Question 5: Multi-Focus — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	The scatterplot reveals a strong, positive, roughly linear association between the chest circumference and weight of tule elk. There are no points that seriously deviate from the straight-line pattern of the points in the plot.	**Essentially correct (E)** if the response provides a description that includes at least three of components 1-4 and component 5: 1. Direction of association (positive or increasing) 2. Strength of association (strong) 3. Form of association (linear or approximately linear) 4. Unusual features (no points with large discrepancies from the straight-line pattern exhibited by most of the points on the plot) 5. Context (association between chest circumference and weight of tule elk) **Partially correct (P)** if the response satisfies only one or two components out of components 1-4 and component 5 *OR* if the response satisfies at least two out of components 1-4 but does not satisfy component 5. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- To satisfy component 4 it is sufficient to simply indicate that there are no unusual features or no outliers.
- To satisfy component 5 it is minimally sufficient for the response to refer to the association or relationship between chest circumference or measurement and weight without explicitly mentioning tule elk.
- The quality of communication in part (a) should be considered if holistic scoring is required.

Model Solution		Scoring
(b)	**(i)** The predicted weight of a male tule elk with a chest circumference of 145.9 cm is $-350.3 + 3.7455(145.9) \approx 196.17$ kg. **(ii)** The residual for a male tule elk with a chest circumference of 145.9 cm with an actual weight of 204.3 kg is $204.3 - 196.17 \approx 8.13$ kg.	**Essentially correct (E)** if the response satisfies the following two components: 1. Provides the correct value of 196.17 kg in part (b-i) with work shown 2. Provides the correct value of 8.13 kg in part (b-ii), or a value consistent with the predicted weight calculated for component 1, with work shown **Partially correct (P)** if the response satisfies only one of the two components required for an E *OR* the response gives both correct values with no work shown. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A response that includes an undefined or incorrect variable to identify work shown (e.g., x or $\hat{p}$) does not satisfy component 1.

	Model Solution	Scoring
(c)	The value of the slope of the least-squares regression line is 3.7455. This value indicates that the predicted weight of a tule elk increases by 3.7455 kilograms for each additional centimeter of chest circumference.	**Essentially correct (E)** if the response satisfies the following three components: 1. Identifies the value of the slope as 3.7455 2. Provides an interpretation that references an increase of a number of kilograms of weight for each one-centimeter increase in chest circumference 3. Indicates that the slope represents a change in a prediction using non-deterministic language such as "predicted," "estimated," "expected," or "average" **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes

- The value of the slope, 3.7455, may be rounded to 3.746, 3.75, or 3.7, but not to 3 or 4, to satisfy the numerical requirement in component 1.
- A response that only contains 3.7455 in the interpretation satisfies component 1.
- A calculation of slope may satisfy component 1 provided that two points from the line are used in the calculation.
- Units of measurements must be correctly specified for both weight and length to satisfy component 2.
- It is not required to refer specifically to the "least-squares regression line."

Model Solution	Scoring
(d) **(i)** The degrees of freedom for the test of slope are $n - 2 = 30 - 2 = 28$. The *t*-table shows that for 28 degrees of freedom, the *p*-value for a one-sided test would be 0.001. Because this is a two-sided test, the *p*-value is $(2)(0.001) = 0.002$. **(ii)** Because the *p*-value $= 0.002$ is less than $\alpha = 0.05$, reject the null hypothesis. There is sufficient statistical evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different from 4.5 kg/cm.	**Essentially correct (E)** if the response satisfies the following three components: 1. Gives a correct *p*-value of 0.002, consistent with the two-sided alternative hypothesis 2. Provides correct comparison of the *p*-value to alpha (*p*-value is less than/greater than alpha) *AND* provides a correct decision about the null and/or alternative hypothesis 3. Provides a conclusion statement in context, consistent with, and in terms of, the alternative hypothesis using non-deterministic language **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes

- The response need not make an explicit decision about the null hypothesis (reject H_o or fail to reject H_o).
- If the conclusion and justification are consistent with an incorrect *p*-value (or an incorrect value of the test statistic, or an incorrect confidence interval, or an incorrect critical value), the response may satisfy component 2.
- To satisfy the *p*-value comparison in component 2, the response can compare the value of the test statistic to the appropriate critical value (e.g., 2.048 or -2.048).
- If an explicit decision is stated and the conclusion statement is inconsistent with the decision, component 2 is not satisfied.
- The decision part of component 2 may be satisfied by implying the decision within the conclusion statement (sufficient evidence/insufficient evidence for the alternative hypothesis).
- If the response includes a statement that is equivalent to accepting the null hypothesis (e.g., "we conclude that the mean population slope is 4.5"), then component 2 is not satisfied.
- A student who doesn't give a *p*-value, but gives a correct conclusion based on a *p*-value $< \alpha$ with a consistent conclusion *OR* a *p*-value $> \alpha$ with a consistent conclusion may be scored P, e.g.,
  - If *p* were less than alpha, then reject H_0 and say there is sufficient evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different than 4.5 kg/cm.
  - If *p* were greater than alpha, then fail to reject H_0 and say there is not sufficient evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different than 4.5 kg/cm.
- If the response includes an incorrect interpretation of the *p*-value (e.g., "this is the probability that the null hypothesis is true"), then component 2 is not satisfied.
- The quality of communication in part (d) should be considered if holistic scoring is required.

Scoring for Question 5

Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.

	Score
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**

If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.

Question 1: Focus on Exploring Data — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	The scatterplot reveals a strong, positive, roughly linear association between the mass and length of bullfrogs. There are no points that seriously deviate from the straight-line pattern of the points in the plot.	**Essentially correct (E)** if the response provides a description that includes at least three of components 1-4 and component 5: 1. Direction of association (positive or increasing) 2. Strength of association (strong) 3. Form of association (linear or approximately linear) 4. Unusual features (no points with large discrepancies from the pattern (straight line) exhibited by most of the points on the plot) 5. Context (association between length and mass of bullfrogs) **Partially correct (P)** if the response satisfies only one or two components out of components 1-4 and component 5 *OR* if the response satisfies at least three out of components 1-4 but does not satisfy component 5. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- To satisfy component 4, it is sufficient to simply indicate that there are no unusual features.
- To satisfy component 5, it is minimally sufficient for the response to refer to the association or relationship between mass and length without explicitly mentioning bullfrogs.
- The strength of the response in part (a) may be considered if holistic scoring is needed.

	Model Solution	Scoring
(b)	The value of the slope of the least-squares regression line is 6.086. This value indicates that the predicted mass of a bullfrog increases by 6.086 grams for each additional millimeter of length.	**Essentially correct (E)** if the response satisfies the following three components: 1. Identifies the value of the slope as 6.086 2. Provides an interpretation that references an increase of a number of grams of mass for each one-millimeter increase in length 3. Indicates that the slope represents a change in a prediction using non-deterministic language such as "predicted," "estimated," "expected," or "average" **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- The value of the slope, 6.086, may be rounded to 6.09 or 6.1, but not to 6, to satisfy the numerical requirement in component 1.
- A response that only contains 6.086 in the interpretation satisfies component 1.
- A calculation of slope may satisfy component 1, provided that two points from the line are used in the calculation.
- Units of measurements must be correctly specified for both mass and length to satisfy component 2.
- It is not required to refer specifically to the "least-squares regression line."

	Model Solution	Scoring
(c)	The coefficient of determination is $r^2 \approx 0.819$. This value indicates that 81.9% of the variation in bullfrog mass can be explained by variation in bullfrog length as described by the least-squares line.	**Essentially correct (E)** if the response provides a correct interpretation of r^2 in context. **Partially correct (P)** if the response provides a generic interpretation (no context) *OR* if the response provides a reasonable but incorrect interpretation of r^2 in context. **Incorrect (I)** if the response does not satisfy the criteria for E or P.

