

Exercise walk-through

2.1.99

Independent transfer: Foundation then Higher

AQA · GCSE · Mathematics

135. Distance-time graphs and contextual intersections · Foundation

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The distance points $(0,0), (2,6), (5,6), (7,10)$, in seconds and metres, give speed $6/2=3$ m/s for the first section, a 3-second stop, then speed $(10-6)/(7-5)=2$ m/s. Average speed for the full interval is $10/7$ m/s, including the stop. Charges $20+3x$ and $44+x$ meet at $x=12$, at cost 56. If speed increases from 0 to 10 m/s over 5 seconds, its average acceleration is $10/5=2$ m/s²; a speed-time graph measures this through its slope.

141. Graphical gradients and area estimates · Higher

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For a distance curve $d=t^2$, the chord from (1,1) to (3,9) has gradient $(9-1)/(3-1)=4$. A tangent at (2,4) passes through (1,0) and (3,8), giving gradient 4 without calculus. A velocity-time trapezium with endpoint velocities 2 and 6 m/s over 3 s has area $(2+6) \times 3/2 = 12$ m. Smaller strips can improve a curved-area estimate; curvature affects over/underestimation. A cost-versus-quantity tangent through (2,12) and (6,28) gives a local rate $(28-12)/(6-2)=4$ yuan per extra item, linking graph slope to a financial interpretation.

153. Numerical iteration for equation roots · Higher

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For $x^2 - x - 2 = 0$, use $x_{\text{next}} = \sqrt{x+2}$ with $x_0 = 1$: $x_1 = \sqrt{3} \approx 1.73205$, $x_2 \approx 1.93185$, $x_3 \approx 1.98289$. The positive fixed point is 2 because $2 = \sqrt{4}$ and $2^2 - 2 - 2 = 0$. The rearrangement only seeks a nonnegative root; the original equation also has root -1. Using $x_{\text{next}} = x^2 - 2$ from 3 instead gives 7 then 47, so a different rearrangement can diverge.

Exercises 2.1·3.1

2.1 Foundation: A taxi charges a fixed 8 CNY plus 3 CNY per kilometre. Form a cost equation. A trip costs 35 CNY; find its distance and check. Then solve the simultaneous equations $2x + y = 11$, $x - y = 1$.

3.1 Higher: Solve $(x^2 - 9)/(x - 3) = x^2 - 5$ on its original real domain. Then solve

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2.1 For distance d in kilometres and cost C in CNY, $C = 8 + 3d$. Rearranging gives $d = (C - 8)/3 = (35 - 8)/3 = 9$ km. Substitution gives $8 + 3(9) = 35$ CNY. Adding the simultaneous equations gives $3x = 12$, so $x=4$ and $y=3$. Both equations check: $2(4) + 3 = 11$, $4 - 3 = 1$. The cost intercept represents the charge at zero distance, not the distance for zero cost.

3.1 Higher: Solve $(x^2 - 9)/(x - 3) = x^2 - 5$ on its original real domain. Then solve

3.1 The equation excludes $x=3$. Cancellation gives $x + 3 = x^2 - 5$, hence $x^2 - x - 8 = 0$ and $x = (1 \pm \sqrt{33})/2$. Neither is 3, so both remain. For the separate inequality, $(x - 5)(x + 1) \leq 0$ gives $-1 \leq x \leq 5$, including both zero endpoints. Completing the square gives $y = (x - 2)^2 - 9$; its turning point is $(2, -9)$, a minimum, with roots -1 and 5. Do not confuse the two different quadratics.

Local guidance: select and explain the relationship; calculate every requested result; check units, scope, conditions and completeness. These are authored classroom criteria.