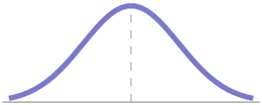
Exploring One-Variable Data

AP Statistics ▪ Unit 1

AP Statistics

What we will cover



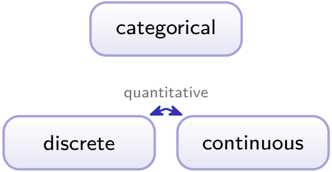
1 Variables

Variables and data

Statistics 统计学 learns from **data** 数据 through **variability** 变异性. Each **variable** 变量 is one of two types:

- **categorical** 分类变量: a group (eye colour)
- **quantitative** 定量变量: a number (height)

> **Descriptive** 描述统计学 statistics summarize a sample; **inference** 推断 generalizes to a population.



The vocabulary the whole course rests on

statistics 统计学	the science of learning from **data 数据** – numbers or labels collected from the real world
a parameter 参数	a numerical summary of a whole population
a statistic 统计量	a numerical summary of a sample – and it varies from sample to sample
a variable 变量	a characteristic that can differ between individuals
categorical	places an individual in a group – shown with **bar charts 条形图** and pie charts
quantitative	a numerical measurement – shown with dotplots, histograms and boxplots

Descriptive statistics summarises the sample; **inferential statistics 推断统计** uses it to say something about the population. The whole course is the second.

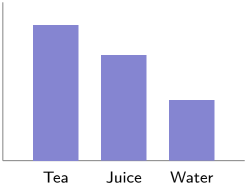
2 Categorical

Displaying categorical data

Summarize with a **frequency table** 频数表 and **relative frequency** 相对频率 (which sums to 1).

- a **bar chart** 条形图 has gaps between bars
- a **two-way table** 双向表 records two variables

> Watch the axis: one not starting at 0 exaggerates tiny differences.



Bars separated by gaps.

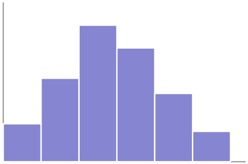
3 Quantitative

Displaying quantitative data

Show *where numbers fall on a scale*:

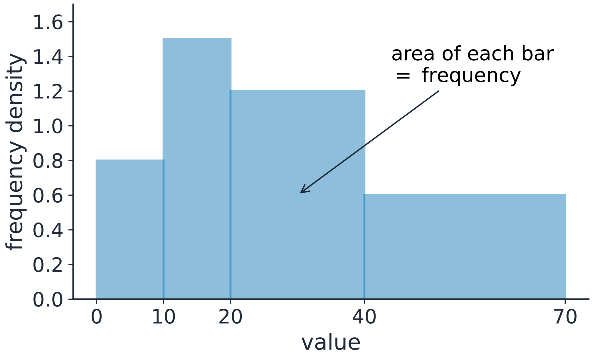
- a **dotplot** 点图 or **stem-and-leaf** 茎叶图 keeps every value
- a **histogram** 直方图 groups values into **bins** 组距
- an **ogive** 累积相对频率图 reads off **percentiles** 百分位数

> Histogram bars touch (a number scale) – the visual difference from a bar chart.



Histogram: bars touch.

Representing a Quantitative Variable with Graphs



On a histogram with unequal class widths the bar area is the frequency

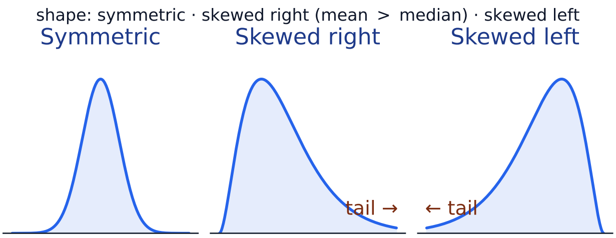
4 Describing

Describing a distribution

Hit four features – **S**hape, **O**utliers, **C**enter, **S**pread – in context.