Additional Notes:

- Correct interpretations of r^2 include the concept that part of the variation in the response (dependent or y) variable is explained by the linear relationship with the explanatory (independent or x) variable. The response can take any of several equivalent forms, such as:
  - The proportion of the total variability in the dependent (response) variable y that is explained by the independent (explanatory) variable x.
  - The proportion of variation in y that is accounted for by the linear model.
  - The proportionate reduction of the total variation of the y-values that is associated with the use of the independent variable x.
  - The proportionate reduction in the sum of the squares of vertical deviations obtained by using the least-squares line instead of the sample mean to predict values of y.
- Correct interpretation of r^2 must explicitly relate to the dependent variable. Mention of the data, predicted values, or no mention of the dependent variable are incorrect interpretations. Common incorrect interpretations include:
  - The percent (or proportion or part of the total) variability in the predicted y-values that is explained by the linear relationship between y and x.
  - The percent (or proportion or part of the total) variability in the data that is explained by the linear relationship between y and x.
  - The percent (or proportion or part of the total) variability that is explained by the linear relationship between y and x.
  - The percent (or proportion or part of the total) variability in y that is on average explained by the linear relationship between y and x.
- A reasonable but incorrect interpretation of r^2 with context might include the following responses:
  - 81.9% of the variation in mass and length can be accounted for by the least-squares regression line.
  - 81.9% of the variability in predicted mass is accounted for by the length.
- For context, the response variable (y) must be identified as mass, and the explanatory variable (x) must be identified as length.
- An interpretation of the correlation between mass and length, $r = \sqrt{0.819} = 0.905$, is not considered a reasonable interpretation of r^2.
- The value of the percentage (81.9%) or proportion (0.819) of variation does not need to be specified, but if an incorrect value is specified, the score is lowered by one level, from E to P or from P to I.
- The strength of the response in part (c) may be considered if holistic scoring is needed.

Model Solution	Scoring
(d) (i) The largest residual in absolute value belongs to the bullfrog with length 162 millimeters and mass 356 grams. (ii) The least-squares regression line overestimates the mass of the bullfrog with length 162 millimeters. Plot 2 shows that the point for the bullfrog with length 162 millimeters is below the least-squares regression line.	**Essentially correct (E)** if the response satisfies the following two components: 1. The response to part (d-i) identifies the correct bullfrog (length between 160 and 165 millimeters, mass between 350 and 375 grams) 2. The response to part (d-ii) explicitly indicates whether the linear model overestimates or underestimates mass for the bullfrog identified in part (d-i) and provides a correct justification based on a comparison of the identified observation to the least-squares regression line **Partially correct (P)** if the response satisfies only one of the two components. **Incorrect (I)** if the response does not satisfy the criteria for E or P.

Additional Notes:

- The comparison of the observation to the regression line in the response to part (d-ii) is satisfied if the response does one of the following:
  - Correctly indicates if the observation is below (above) the least-squares regression line in Plot 2.
  - Notes that observed mass is smaller (larger) than the mass predicted by the least-squares regression line.
  - Marks the observation selected in part (d-i) on Plot 2 with an indication of the vertical distance from the least-squares regression line.
  - Notes the correct sign of the residual.
- Numerical values are not required in the response to part (d-ii). If a numerical value is given for the predicted mass, however, it must be reasonable. A numerical value for the predicted mass could be computed with the formula given in the stem, e.g., $-546 + (6.086)(162) = 439.9$ grams, for a bullfrog of length 162 millimeters, or a value can be read from the line shown in Plot 2. Any value between 425 and 450 should be considered a reasonable value. Showing work is not required.
- The word overestimate with the calculated predicted value of mass is enough to satisfy component 2.
- If the wrong observation is identified in part (d-i), the response to part (d) may be scored P if the response to part (d-ii) correctly compares that observation to the least-squares regression line and states the correct conclusion about overestimating or underestimating mass with justification.
- It is not required to refer specifically to the "least-squares regression line."

Scoring for Question 1
Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.

	Score
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**

If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.

Question 6: Investigative Task — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	The boxplots reveal that the team tended to have a higher average per-game attendance during years in the new stadium than during years in the old stadium because the median value of about 25,000 attendees per game during years in the new stadium is greater than the median value of about 16,000 attendees per game during years in the old stadium. The interquartile ranges (IQRs) are similar, which indicates that variability in average per-game attendance is similar during the years in the two stadiums, but the range of average per-game attendance is slightly larger during the years in the new stadium. There are no years with average per-game attendance identified as an outlier for either stadium.	**Essentially correct (E)** if the response satisfies the following three components: 1. Indicates that the median average per-game attendance is greater in the new stadium *OR* Indicates that the average per-game attendance is usually (typically) greater in the new stadium 2. Indicates that the variability in average per-game attendance is greater in the new stadium *OR* Indicates that the variability in the two stadiums is roughly the same according to the IQRs 3. Provides sufficient context by including the response variable (average attendance or attendance) or the units of the response variable (thousands of people or people) **Partially correct (P)** if the response satisfies only two of the three components *OR* if the response satisfies only component 1 *OR* if the response satisfies only component 2. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- To satisfy component 1, the response may refer only to "center" and does not need to specify which measure of center (e.g., median, mean) is being used.

- To satisfy component 2, a response may refer only to "variability" or "spread" and does not need to specify which measure of variability (e.g., range or IQR) is being used. However, if the response states that the variability is about the same, the response must explicitly refer to the IQRs.
- To satisfy components 1 and 2, the stadiums must be identified (e.g., "old," "new"), and an explicit comparison phrase (e.g., "greater than," "about the same as") must be used. Separate lists of characteristics alone or summary statistics alone do not count as a comparison.
- To satisfy components 1 and 2, numerical values are not required. However, if they are included, they should be reasonably correct. Numerical values can be reported in units of people (e.g., median = 16,000) or in thousands of people (e.g., median = 16).
- Any mention of shape is ignored in the scoring of part (a) because complete shape information cannot be obtained from a boxplot. Statements about shape not clearly supported by the boxplots (e.g., "the old stadium distribution is approximately normal") should be considered a negative in terms of holistic scoring. However, statements about shape that are supported by the boxplots (e.g., "the old stadium distribution is roughly symmetric," "the new stadium distribution is skewed to the left") should be considered a positive.

	Model Solution	Scoring
(b)	During the years in the new stadium, the average per-game attendance increases linearly, from about 16,000 people in 2000 to about 27,000 people in 2016. However, during the years in the old stadium, there is no obvious increasing or decreasing trend over time for the average per-game attendance. The average attendance appears to vary about an average of approximately 16,000 attendees per game from 1970 to 1999.	**Essentially correct (E)** if the response satisfies the following three components: 1. Describes the direction of the trend in average per-game attendance in the new stadium as increasing (positive) 2. Describes the direction of the trend in average per-game attendance in the old stadium as relatively constant (e.g., "no association," "flat") *OR* Describes the direction of the trend in the old stadium as positive but less steep (e.g., "less positive," "flatter") than the trend in the new stadium 3. Provides sufficient context by including the two groups (old stadium, new stadium) AND the explanatory variable (time or year) AND the response variable (average attendance or attendance) or the units of the response variable (thousands of people or people) **Partially correct (P)** if the response satisfies only two of the three components **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Only describing an association or correlation as "strong" or "weak" addresses strength and not the direction of the trend and does not satisfy components 1 or 2.
- Only describing an association as "linear" or "non-linear" addresses form and not the direction of the trend and does not satisfy components 1 or 2.
- Numerical values, including years (e.g., "from 2000 to 2016"), are not required for any component. However, a response that includes years in numerical form (e.g., "1970") satisfies the context requirement for the explanatory variable in component 3.
- Component 1 can also be satisfied if the response provides an estimated value for the correlation for the new stadium that is positive.
- Component 2 can also be satisfied if the response provides an estimated value for the correlation for the old stadium of 0 (or approximately 0). Providing positive correlations for both stadiums does not satisfy component 2 because it is impossible to compare the steepness of the trends using their correlations.

Model Solution	Scoring
(c) **(i)** Graph I indicates a strong, positive, linear relationship between average per-game attendance and the number of games won during the 47 years of the team's existence. Average per-game attendance increases linearly, with an average increase of about 500 attendees per game for each additional game won. Variation about the linear trend in attendance is relatively small and about the same for any number of games won. **(ii)** No. Graph II suggests that the rates at which average per-game attendance increases as the number of games won increases are about the same for the two stadiums. A line drawn through the points for the old stadium has about the same slope as (or may have a slightly larger slope than) a line drawn through the points for the new stadium.	**Essentially correct (E)** if the response satisfies four or five of the following five components: 1. In part (c-i) describes the direction of the relationship as positive 2. In part (c-i) describes the form of the relationship as linear or nearly linear 3. In part (c-i) describes the strength of the relationship as very strong, strong, or moderately strong 4. In part (c-ii) indicates that the rates are about the same for the two stadiums (or slightly larger for the old stadium) 5. In part (c-ii) provides an explanation that indicates that if a line were drawn through the points for the old stadium, the slope would be roughly the same (or slightly greater than) the slope of a line through the points for the new stadium **Partially correct (P)** if the response satisfies only three of the five components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response that provides an estimated value of the correlation satisfies component 1 if the estimated correlation is positive, but an estimated correlation cannot satisfy components 2 or 3.
- A response need not include the word "positive" to satisfy component 1. For example, "the average attendance is higher when the team has more wins" satisfies component 1. Likewise, a response need not include the word "strong" to satisfy component 3. For example, "variation about the linear trend in attendance is relatively small" satisfies component 3.
- Correct comments on homogeneous variation in part (c-i) (i.e., the variation about the linear trend in attendance is about the same for any number of games won) should be considered a positive in terms of holistic scoring.
- Responses that satisfy all 5 components should be considered a positive in terms of holistic scoring.
- Context is not required in part (c) because it has already been assessed in parts (a) and (b).

