

Exercise walk-through

# 2.4.9

## Circle Coordinates

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AQA · GCSE · Mathematics

## 21. Quadratics and inequalities · Higher

### Quadratics and inequalities · Higher

The equation  $x^2 - 10x + 21 = 0$  factorises as  $(x-3)(x-7) = 0$ , giving roots 3 and 7. Complete the square:  $x^2 - 10x + 21 = (x-5)^2 - 4$ , giving turning point  $(5, -4)$  and symmetry line  $x = 5$ . The formula  $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$  also gives  $(10 \pm 4) / 2 = 3, 7$ . For  $2x^2 + 5x - 3 = 0$ , the discriminant is 49 and roots are  $(-5 \pm 7) / 4 = 1/2, -3$ . A graph gives approximate roots when exact factorisation is inconvenient. For an enclosure with area  $A = x(10 - x)$ , complete the square to get  $A = 25 - (x-5)^2$ . The greatest area is 25 at  $x = 5$ , within  $0 < x < 10$ .

## 141. Graphical gradients and area estimates · Higher

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For a distance curve  $d=t^2$ , the chord from (1,1) to (3,9) has gradient  $(9-1)/(3-1)=4$ . A tangent at (2,4) passes through (1,0) and (3,8), giving gradient 4 without calculus. A velocity-time trapezium with endpoint velocities 2 and 6 m/s over 3 s has area  $(2+6) \times 3/2 = 12$  m. Smaller strips can improve a curved-area estimate; curvature affects over/underestimation. A cost-versus-quantity tangent through (2,12) and (6,28) gives a local rate  $(28-12)/(6-2)=4$  yuan per extra item, linking graph slope to a financial interpretation.

## 183. Reasoning and check

### Reasoning and check

For  $F(x) = (x^2 - 9)/(x - 3)$ , factor  $x^2 - 9 = (x - 3)(x + 3)$ . Thus  $F(x) = x + 3$  for  $x \neq 3$ ;  $F(5) = 8$ , but  $F(3)$  is undefined, not 6. For addition,  $1/(x - 1) + 2/(x + 1) = [(x + 1) + 2(x - 1)]/[(x - 1)(x + 1)] = (3x - 1)/(x^2 - 1)$ , with  $x \neq \pm 1$ . At  $x = 3$ , both forms give 1. For subtraction, the numerator is  $(x + 1) - 2(x - 1) = 3 - x$ , so the minus sign acts on both terms. Multiplication  $[(x - 1)/(x + 2)] \times [(x + 2)/(x + 1)]$  gives  $(x - 1)/(x + 1)$ , but the original excludes  $x = -2$  and  $x = -1$ . Division  $[(x^2 - 1)/(x^2 + 3x + 2)] \div [(x - 1)/(x + 2)]$  gives 1, yet  $x = -2, -1, 1$  are excluded:  $x = 1$  makes the divisor zero. Finally  $F(x) = 6$  reduces to  $x + 3 = 6$ .

## Exercises 2.1·2.3

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**2.1** (Higher) Find the radius of the circle through  $(3,4)$ .

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**2.2** (Higher) Find the tangent gradient at  $(3,4)$ .

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**2.3** (Higher) For tangent  $3x+4y=25$ , find  $x$  when  $y=0$ .

## 3 Exam-style questions

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## Exercises 2.1·2.3

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**2.1** (Higher) Find the radius of the circle through (3,4).

**2.1**  $\sqrt{(3^2+4^2)}=5$ .

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**2.2** (Higher) Find the tangent gradient at (3,4).

**2.2** The negative reciprocal of  $4/3$  is  $-3/4$ .

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**2.3** (Higher) For tangent  $3x+4y=25$ , find  $x$  when  $y=0$ .

## 3 Exam-style questions

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**2.3** Set  $y=0$ , then divide 25 by 3.

## Exercise 3.1

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**3.1** Decide whether this claim is true. Justify with a concrete calculation or counterexample: The tangent at a circle point has the same gradient as the radius to that point.

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**3.1** Decide whether this claim is true. Justify with a concrete calculation or counterexample: The tangent at a circle point has the same gradient as the radius to that point.

**3.1** False. On  $x^2+y^2=25$  at  $(3,4)$ , the radius gradient is  $4/3$  and the tangent gradient is  $-3/4$ . The radius and tangent are perpendicular.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.