In a 30° – 60° – 90° triangle with opposite sides $1, \sqrt{3}, 2$, the correct ratios are $1/\sin 30^\circ = 2$ and $2/\sin 90^\circ = 2$. Pairing side 1 with 60° instead gives $2/\sqrt{3}$, which is different.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.