Model Solution	Scoring
(d) The number of games won could be a confounding variable for assessing the potential effect of opening the new stadium on average per- game attendance. The boxplots in part (a) show that average per-game attendance tended to be higher for games in the new stadium than for games in the old stadium, but the cause of the increase may actually be that attendees were more excited about attending games for teams that were better at winning. The scatterplots in part (c) show that average per-game attendance has a strong positive correlation with games won, and the team tended to win more games playing in the new stadium than in the old stadium.	**Essentially correct (E)** if the response provides an explanation that satisfies the following four components: 1. States that there is an association between attendance and one of the explanatory variables (stadium, year, wins) 2. States that there is an association between attendance and a different one of the explanatory variables (stadium, year, wins) 3. States that there is an association between the two explanatory variables (stadium, year, wins) identified in components 1 and 2 4. Explains the idea of confounding by describing that the variable identified as a potential confounding variable could be the cause of the association between attendance and the other explanatory variable identified in components 1 and 2 *OR* Explains the idea of confounding by stating that it is impossible to know which of the two explanatory variables identified in components 1 and 2 may be the cause of the increase in attendance **Partially correct (P)** if the response satisfies only three of the four components *OR* if the response satisfies only two of the four components and justifies at least one of the statements in components 1 through 3 by referring to the appropriate graph from parts (a) through (c) (e.g., "based on the boxplots," "in part (b)"). **Incorrect (I)** if the response does not otherwise meet the criteria for E or P.

Additional Notes:

- The response can use any combination of 2 of the 3 explanatory variables (stadium and wins, stadium and year, year and wins). The response cannot introduce a new variable (e.g., weather, having popular players) to satisfy any component.
- An incorrectly described association cannot be used to satisfy components 1 through 3.
- To satisfy component 4 the response must discuss all three variables: the response variable (attendance)

- The strength of the response to part (d), especially communication in component 4, can be considered in holistic scoring. A well-communicated response to part (d) should be considered a positive. A poorly-communicated response to part (d), should be considered a negative.

Scoring for Question 6 Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.	**Score**
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**
If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.	

AP® Statistics
2016 Scoring Guidelines

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe the distribution of a quantitative variable based on a histogram and (2) determine the effect of changing one data value on mean and the median.

Solution

Part (a):

The distribution of Robin's tip amounts is skewed to the right. There is a gap between the largest amount (in the \$20 to \$22.50 interval) and the second largest tip amount (in the \$12.50 to \$15 interval), and the largest tip amount appears to be an outlier. The median tip amount is between \$2.50 and \$5.00. Robin's tip amounts vary from a minimum of between \$0 and \$2.50 to a maximum of between \$20.00 and \$22.50. About 78 percent of the tip amounts are between \$0 and \$5.

Part (b):

ɜ mean: If the \$8 tip had been \$18, the mean would increase by \$10 divided by 60, or $\$\frac{1}{6}$, or about 17 cents.

ɜ median: If the \$8 tip had been \$18, the median would not change because the current median is between \$2.50 and \$5.00, and both \$8 and \$18 are greater than that.

Scoring

Parts (a) and (b) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes reasonable comments on the following five components:

1. Shape (skewed right)
2. Outlier (at least one) *OR* gap (one tip amount greater than \$20, next highest at most \$15)
3. Center between \$2.50 and \$5.00 (median) or between \$2.62 and \$5.13 (mean)
4. Variability, by noting that the tip amounts vary from about \$0 to at most \$22.50, or that a majority of tip amounts are between \$0 and a value greater than or equal to \$5, or by providing a correct numerical approximation of a measure of variability
5. Context (tip amounts)

Partially correct (P) if the response includes only three or four of the five components.

Incorrect (I) if the response includes at most two of the five components.

SCORING GUIDELINES

Question 1 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response includes the following four components:
1. Comments that the mean will increase
2. Correctly justifies why the mean will increase
3. Comments that the median will not change
4. Correctly justifies why the median will not change

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes at most one of the four components.

4 Complete Response

Both parts essentially correct

3 Substantial Response

One part essentially correct and one part partially correct

2 Developing Response

One part essentially correct and one part incorrect

OR

Both parts partially correct

1 Minimal Response

One part partially correct and one part incorrect

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question and (2) assess the effectiveness of treatments in a controlled experiment.

Solution

Part (a):

Step 1: States a correct pair of hypotheses.

H_0 : The proportion of children who would choose each snack is the same regardless of which type of ad is viewed.
H_a : The proportion of children who would choose each snack differs based on which type of ad is viewed.

Step 2: Identifies a correct test procedure (by name or formula) and checks appropriate conditions.

The appropriate procedure is a chi-square test of homogeneity.

The conditions for this test are satisfied because (1) the question states that the children were randomly assigned to groups, and (2) expected counts for the six cells of the table are all at least 5, as seen in the following table that lists expected counts beside observed counts.

Group	Choco-Zuties	Apple-Zuties	Total
A	21 (18.67)	4 (6.33)	25
B	13 (18.67)	12 (6.33)	25
C	22 (18.67)	3 (6.33)	25
Total	56	19	75

Step 3: Calculates the appropriate test statistic and p-value.

The test statistic is calculated as $\chi^2 = \sum \frac{(O-E)^2}{E}$, which is

$$\chi^2 \approx 0.292 + 0.860 + 1.720 + 5.070 + 0.595 + 1.754 \approx 10.291.$$

The p-value is $P(\chi^2_{df2} \geq 10.291) \approx 0.006$.

Step 4: States a correct conclusion in the context of the study, using the result of the statistical test

Because the value is very small (for instance, much smaller than $\alpha = 0.05$ we reject the null hypothesis at the 05 level (and at the 01 level). The data provide convincing statistical evidence that the proportions who would choose each snack differ based on which ad is viewed.

SCORING GUIDELINES

Question 2 (continued)

Part (b):

When neither ad was viewed, $\frac{22}{25}$ or 88 percent of the children chose Choco-Zuties whereas only
)ercent chose Apple-Zuties.

When the Choco-Zuties ad was viewed, 84 percent of the children chose Choco-Zuties, which was very similar to the percentage that chose them without viewing any ad. So watching the Choco Zuties ad did not affect the snack choice very much.

:n the Apple-Zuties ad was viewed, only $\frac{13}{25}$ or 52 percent of the children chose Choco-Zuties, and
)ercent chose Apple-Zuties. Watching the Apple-Zuties ad seemed to increase the proportion of children choosing Apple-Zuties.

Scoring

This question is scored in four sections. Section 1 consists of steps 1 and 2 in part (a); section 2 consists of step 3 in part (a); section 3 consists of step 4 in part (a); and section 4 consists of part (b). Sections 1, 2, 3 and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following three components:
1. Both hypotheses are stated correctly with at least one in context.
2. Identifies the correct test procedure (by name or by formula).
3. The technical conditions are checked (all expected counts are greater than or equal to 5).

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response correctly includes at most one of the three components.

Notes for component 1:
- It is acceptable if hypotheses are stated in terms of association/independence instead of proportions.
- Context must be included in at least one of the hypotheses, but is not required to be in both.
- If the hypotheses contain language that suggests that the response refers to the sample data, the component is not satisfied.

Note for component 2: It is acceptable if the test is identified as a test of independence instead of a test of homogeneity.

Notes for component 3:
- The random assignment condition was stated so need not be explicitly mentioned.
- Stating the expected count condition is met is not sufficient for component 3. The condition must be checked by reporting the expected counts and either:
    - noting that all are greater than or equal to 5;

SCORING GUIDELINES

Question 2 (continued)

    - noting that all are greater than or equal to 1 and at most 20 percent are less than 5.
- Noting that the smallest expected count is 6.33 and that it is greater than or equal to 5 is sufficient to satisfy this component.
- Component 3 is not satisfied if the expected counts are reported as integers.
- This component is not satisfied if the response includes any of the following inappropriate conditions:
    - The response implies that a random sample was taken, e.g., "SRS – check."
    - The response refers to independence of groups or independence of ads as a required condition.
    - The response indicates that a sample size greater than 30 ensures normality or the response implies normality as a condition.
- A response stating that children are independent can be ignored in the scoring of this component.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following three components:
1. Correct chi-square test-statistic
2. Correct degrees of freedom
3. Correct *p*-value

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response correctly includes at most one of the three components.