- **symmetric** 对称 or **skewed** 偏
- skew is named for where the *tail* points

> Always mention **outliers** 离群值: they may be errors – or the whole story.



Describing a distribution, in detail

shape	symmetric, or **skewed** 偏斜 left or right – the tail names the direction
peaks	one main peak is unimodal; two is **bimodal** 双峰, and usually means two groups are mixed
centre	the mean for a symmetric distribution, the median when it is skewed
spread	the standard deviation with a mean; the **interquartile range** 四分位距 with a median
outliers	more than $1.5 \times$ IQR beyond a quartile
a cumulative relative frequency graph 累积相对频率图	reads off percentiles directly

Answer all four, in context, every time. Naming the shape without the context is the commonest way to lose a description mark.

5 Summaries

Summary statistics

Center: mean 均值 $\bar{x}$ vs **median** 中位数.
The median is **resistant** 稳健 to outliers.

$$\bar{x} = \frac{1}{n}\sum x_i \qquad s_x = \sqrt{\frac{1}{n-1}\sum(x_i - \bar{x})^2}$$

Spread: range 极差, **IQR** 四分位距 $= Q_3 - Q_1$ (resistant), **standard deviation** 标准差.

> Symmetric $\Rightarrow$ mean & SD. Skewed $\Rightarrow$ median & IQR.

The z-score

$$z = \frac{x - \bar{x}}{s_x}$$

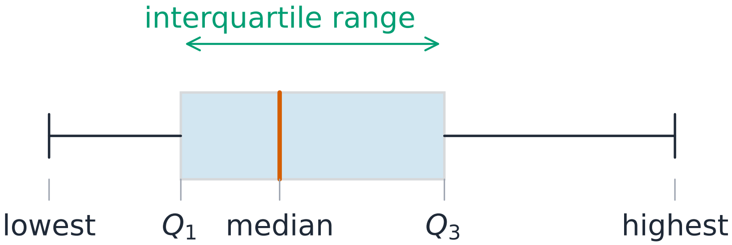
How many **standard deviations** 标准差 a value sits from the mean – so we can compare different scales.

Describing a distribution, and one more display

Shape words	Skewed left or right, symmetric, **uniform**, or bimodal – name the shape before any number.
Variance	The **variance** is the mean squared deviation, s^2; the standard deviation is its square root, and it carries the data's own units.
Variation	Talking about spread *is* talking about **variation** – the range, the IQR and the standard deviation all measure it differently.
Stem-and-leaf plot	A **stem-and-leaf plot** keeps every original value while showing the shape, so the median is read directly off it.

> Shape, centre, spread, and unusual features – in context. Four things, every time a distribution is described.

Graphical Representations of Summary Statistics



A box-and-whisker plot shows the quartiles and the range

6 Boxplots

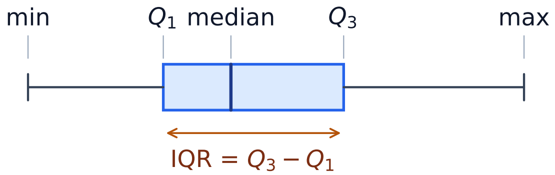
Boxplots and the five-number summary

The **five-number summary** 五数概括: min, Q_1, median, Q_3, max.

- the box length *is* the IQR
- outlier if beyond $Q_{1,3} \mp 1.5\,\mathrm{IQR}$

> A **boxplot** 箱线图 hides peaks – pair it with a histogram when shape matters.

Boxplots and the five-number summary, in detail



five-number summary: min, Q_1, median, Q_3, max · IQR $= Q_3 - Q_1$

The **five-number summary** 五数概括: min, Q_1, median, Q_3, max.

Worked example – is it an outlier?

