

Exercise walk-through

2.4.8

Graph Rates

AQA · GCSE · Mathematics

21. Quadratics and inequalities · Higher

Quadratics and inequalities · Higher

The equation $x^2 - 10x + 21 = 0$ factorises as $(x-3)(x-7) = 0$, giving roots 3 and 7. Complete the square: $x^2 - 10x + 21 = (x-5)^2 - 4$, giving turning point $(5, -4)$ and symmetry line $x = 5$. The formula $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$ also gives $(10 \pm 4) / 2 = 3, 7$. For $2x^2 + 5x - 3 = 0$, the discriminant is 49 and roots are $(-5 \pm 7) / 4 = 1/2, -3$. A graph gives approximate roots when exact factorisation is inconvenient. For an enclosure with area $A = x(10 - x)$, complete the square to get $A = 25 - (x-5)^2$. The greatest area is 25 at $x = 5$, within $0 < x < 10$.

135. Distance-time graphs and contextual intersections · Foundation

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The distance points $(0,0), (2,6), (5,6), (7,10)$, in seconds and metres, give speed $6/2=3$ m/s for the first section, a 3-second stop, then speed $(10-6)/(7-5)=2$ m/s. Average speed for the full interval is $10/7$ m/s, including the stop. Charges $20+3x$ and $44+x$ meet at $x=12$, at cost 56. If speed increases from 0 to 10 m/s over 5 seconds, its average acceleration is $10/5=2$ m/s²; a speed-time graph measures this through its slope.

141. Graphical gradients and area estimates · Higher

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For a distance curve $d=t^2$, the chord from (1,1) to (3,9) has gradient $(9-1)/(3-1)=4$. A tangent at (2,4) passes through (1,0) and (3,8), giving gradient 4 without calculus. A velocity-time trapezium with endpoint velocities 2 and 6 m/s over 3 s has area $(2+6) \times 3/2 = 12$ m. Smaller strips can improve a curved-area estimate; curvature affects over/underestimation. A cost-versus-quantity tangent through (2,12) and (6,28) gives a local rate $(28-12)/(6-2)=4$ yuan per extra item, linking graph slope to a financial interpretation.

Exercises 2.1·2.3

2.1 (Higher) Find the chord gradient from $(1,1)$ to $(3,9)$.

2.2 (Higher) Find the tangent gradient from $(1,0)$ to $(3,8)$.

2.3 (Higher) Find the velocity trapezium displacement for 2 and 6 m/s over 3 s.

3 Exam-style questions

Exercises 2.1·2.3

2.1 (Higher) Find the chord gradient from (1,1) to (3,9).

$$\mathbf{2.1} \ (9-1)/(3-1)=4.$$

2.2 (Higher) Find the tangent gradient from (1,0) to (3,8).

$$\mathbf{2.2} \ (8-0)/(3-1)=4.$$

2.3 (Higher) Find the velocity trapezium displacement for 2 and 6 m/s over 3 s.

3 Exam-style questions

2.3 Average endpoint height 4 times width 3 gives 12.

Exercise 3.1

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Area under any graph always means the distance travelled.

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3.1 False. Area under a speed-time graph gives distance, but a constant force 2 N over displacement 3 m gives area 6 J of work. Interpret the axis units first.

Local review guidance: show the chosen method; give the checked result; explain the condition or failed shortcut. This is authored classroom guidance.