Notes:
- If the response makes an error in one calculation, future calculations are considered correct if they follow correctly from the initial miscalculation.
- A chi-square critical value approach is acceptable: The critical value for $\alpha = 0.05$ is $\chi^2 = 5.99$.

Section 3 is scored as follows:

Essentially correct (E) if the response provides a correct conclusion about the alternative hypothesis context, *AND* provides justification based on linkage between the *p*-value and conclusion.

Partially correct (P) if the response provides a correct conclusion in context, but without justification based on linkage to the *p*-value;
OR
if the response provides a correct conclusion, with linkage to the *p*-value, but not in context;
OR
if the response provides a correct decision stated in terms of the null hypothesis in context, with linkage to the *p*-value, but no conclusion is made about the alternative hypothesis.

Incorrect (I) if the response does not meet the criteria for E or P.

SCORING GUIDELINES

Question 2 (continued)

Notes:

- If the conclusion is consistent with the *p*-value from section 2, and also in context with justification based on linkage to the *p*-value, section 3 is scored as E.
- If no alpha level is given, the solution must be explicit about the linkage by giving a correct interpretation of the *p*-value or by explaining how the conclusion follows from the *p*-value such as saying: "Because the *p*-value is small, we reject the null hypothesis." or "Because the alue is large, we do not reject the null hypothesis."
- If a conclusion contains language that suggests that the response refers to the sample data, conclusion component is not correct, unless the same error occurred in the statement of hypotheses in section 1.

Section 4 is scored as follows:

Essentially correct (E) if the response, in context, concludes that ad type A had little effect id type B had an effect, both supported by the observed proportions (or counts) from the table.

Partially correct (P) if the response correctly concludes that ad type A had little effect, and ad type B had an effect but does not provide correct numerical justification;
OR
if the response compares all of the proportions (or counts) as required, but without correctly concluding ad effectiveness;
OR
if the response correctly describes the effect of one of the ad types A or B (in context, with correct numerical justification) but not the other;
OR
if the response compares all of the proportions (or counts) as required, but not in context.

Incorrect (I) if the response does not meet the criteria for E or P.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as ½ point.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether score up or down, depending on the strength of the response and communication.

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) identify explanatory and response variables from the description of a research study; (2) indicate and justify whether a study is observational or experimental; and (3) explain what confounding means in the context of a particular study with a specific confounding variable.

Solution

Part (a):

The explanatory variable is the person's degree of cigarette smoking. The response variable is whether the person develops Alzheimer's disease during the course of the study.

Part (b):

This is an observational study because the people in the study were not assigned to a certain degree of cigarette smoking. Rather, the degree of cigarette smoking for each person was passively observed and recorded, not manipulated by the researchers.

Part (c):

nfounding variable is one that is related to the explanatory variable and possibly influences response variable. In this case it seems plausible that people who exercise more regularly might be more health conscious, therefore, less likely to smoke cigarettes than people who do not exercise regularly. Similarly, it's possible that people who exercise more regularly are less likely to develop Alzheimer's disease than people who do not exercise regularly. If both of these relationships turn out to be true, then smoking cigarettes would be associated with developing Alzheimer's disease due to the association of both variables with exercise, even if there were no cause-and-effect relationship between smoking cigarettes and developing Alzheimer's disease.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if both variables are described correctly. A correct description includes some degree of status of the variables, such as smoking versus not smoking and developing Alzeimer' versus not developing Alzheimer's.

Partially correct (P) if one variable is described correctly and one is not described correctly;
OR
ıeither variable is described correctly but smoking is mentioned as the explanatory variable, and Alzheimer's is mentioned as the response variable;
OR
the explanatory and response variables are correctly described but are interchanged.

Incorrect if the response does not meet the criteria for E or P.

SCORING GUIDELINES

Question 3 (continued)

Note: Describing the variables only as smoking and Alzheimer's without a reference to levels/status not sufficient for E. But making the connection between smoking and explanatory and between Alzheimer's and response is sufficient for P.

Part (b) is scored as follows:

Essentially correct (E) if the response selects the correct type of study (observational) and provides the correct explanation that smoking status was not assigned *OR* that smoking status was only observed.

Partially correct (P) if the response selects the correct type of study (observational) and provides correct explanation but does not refer to smoking status as the variable that would have been assigned had it been an experiment; for instance, by simply stating that treatment was not assigned;
OR
if the response selects the correct type of study (observational) and provides an explanation that says that smoking is observed (or that smoking status and Alzheimer's are observed) without a modifier for observed (such as, only, just, merely, simply) *AND* without indicating that treatments were not assigned.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- A response that states the explanatory variable was not assigned without naming smoking status is sufficient for E if the explanatory variable is correctly defined in part (a).
- A response that states that smoking status is ONLY observed (or that smoking status and Alzheimer's are ONLY observed) is sufficient for E.
- A response that says that Alzheimer's is observed without mentioning smoking status is scored I.
- A response that provides an incorrect statistical explanation (such as, "the study is observational because an experiment must have a control group") lowers the score in part (b) by one level (from E to P or from P to I).
- If Alzheimer's is given as the explanatory variable and smoking is given as the response variable in part (a), then part (b) should be scored accordingly with the two variables interchanged.
- Any phrase that refers to the "effect of smoking on Alzheimer's" or "the association between smoking and Alzheimer's" (rather than smoking status) should be ignored.

Part (c) is scored as follows:

Essentially correct (E) if the response includes the following two components:
1. Provides a reasonable explanation that exercise status is related to smoking status.
2. States that exercise status might influence whether the person develops Alzheimer's disease.

Partially correct (P) if the response describes only one of the two components;
OR
if the response only describes how smoking and exercise jointly influence whether the person develops Alzheimer's;

SCORING GUIDELINES

Question 3 (continued)

if the response mentions both of the associations in each component, but also includes an incorrect statistical statement. For instance, part (a) is correct, but in part (c), the explanatory and response variables are interchanged.

Incorrect (I) if the response does not meet the criteria for E or P.

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

SCORING GUIDELINES

Question 4

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a probability using basic probability rules or the geometric distribution; (2) recognize that a probability calculation for independent events does not depend on the previous outcomes of those events; and (3) assess whether a claim about the probability of a single event is reasonable based on a calculated probability of a series of those events.

Solution

Part (a):

If the failure rate for the super igniters is 15 percent, then the probability that each igniter fails is 0.15, and the probability that it does not fail is 0.85. Therefore the probability that the first 30 igniters tested do not fail is $(0.85)^{30} \approx 0.0076$. The solution can also be written as $(1 - 0.15)^{30} \approx 0.0076$.

Part (b):

Given that there are no failures in the first 30 trials, the probability that the first failure occurs on the trial is 0.15, and the probability that it does not occur on the 31st but occurs on the 32nd trial $(0.85)(0.15) = 0.1275$. Therefore the probability that the first failure occurs on the 31st or 32nd super igniter tested is $0.15 + 0.1275 = 0.2775$.

Note that this is equivalent to asking for the probability that the first failure occurs on the first or second trial, which is $0.15 + (0.85)(0.15) = 0.2775$.

Part (c):

The result of the probability calculation in part (a) provides a reason to believe that the failure rate of the super igniters is less than 15 percent. The calculated probability of 0.0076 shows that there is less than a 1 percent chance that 30 or more igniters in a row would not fail if the failure rate was ercent. This probability is smaller than conventional significance levels such as $\alpha = 0.05$ $\alpha = 0.01$, and thus is small enough to make it reasonable to believe that the failure rate of the super igniters is less than 15 percent.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response gives the correct probability *AND* correct justification.

Partially correct (P) if the response correctly notes that the answer is the probability that there will be 30 successes in 30 attempts, but does not carry out a correct probability calculation;
OR

SCORING GUIDELINES

Question 4 (continued)

if the response defines the random variable X as the trial with the first failure, identifies X as having a geometric distribution with $p = 0.15$, and writes the desired probability as $P(X > 30)$, but does not carry out a correct probability calculation;

OR

if the response defines the random variable X as the number of failures in the first 30 attempts, identifies X as a binomial random variable with $p = 0.15$ and $n = 30$, and writes the desired probability as $P(X = 0)$, but does not carry out a correct probability calculation;

OR

if the response gives the correct probability but, in specifying a geometric or binomial distribution, has an incorrect or incomplete definition of parameters or value(s) of the random variable.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Justification can be given using the multiplication rule; *OR* by defining X to be the trial with the first failure, recognizing that X has a geometric distribution, and using that information to find $P(X > 30)$; *OR* by defining X to be the number of failures in the first 30 attempts, and then finding $P(X = 0)$ using either probability rules or the binomial distribution with $n = 30$ and $p = 0.15$.