Salaries ($000s):
$28,\ 31,\ 34,\ 36,\ 39,\ 42,\ 47,\ 96$. **Is** 96 **an outlier?**

$Q_1 = \frac{31+34}{2} = 32.5, \quad Q_3 = \frac{42+47}{2} = 44.5$

$\text{IQR} = 44.5 - 32.5 = 12, \quad 1.5 \times 12 = 18$

upper fence $= 44.5 + 18 = 62.5$

$96 > 62.5$, so 96 is an outlier.

The fence comes off Q_3, not the median or the mean. And an outlier is not an error – here it is a real salary that makes the mean (44.1) a worse summary than the median (37.5).

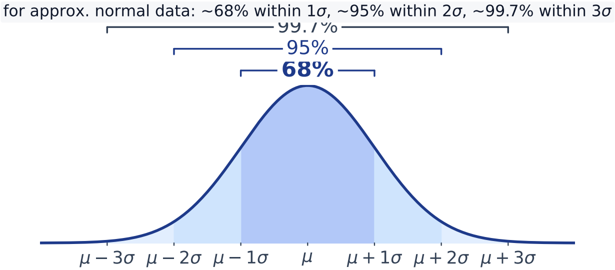
7 Normal

The normal distribution

The **normal distribution** 正态分布 is set by μ and σ. The **empirical rule** 经验法则 (68–95–99.7):

- 68% within $\mu \pm \sigma$
- 95% within $\mu \pm 2\sigma$; 99.7% within 3σ

> The **proportion** 比例 below a value $=$ the **area** 面积 to its left, found from *z*.



Worked example – the empirical rule and z

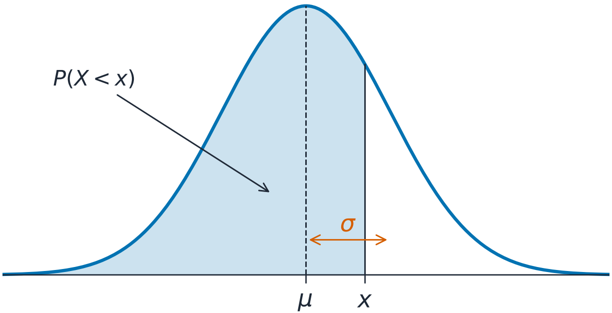
Heights are normal with $\mu = 170$ cm, $\sigma = 8$ cm. (a) What percent are between 154 and 186? (b) Where does 182 cm sit?

(a) $154 = \mu - 2\sigma$ and $186 = \mu + 2\sigma$, so by the empirical rule 95%.

(b) $z = \dfrac{182 - 170}{8} = 1.5$ – one and a half SDs above the mean.

The empirical rule only works at whole multiples of σ. 182 is not one, so it needs z – and z answers "how many SDs from the mean", which is what every later unit builds on.

The Normal Distribution, continued



The normal curve: a probability is the area under it, centred on the mean

Exam tips

- Describe a distribution by **shape, center, spread, and outliers** (SOCS) – always in context.
- The **mean** is pulled by outliers; the **median** resists them, so prefer the median for skewed data.
- For a **normal** distribution use the 68–95–99.7 rule and z-scores $z = \frac{x-\mu}{\sigma}$.
- Compare distributions with side-by-side boxplots and comment on center, spread, and shape.
- Standard deviation measures a typical distance from the mean; the IQR pairs with the median.

8 Recap

Unit 1 – key takeaways

1 Variables

categorical vs quantitative decides every graph

2 Describe

Shape, Outliers, Center, Spread – in context

3 Summaries

median/IQR resistant; mean/SD for symmetric; z-score

4 Normal

68–95–99.7 rule; area to the left = proportion

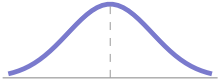
Check yourself

1. Is a zip code categorical or quantitative?

2. Which center is resistant to outliers?

3. A right-skewed tail points toward high or low values?

4. What percent of a normal lies within 2σ?



Answers **1.** categorical (a label) **2.** median **3.** high values **4.** about 95%