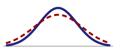
Inference for Means

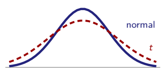
AP Statistics ▪ Unit 7

AP Statistics



What we will cover

1. The t-distribution
2. Confidence interval for a mean
3. Test for a mean
4. Difference of two means
5. Two-mean test and paired data
6. Choosing a procedure



1 t

The t-distribution

We almost never know σ, so we use s_x instead. That extra guessing means the **t-distribution** t 分布, not the normal.

- heavier tails $\Rightarrow$ a little more caution
- **degrees of freedom** 自由度 $= n - 1$

> **Sampling variability** 抽样变异性
> is honest bounce – not a mistake a calculation can fix.

Two group comparison

2 Interval

Confidence interval for a mean

With σ unknown, use t^* with $df = n - 1$:

$$\bar{x} \pm t^* \frac{s_x}{\sqrt{n}}$$

- **standard error** 标准误 $= s_x/\sqrt{n}$
- **margin of error** 误差幅度 $= t^* \cdot \mathrm{SE}$

> Check: random, $\leq 10\%$ of population, roughly normal (or $n \geq 30$).

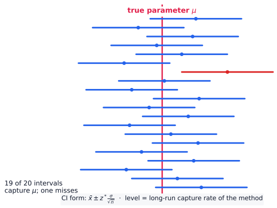
Batteries

$n = 25,\ \bar{x} = 50,\ s_x = 8;\ t^* = 2.064\ (df = 24)$:

$$50 \pm 2.064 \cdot \tfrac{8}{5} = 50 \pm 3.30$$

$= (46.70,\ 53.30)$ hours.

"95% confident" describes the method



> "95% confident" describes the method, not one interval: over many samples about 95% of the intervals contain μ and about 5% miss it.

3 Test

Test for a mean

A **one-sample t-test** 单样本 t 检验:

- **null** 原假设 $H_0 : \mu = \mu_0$
- **alternative** 备择假设 $H_a : \mu >, <,$ or $\neq \mu_0$

$$t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$$

> Find the p-value from the t-distribution with $df = n - 1$; reject if p $\leq \alpha$.

Batteries

Claim $\mu = 50$; sample $\bar{x} = 53$, $s_x = 8$, $n = 25$, $H_a : \mu > 50$:

$$t = \frac{53 - 50}{8/5} = 1.875$$

p $\approx 0.037 < 0.05$: reject.

4 Two means

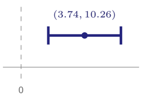
Difference of two means

The design decides the tool:

- **independent samples** 独立样本 $\Rightarrow$ **two-sample t-interval** 双样本 t 区间
- **paired** 配对 data $\Rightarrow$ one-sample t on the differences

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

> Variances add under the root. If the interval excludes 0, the means differ.



Excludes 0 $\Rightarrow$ Brand 1 longer.

5 Two-mean test

Two-mean test and paired data

A **two-sample t-test** 双样本 t 检验 of $H_0 : \mu_1 = \mu_2$ – do not pool (unlike proportions):

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$$

Same subjects measured twice? Use a **paired t-test** 配对 t 检验: reduce each pair to a difference d, then a one-sample t.

Brands

$\bar{x}_1 = 85, \bar{x}_2 = 78$; SE= $\sqrt{2}$:

$$t = \frac{7}{1.414} \approx 4.95$$

$df = 8$, p ≈ 0.001: reject.

6 Choose

Choosing a procedure

Two questions point to the right tool:

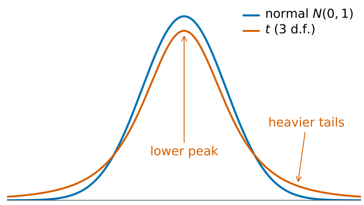
Goal?
estimate ⇒ interval;
weigh a claim ⇒ test

How many groups?
one ⇒ one-sample; two
⇒ two-sample or paired

Mean or proportion?
mean ⇒ t; proportion ⇒ z

Then **select** 选择 the formula, **implement** 实施 with conditions checked, and **communicate** 表达 in context.

Confidence Interval for a Mean, continued



The t-distribution has a lower peak and heavier tails than the normal

Worked example – a one-sample t-test for a mean

Test $H_0 : \mu = 45$ against $H_a : \mu \neq 45$, given $\bar{x} = 50$, $s = 8$, $n = 25$.

- $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{50-45}{8/5} = \frac{5}{1.6} = 3.125$
- $df = n - 1 = 24$
- two-sided $p \approx 0.005$, so reject H_0 at the 5% level
- in context: the evidence says the true mean is *not* 45

As with proportions: a claimed mean inside the interval is plausible; outside it, the data give evidence against the claim. The interval and the test agree by construction.

Should I Worry About Error?

Inference for a **mean** works like inference for a proportion, with one change: we rarely know the population standard deviation σ, so we estimate it with the sample s.

That extra uncertainty means we use the *t*-**distribution** instead of the normal – a **distribution 分布** that is bell-shaped but with heavier tails.

Exam tips

- Use **t-procedures** for means (population σ unknown) – the t-distribution has heavier tails than normal.
- Check conditions: random, independent, and roughly normal (or large n).
- Interpret an interval and a test in context, always tied to the parameter (the true mean).
- Match the right procedure: one-sample, two-sample, or paired (look for a natural pairring).
- State the degrees of freedom; for a two-sample t-test use technology's value (or, by hand, the conservative smaller $n-1$).

7 Recap

Unit 7 – key takeaways

1 The t-model

unknown $\sigma \Rightarrow$ use s_x and t; $df = n-1$

2 One mean

$\bar{x} \pm t^* s_x/\sqrt{n}$; $t = (\bar{x} - \mu_0)/(s_x/\sqrt{n})$

3 Two means

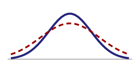
add variances; don't pool; paired $\rightarrow$ one-sample t

4 Choosing

goal + groups + mean/proportion select the test

Check yourself

1. Why use t instead of z for a mean?
2. What are the degrees of freedom for a one-sample t?
3. Do you pool for a two-sample t-test of means?
4. Before-and-after on the same subjects calls for which test?



Answers **1.** σ is unknown (use s_x) **2.** $n-1$ **3.** no **4.** a paired t-test