Part (b) is scored as follows:

Essentially correct (E) if the response gives the correct probability AND correct justification.

Partially correct (P) if the response makes a reasonable attempt to calculate a geometric, binomial, or conditional probability, but does not successfully carry out the calculation;

OR

if the response gives the correct probability but, in specifying a geometric or binomial distribution, has an incorrect or incomplete definition of parameters or value(s) of the random variable.

Incorrect (I) if the response finds an incorrect probability resulting from an unreasonable attempt to calculate a geometric, binomial, or conditional probability or otherwise does not meet the criteria for E or P.

Note: Similar to part (a) justification can be given using probability rules; *OR* by stating that X geometric where X is the trial with the first failure, then finding $P(X = 1 \text{ or } X = 2)$; *OR* by stating that s the number of failures in two trials and finding $1 - P(X = 0)$ or $P(X = 1 \text{ or } X = 2)$ using the binomial distribution.

Part (c) is scored as follows:

Essentially correct (E) if the response states that it is reasonable to believe that the failure rate is less than 15 percent *AND* bases this decision on the fact that the probability of 30 consecutive successful launches with a failure rate of 15 percent (that is, answer from part (a)) is small *AND* does so in the context of the situation.

Partially correct (P) if the response otherwise satisfies the criteria for an (E) but does so without any context;

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Question 4 (continued)

if the response states a significance level and makes a decision in a context that is appropriate to the given probability in part (a) and the stated significance level but does not explicitly compare the probability and the significance level (no linkage).

Incorrect (I) if the response does not explicitly make a decision about whether it is reasonable to conclude that the failure rate is less than 15 percent (For example: "As seen in Part (a), if the failure rate is 15 percent then the probability of 30 successful launches in a row is very small.");
OR
if the response otherwise does not meet the criteria for E or P.

Notes:

- Justification based on the probability can come by stating a significance level and noting that the probability is smaller than the significance level *OR* by simply stating that the probability of 0.0076 is small *OR* by referring to the expected number of failures (4.5) as being very unlikely because zero failures is more than two standard deviations below 4.5.
- If the response bases the decision on the expected number of failures (4.5) for $n = 30$ and $p = 0.15$ without referencing why zero failures would be considered to be too far below 4.5 to give reason to doubt the stated 15 percent failure rate, the response is scored P.
- If the calculation in part (a) is incorrect, the answer in part (c) needs to be consistent with the answer in part (a), unless the value is recalculated in part (c).

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct
OR
One part essentially correct and one or two parts partially correct
OR
Three parts partially correct

1 Minimal Response

One part essentially correct
OR
No parts essentially correct and two parts partially correct

SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) construct and interpret a confidence interval for a population proportion; (2) explain why one of the conditions for inference necessary; and (3) explain why a suggested procedure for constructing a confidence interval is incorrect.

Solution

Part (a):

The appropriate procedure is a one-sample z-interval for a population proportion. The problem stated the conditions for inference have been met, so they do not need to be checked. A 95 percent confidence interval for the population proportion is given as $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, which is

$$0.37 \pm 1.96\sqrt{\frac{(0.37)(0.63)}{1{,}048}} \approx 0.37 \pm 0.03 = (0.34, 0.40).$$

We are 95 percent confident that the population proportion of all adults in the U.S. who would have chosen the economy statement is between 0.34 and 0.40.

Part (b):

The condition is necessary because the formula for the confidence interval relies on the fact that the binomial distribution can be approximated by a normal distribution which then results in the sampling distribution of $\hat{p}$ being approximately normal. The approximation does not work well unless both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10.

Part (c):

The suggested procedure is not appropriate because one of the requirements for using a two-sample interval for a difference between proportions is that the two proportions are based on two independent samples. In the situation described the two proportions come from a single sample and thus are not independent.

Scoring

This question is scored in three sections. Section 1 consists of the mechanics of the confidence interval in part (a), section 2 consists of interpreting the confidence interval in part (a), and section 3 consists of parts (b) and (c).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following two components:
1. States the correct procedure by name or formula
2. Calculates the confidence interval

Partially correct (P) if the response includes only one of the two components.

Incorrect (I) if the response does not meet the criteria for E or P.

SCORING GUIDELINES

Question 5 (continued)

Notes:

- A formula with correct values is sufficient for component 2.
- Component 2 can never be satisfied if the response contains an incorrect formula.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following four components for the interval interpretation:

1. Estimates a proportion
2. Infers about the population
3. States 95 percent confidence
4. Includes context

Partially correct (P) if the response satisfies components 1 and 2 *AND* satisfies only one of components 3 or 4;
OR
if the response gives a correct interpretation of the confidence *level* in context without interpreting the specific interval.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Any indication of an inference to the adults sampled rather than the population does not satisfy component 2 and is scored as I.
- Stating values that are unrealistic as proportions (including blanks) in the interpretation lower the score by one level (from E to P or from P to I).
- When both the interpretation of the interval and the level are given, only the interpretation of the interval is scored.

Section 3 is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Part (b) states that the condition implies the sampling distribution of $\hat{p}$ is approximately normal *OR* the normal approximation to the binomial distribution is appropriate.
2. Part (c) indicates the procedure is not appropriate because the two proportions come from single sample (dependent) rather than two (independent) samples.

Partially correct (P) if the response includes only one of the two components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Referring to the sampling distribution needing to be normal without explicitly stating $\hat{p}$ satisfies component 1.
- Referring to normal approximation must specify "to binomial" to satisfy component 1.
- Discussing the two ample nterval versus the two roportion nterval is extraneous information.

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Question 5 (continued)

- In part (c) component 2 can never be satisfied if a procedure that is not possible for one sample of categorical data is suggested.

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use a scatterplot to comment on eport about the relationship between two variables and interpret the slope for the least-squares regression line summarizing this relationship; (2) describe the relationship between two variables scatterplot when a categorical variable is introduced and compare a characteristic of the distribution variable for different categories of individuals in a scatterplot; and (3) describe how the associations between two variables for each category of individuals in a scatterplot differ from the overall association i the same scatterplot.

Solution

Part (a):

The scatterplot supports the newspaper report about number of semesters needed to complete an academic program and starting salary because it shows a positive association between these two variables.

Part

The slope is 1.1594. For each additional semester needed to complete an academic program, predicted starting salary increases by €1,159.40.

Part (c):

For the business majors alone, there is a strong, negative, linear association between number of semesters and starting salary. Business majors who need a greater number of semesters to complete an academic program tend to have lower starting salaries.

Part (d):

Business majors have the lowest median starting salary at around €38,000, followed by physics majors at around €48,000, and then chemistry majors with the highest median starting salary at around €55,000.

Part (e):

The newspaper report should be modified to account for major. Overall, majors that take longer to complete tend to have higher starting salaries, with chemistry the highest, physics the next highest, and business the lowest. However, within a major, students who take a greater number of semesters tend to have lower starting salaries.

Scoring

This question is scored in three sections. Section 1 consists of parts (a) and (b), section 2 consists of parts (c) and (d), and section 3 consists of part (e). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

SCORING GUIDELINES

Question 6 (continued)

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following five components:

1. In part (a) the response addresses the positive association.
2. In part (a) the response uses the positive association to justify that the scatterplot support newspaper report.
3. In part (b) the response correctly identifies the numerical value of the slope from the computer output.
4. In part (b) the response interprets the slope as the change in starting salary for each additional semester, in context.
5. In part (b) the interpretation of slope includes nondeterministic language (e.g., "predicted starting salary" or equivalent) when interpreting the slope.

Partially correct (P) if the response includes three or four of the five components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In part (a) the response can use phrases such as "positive association (correlation, relationship)," "increasing relationship," or describe a positive association (e.g., "the starting salaries are higher when there is a greater number of semesters") to satisfy component 1 However, describing the relationship between only two points does not satisfy this component.
- In part (a) comments about linearity and strength should be ignored, even if the response implies these are required (e.g., "Yes, because the relationship is strong, positive, and linear.").
- In part (a) responses that answer "no," "maybe," "kind of," "somewhat," or equivalent do not satisfy component 2.
- In part (a) a response that says "no, because the three clusters of points each have a negative association" does not satisfy component 1 or component 2.
- In part (a) no context is required, but variable names are required in part (b) to satisfy component 4.
- In part (b) a response that states the equation $\hat{y} = 34.018 + 1.1594x$ does not satisfy component 3 unless the slope is specifically identified or used in the interpretation.
- In part (b) a response that incorrectly identifies the numerical value of the slope can still satisfy components 4 and 5 using the incorrect value.
- In part (b) the 1-unit increase in number of semesters must be stated or implied to satisfy component 4 (e.g., for each semester, for every semester). A response that states or implies an unspecified number of semesters does not satisfy this component (for example, as semesters increase).
- In part (b) examples of nondeterministic language include "predicted starting salary," "expected starting salary," "estimated starting salary," "typical starting salary," "average starting salary," "starting salary, on average," "our model predicts," and so on. However, "about," "approximately," and "according to the model" do not satisfy component 5.
- In part (b) no units are required for the change in predicted starting salary (which means OK to say 1.1594 euros, 1159.40, 1.1594, 1,159.40 dollars, and so on).
- In all parts it is acceptable if a response refers to salary rather than starting salary.

SCORING GUIDELINES

Question 6 (continued)

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following five components:

1. In part (c) the response states that the association is negative.
2. In part (c) the response states that the association is strong *OR* linear *OR* both.
3. In part (c) the response refers to both variables (semesters, salary) in context.
4. In part (d) the response correctly compares the three majors.
5. In part (d) the response provides reasonable values for the median salaries or refers to "median starting salaries" when describing a characteristic of the graph.

Partially correct (P) if the response includes three or four of the five components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In part (c) the response can use phrases such as "negative association (correlation, relationship)," "decreasing relationship," "inversely related," or describe a negative association (for example, "the starting salaries are smaller when there is a greater number of semesters") satisfy component 1. However, describing the relationship between only two points does not satisfy this component.
- In part (c) responses that describe the relationship incorrectly as weak or nonlinear do satisfy component 2, even if the other characteristic is described correctly.
- Only drawing a line on the scatterplot does not satisfy component 2.
- In part (d) a response that says "chemistry has the highest median starting salary and business has the lowest median starting salary" (or the equivalent) implies that physics is in the middle, and satisfies component 4.
- In part (d) if no values are provided for the medians, the response must use the phrase "median starting salary" at least once to satisfy component 5.
- In all parts it is acceptable if a response refers to salary rather than starting salary.

Section 3 is scored as follows:

Essentially correct (E) if the response states that there is a negative association for each of the majors *AND* the response notes the overall positive association.

Partially correct (P) if the response states that there is a negative association for each of the majors BUT does not note the overall positive association;
OR
if the response states that there is a negative association for one or two specific majors (for example, for business majors) *AND* the response notes the overall positive association.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Additional ways to note the overall positive association include stating that the original report was correct, stating that majors that take longer to complete tend to have higher starting salaries, or the equivalent.

SCORING GUIDELINES

Question 6 (continued)

- A response that does not explicitly name the majors or refer to the three majors but makes a general statement such as "However, for an academic major, there is a negative association" satisfies the requirement to state the negative association for each of the majors.
- A response that states the original report was wrong, or incorrect, or should be retracted, cannot satisfy the requirement to note the overall positive association. However, a response that says the original report might be misleading (or the equivalent) but still notes the overall positive association satisfies the requirement to note the overall positive association.
- In all parts it is acceptable if a response refers to salary rather than starting salary.

4 Complete Response

All three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

OR

Sections 1 and 2 partially correct and section 3 essentially correct

OR

Sections 1 and 3 essentially correct and section 2 incorrect

OR

Sections 2 and 3 essentially correct and section 1 incorrect

2 Developing Response

Sections 1 and 2 essentially correct and section 3 incorrect

OR

Section 1 essentially correct and sections 2 and 3 partially correct

OR

Sections 1 and 3 partially correct and section 2 essentially correct

OR

One section essentially correct and one section partially correct

OR

All three sections partially correct

1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect

OR

One section partially correct and two sections incorrect

AP® Statistics
2015 Scoring Guidelines

2015 SCORING GUIDELINES

Question 1

Intent of Question

The primary goals of this question were to assess a student's ability to (1) compare features of two distributions of data displayed in boxplots and (2) identify statistical measures that are important in making decisions based on data sets.

Solution

Part (a):

The median salary is approximately the same for both corporations. The range and interquartile range of the salaries are greater for Corporation A than for Corporation B. The two highest salaries at Corporation A are outliers while Corporation B has no outliers.

Part (b):

(i) Five years after starting, at least 3 out of 30 (10%) of the salaries at Corporation A are greater than the maximum salary at Corporation B. If I accept the offer from Corporation A, I might be able to make a higher salary at Corporation A than at Corporation B.

(ii) Five years after starting, the minimum salary at Corporation B is greater than at Corporation A. In fact, at Corporation A it looks like some people are still making the starting salary of $36,000 and never received a raise in the five years since they were hired. So if I work at Corporation A, I might never receive a raise in salary.

Scoring

Parts (a) and (b) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes the following four components:
1. A correct comparison of center.
2. A correct comparison of spread.
3. A discussion of the outliers for Corporation A.
4. The response is in context.

Partially correct (P) if the response includes only three of the four components.

Incorrect (I) if the response includes at most two of the four components.

Note: Any mention of shape should be ignored because complete shape information cannot be determined from a boxplot.

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Question 1 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response includes the following four components:

1. In part (b-i) a relevant statistical measure is identified (or described) or a relevant statistical comparison is provided that supports the choice of Corporation A.
2. In part (b-i) an explanation is provided for why the measure or comparison is relevant.
3. In part (b-ii) a relevant statistical measure is identified (or described) or a relevant statistical comparison is provided that supports the choice of Corporation B.
4. In part (b-ii) an explanation is provided for why the measure or comparison is relevant.

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes none or one of the four components.

Note: If a response does not provide a statistical measure or comparison in part (b-i) or (b-ii), the second and fourth components can still be satisfied if an acceptable explanation is provided that would follow from a relevant statistical measure or comparison. For example, if the response in part (b-i) only states "At Corporation A, I have the potential to earn a higher salary," the second component is satisfied.

4 Complete Response

Both parts essentially correct

3 Substantial Response

One part essentially correct and one part partially correct

2 Developing Response

One part essentially correct and one part incorrect

OR

Both parts partially correct

1 Minimal Response

One part partially correct and one part incorrect

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Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use confidence intervals to test a question about a proportion and (2) understand the relationship between sample size and margin of error in a confidence interval for a proportion.

Solution

Part (a):

(i) No. The confidence interval is $(0.09, 0.21)$, which includes the value of 0.20. Therefore, it is plausible that the computer program is generating discounts with a probability of 0.20, and the confidence interval does not provide convincing statistical evidence that the program is not working as intended.

(ii) No. The confidence interval includes values from 0.09 to 0.21, so any value in that interval is a plausible value for the probability that the computer is using to generate discounts.

Part (b):

The formula for computing the margin of error for a proportion includes the square root of the sample size in the denominator. For a random sample that is four times the size of the original sample, the margin of error can be determined by dividing the margin of error of the original sample by two. Therefore, the new margin of error is 0.03.

Part (c):

Using the margin of error of 0.03 obtained from the second sample, the confidence interval for p is 0.15 ± 0.03 or $(0.12, 0.18)$. The interval does not include 0.20, and therefore, there is convincing evidence that the computer program is not working as intended and is not generating discounts with a probability of 0.20.

Scoring

This question is scored in four sections. Section 1 consists of part (a-i); section 2 consists of part (a-ii); section 3 consists of part (b); and section 4 consists of part (c). Sections 1, 2, 3, and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response states that because the interval contains 0.20, it does not provide convincing statistical evidence that the computer program is not working as intended.

Partially correct (P) if the response indicates that it is necessary to check whether the value of 0.20 is in the computed interval, but there are errors in implementation. Examples of errors include:

- The response notes that 0.20 is within the interval but does not draw a conclusion.
- The response has an arithmetic error in the computation of the endpoints of the interval but provides a correct conclusion with justification that is consistent with the computed interval.

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Question 2 (continued)

Incorrect (I) if the response does not recognize how to use the confidence interval to check whether the computer is working correctly;

OR

if the response states that the interval shows that the proportion is equal to 0.20;

OR

if the response notes that 0.20 is within the interval and concludes that the program is not working as intended;

OR

if the response otherwise does not meet the criteria for E or P.

Section 2 is scored as follows:

Essentially correct (E) if the response concludes that there is not convincing statistical evidence that the computer program generates the discount with a probability of 0.20 *AND* justifies the conclusion by noting that there are values other than 0.20 in the interval.

Partially correct (P) if the response correctly concludes that there is not convincing evidence that the computer program generates the discount with a probability of 0.20, but provides incomplete reasoning to justify the conclusion.

Examples of incomplete reasoning include:

- stating that 0.20 is a plausible value for the proportion of discounts without giving further explanation;
- indicating that having 0.20 in the interval does not prove that 0.20 is the true proportion; and
- providing a generic statistical argument, such as reference to the fact that the null hypothesis should never be accepted, or stating that not rejecting the null hypothesis is not proof that 0.20 is the true proportion of bills discounted.

Note: If an incorrect interval is computed in part (a-i) that does not contain 0.20, and in part (a-ii) the response concludes that the program is not generating discounts with a probability of 0.20 because 0.20 is not in the interval, section 2 is scored as P.

Incorrect (I) if the response states that there is evidence that the computer program generates the discount with a probability of 0.20;

OR

if the response correctly concludes that there is not convincing evidence that the computer program generates the discount with a probability of 0.20 *AND* provides incorrect or no justification;

OR

if the response otherwise does not meet the criteria for E or P.

Section 3 is scored as follows:

Essentially correct (E) if the response gives the correct value of 0.03 as the new margin of error by using the correct formula or by providing a correct explanation that recognizes that quadrupling the sample size divides the margin of error from the original sample by two.

Note: If the response relies on the margin of error formula and calculates a value that would round to 0.030, the response is scored as E.

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Question 2 (continued)

Partially correct (P) if the response uses the correct margin of error formula or provides a correct explanation that recognizes that quadrupling the sample size divides the margin of error from the original sample by two, but calculates an incorrect margin of error that is less than 0.06;
OR
if the response does not calculate a margin of error but provides a correct explanation that recognizes that quadrupling the sample size divides the margin of error from the original sample by two;
OR
if the response gives the correct margin of error without correct justification.

Incorrect (I) if the response does not recognize in any way that the new margin of error depends on the square root of the sample size;
OR
if the response calculates a margin of error that is greater than or equal to 0.06;
OR
if the response otherwise does not meet the criteria for E or P.

Section 4 is scored as follows:

Essentially correct (E) if the conclusion states that there is convincing evidence that the computer program is not working as intended because, based on the new margin of error, the interval does not contain 0.20;
OR
if the conclusion states the intended value of 0.20 is greater than the upper boundary of 0.18;
OR
if the conclusion states the margin of error is smaller than the difference between the sample proportion and the intended long-run proportion of 0.20.

Notes:

- If the margin of error was computed incorrectly in part (b), but a correct answer to part (c) is consistent with this incorrect margin of error, section 4 is scored as E.
- If no specific margin of error or confidence interval was given in parts (b) or (c), but the response provides a complete and correct conclusion with reference to a smaller margin of error (or narrower confidence interval), section 4 is scored as E.

Partially correct (P) if an interval is incorrectly constructed using the margin of error from part (b), but a correct conclusion is given and justified for the computed interval;
OR
if an interval is computed correctly using the margin of error from part (b) and an argument is made based on whether or not 0.20 is in the interval, but the conclusion is incorrect;
OR
if no specific margin of error or confidence interval has been given in parts (b) or (c), but the response concludes that because the margin of error has decreased (or the confidence interval is narrower), 0.20 is not in the interval.

Incorrect (I) if an interval is computed correctly using the margin of error from part (b) and a correct conclusion is given, but no argument is made based on whether or not 0.20 is in the interval;
OR
if the response otherwise does not meet the criteria for E or P.

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Question 2 (continued)

Notes:

- If a response includes a confidence level, the level can be ignored because no confidence level was provided.
- If the response bases a conclusion on the relative location of 0.20 within the interval (for example, 0.20 is near the edge of the interval), the response is scored as I.
- A response that provides additional incorrect explanation lowers the score in section 4 by one level (that is, from E to P, or from P to I).

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as ½ point.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) perform a probability calculation from a discrete random variable; (2) calculate the expected value of a discrete random variable; (3) perform a conditional probability calculation from a discrete random variable; and (4) use probabilistic thinking to make a prediction about how an expected value will change given a condition about the random variable.

Solution

Part (a):

The probability that at least one ATM is working when the mall opens is:

$$P(X \geq 1) = 0.21 + 0.40 + 0.24 = 0.85.$$

Part (b):

The expected value of the number of ATMs that are working when the mall opens is:

$$E(X) = 0(0.15) + 1(0.21) + 2(0.40) + 3(0.24) = 1.73 \text{ machines.}$$

Part (c):

The probability that all three ATMs are working when the mall opens, given that at least one ATM is working is:

$$P(X = 3 \mid X \geq 1) = \frac{P(X = 3 \text{ and } X \geq 1)}{P(X \geq 1)} = \frac{P(X = 3)}{P(X \geq 1)} = \frac{0.24}{0.85} \approx 0.282$$

Part (d):

Given that at least one ATM is working when the mall opens, the expected value of the number of working ATMs would be greater than the expected value calculated in part (b). By eliminating the possibility of 0 working ATMs, the probabilities for 1, 2, and 3 working ATMs all increase proportionally, so the expected value must increase.

Scoring

(b), (c), and (d) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the probability is computed correctly with work shown.

Partially correct (P) if the correct answer is given, but no work is shown;
OR
appropriate work is shown but the answer is incorrect or missing;
OR

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Question 3 (continued)

one of the incorrect cumulative probabilities $P(X < 1)$, $P(X \leq 1)$, or $P(X > 1)$ is stated *AND* computed correctly.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: The probability can be calculated as $1 - P(X = 0) = 1 - 0.15 = 0.85$.

Part (b) is scored as follows:

Essentially correct (E) if the expected value is computed correctly with work shown.

Partially correct (P) if the correct answer is given, but no work is shown;
OR
if appropriate work is shown but the answer is incorrect or missing.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (c is scored as follows:

Essentially correct (E) if the probability is computed correctly, with work shown that includes correct numerical values for both the numerator and denominator.

Partially correct (P) if the response includes a numerator and denominator in calculating the conditional probability, with one (numerator or denominator) correct in numerical value and the other incorrect;
OR
if the correct answer is given, but no work is shown.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (d) is scored as follows:

Essentially correct (E) if the response provides the correct answer (greater than) with a reasonable explanation based on the fact that with $X = 0$ eliminated, the probabilities for $X = 1$, $X = 2$, $X = 3$ *all* increase;
OR
if the response provides the correct answer (greater than) with a reasonable explanation based on fact that with $X = 0$ eliminated, the balance point of the distribution increases;
OR
if the response provides the correct answer (greater than) and the conditional expected value is computed correctly with work shown.

Partially correct (P) if the response provides the correct answer (greater than) with a weak explanation, such as "Yes, because 0 is eliminated";
OR

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Question 3 (continued)

if the response provides the correct answer (greater than) with a reasonable but incorrect attempt to calculate the conditional expected value using a revised probability distribution with only the values $X = 1$, $X = 2$, and $X = 3$, and corresponding probabilities greater than or equal to those in the original probability distribution.

Incorrect (I) if the response does not provide the correct answer (greater than);
OR
if the response provides the correct answer (greater than) with an incorrect explanation or no explanation.

Note: The conditional expected value is:

$$1\left(\frac{0.21}{0.85}\right) + 2\left(\frac{0.40}{0.85}\right) + 3\left(\frac{0.24}{0.85}\right) \approx 1(0.247) + 2(0.471) + 3(0.282) = 2.04 \text{ machines.}$$

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether score up or down, depending on the overall strength of the response and communication.

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Question 4

Intent of Question

The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and *p*-value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

Solution

Step 1: States a correct pair of hypotheses.

Let p_{asp} represent the population proportion of adults similar to those in the study who would have developed colon cancer within the six years of the study if they had taken a low-dose aspirin each day. Similarly, let p_{plac} represent the population proportion of adults similar to those in the study who would have developed colon cancer within the six years of the study if they had taken a placebo each day.

The hypotheses to be tested are $H_0 : p_{asp} = p_{plac}$ versus $H_a : p_{asp} < p_{plac}$ or equivalently, $H_0 : p_{asp} - p_{plac} = 0$ versus $H_a : p_{asp} - p_{plac} < 0$.

Step 2: Identifies a correct test procedure (by name or by formula) and checks appropriate conditions.

The appropriate procedure is a two-sample *z*-test for comparing proportions.

Because this is a randomized experiment, the first condition is that the volunteers were randomly assigned to one treatment group or the other. The condition is satisfied because we are told that the volunteers were randomly assigned to take a low-dose aspirin or a placebo.

The second condition is that the sample sizes are large, relative to the proportions involved. The condition is satisfied because all sample counts are large enough; that is, 15 with colon cancer in aspirin group, 26 with colon cancer in placebo group, $500 - 15 = 485$ cancer-free in aspirin group, and $500 - 26 = 474$ cancer-free in placebo group.

Step 3: Calculates the appropriate test statistic and *p*-value.

The sample proportions who developed colon cancer are $\hat{p}_{asp} = \frac{15}{500} = 0.030$ and $\hat{p}_{plac} = \frac{26}{500} = 0.052$.

The combined sample proportion who developed colon cancer is $\hat{p}_{combined} = \frac{15 + 26}{500 + 500} = 0.041$.

The test statistic is $z = \frac{0.030 - 0.052}{\sqrt{0.041(1 - 0.041)\left(\frac{1}{500} + \frac{1}{500}\right)}} \approx -1.75$ (-1.7542 from calculator).

The *p*-value is $P(Z \le -1.75) = 0.0401$ (0.0397 from calculator), where Z has a standard normal distribution.

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Question 4 (continued)

Step 4: States a correct conclusion in the context of the study, using the result of the statistical test.

Because the *p*-value is less than the given significance level of $\alpha = 0.05$, we reject the null hypothesis. The data provide convincing statistical evidence that the proportion of all adults similar to the volunteers who would develop colon cancer if they had taken a low-dose aspirin every day is less than the proportion of all adults similar to the volunteers who would develop colon cancer if they had not taken a low-dose aspirin every day.

Scoring

Steps 1, 2, 3, and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Step 1 is scored as follows:

Essentially correct (E) if the response identifies correct parameters *AND* both hypotheses are labeled and state the correct relationship between the parameters.

Partially correct (P) if the response identifies correct parameters *OR* states correct relationships, but not both.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Either defining the parameters in context, or simply using common parameter notation with subscripts clearly relevant to the context, such as p_{asp} and p_{plac}, is sufficient.

Step 2 is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:
1. Identifies the correct test procedure (by name or by formula).
2. Notes that the use of random assignment satisfies the randomness condition.
3. Checks for approximate normality of the test statistic by citing that all four counts are larger than some standard criterion such as 5 or 10.

Partially correct (P) if the response correctly includes only two of the three components.

Incorrect (I) if the response correctly includes at most one of the three components.

Notes:
- For the randomness component, it is (minimally) acceptable to say "random assignment — check" but not acceptable to say "random — check" or "SRS — check." The important concept here is that it is random assignment, and not random sampling, that is required. If the response implies that the study used a random sample, the randomness component is not satisfied, regardless of whether random assignment is correctly addressed.
- The normality check may use the expected counts under the null hypothesis in place of observed counts.

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Question 4 (continued)

Step 3 is scored as follows:

Essentially correct (E) if the response correctly calculates both the test statistic and a *p*-value that is consistent with the stated alternative hypothesis.

Partially correct (P) if the response correctly calculates the test statistic but not the *p*-value;
OR
if the response calculates the test statistic incorrectly but then calculates the correct *p*-value for the computed test statistic;
OR
if the response reports the correct *p*-value but no calculations or test statistic are shown.

Incorrect (I) if the response fails to meet the criteria for E or P.

Note: The *p*-value is considered correct if it is consistent with the alternative stated in the response and the calculated test statistic, even if those are incorrect.

Step 4 is scored as follows:

Essentially correct (E) if the response provides a correct conclusion in context, with justification based on linkage between the *p*-value and the given $\alpha = 0.05$.

Partially correct (P) if the response provides a correct conclusion, with linkage to the *p*-value, but not in context;
OR
if the response provides a correct conclusion in context, but without justification based on linkage to the *p*-value.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- The conclusion must be related to the alternative hypothesis.
- If the *p*-value is incorrect, then step 4 is scored as E if the response includes proper linkage and a conclusion in context consistent with that *p*-value.
- If the *p*-value is less than 0.05, wording that states or implies that the alternative hypothesis is *proven* lowers the score one level (that is, from E to P or P to I) in step 4.
- If the *p*-value is incorrect and greater than 0.05, wording that states or implies that the null hypothesis is *accepted* lowers the score one level (that is, from E to P or P to I) in step 4.

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Question 4 (continued)

Each essentially correct (E) step counts as 1 point. Each partially correct (P) step counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use the information provided by a scatterplot to describe the relationship between two quantitative variables; (2) interpret and use the information given by lines displayed on a scatterplot; and (3) use a regression equation to estimate a predicted value of y for a given x value.

Solution

Part

There is a moderately strong, positive, linear relationship between height and arm span so that taller students tend to have longer arm spans.

Part (b):

The line in Graph 2 is the one that is helpful. For each student, the graph illustrates whether arm is equal to height (points on the line), arm span is greater than height (points above the line), or arm span is less than height (points below the line).

Classification	Square	Tall Rectangle	Short Rectangle
Frequency	3	4	5

Part (c):

The predicted arm span is $\hat{y} = 11.74 + 0.8247x = 11.74 + 0.8247(61) = 62.05$ inches.

Scoring

b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes the following four components:

1. The relationship is approximately linear.
2. The relationship is positive.
3. There is a moderate to strong association (or relationship).
4. The response is given in context.

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes at most one of the four components.

SCORING GUIDELINES

Question 5 (continued)

Notes:

- Correlation alone is not sufficient to indicate a linear relationship. However, linear correlation sufficient to indicate a linear relationship.
- Reporting that the correlation coefficient is 0.81, the correlation coefficient is close to 1.0, or some other value or range of values for the correlation coefficient is not sufficient to satisfy component 3.

Part (b) is scored as follows:

Essentially correct (E) if the response contains the following four components:

1. The line in Graph 2 is selected.
2. A reasonable explanation for selecting the line in Graph 2 that links the body shapes to the regions defined by the line is given. The explanation should indicate that square shapes (arm span is equal to height) are represented by points on the line, tall rectangle shapes (arm spa less than height) are represented by points below the line, and short rectangle shapes (arm span is greater than height) are represented by points above the line.
3. Correct counting of body types demonstrated by reporting correct frequencies (3, 4, and 5) reporting correct proportions $\left(\frac{1}{4} = \frac{3}{12}, \frac{1}{3} = \frac{4}{12}, \text{ and } \frac{5}{12}\right)$ for the square, tall rectangle, and short rectangle body types, respectively.
4. The correct frequencies (3, 4, and 5) are reported in the table.

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes at most one of the four components.

Notes:

- Selecting the regression line on Graph 1 cannot satisfy either component 1 or component 2.
- To satisfy component 1, it is sufficient to refer to the $y = x$ line without explicitly mentioning Graph 2.
- Use of incorrect labels, such as regression line or least-squares regression line, in referring to the $y = x$ line in Graph 2 is an incorrect use of terminology that should be strongly discouraged, but it is ignored in this scoring rubric.
- The explanation for selecting Graph 2 is acceptable if it explicitly includes at least two of the following: (i) square body shapes (arm span equal to height) are represented by points line, (ii) tall rectangle body shapes (arm span less than height) are represented by points *below* the line, or (iii) short rectangle body shapes (arm span greater than height) are represented by points *above* the line.
- Incorrectly counting the points on the $y = x$ line in Graph 2 is considered a minor error. Frequencies (2, 4, and 5) are accepted for both component 3 and component 4. Reporting corresponding proportions $\left(\frac{2}{11}, \frac{4}{11}, \frac{5}{11}\right)$ or $\left(\frac{2}{12}, \frac{4}{12}, \frac{5}{12}\right)$ in the table satisfies component does not satisfy component 4.
- Frequencies reported in the table as (2, 5, 4) or (3, 5, 4), interchanging the counts for tall and short rectangle body shapes, satisfies component 3, but does not satisfy component 4. Reporting corresponding proportions, $\left(\frac{2}{11}, \frac{5}{11}, \frac{4}{11}\right)$ $\left(\frac{3}{12}, \frac{5}{12}, \frac{4}{12}\right)$, in the table does not satisfy either component 3 or component 4.

SCORING GUIDELINES

Question 5 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response contains the following three components:
1. Correct formula for predicting arm span with 61 inserted for x.
2. Correct value for the predicted arm span.
3. Units for the predicted arm span given as inches.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:
- Any value for the predicted arm span between 61.5 and 62.5 is acceptable; values outside th interval do not satisfy component 2. This allows for inaccuracy in obtaining a value for the predicted arm span directly from the line displayed on Graph 1 and for rounding in applying the prediction formula; but it excludes predictions based on the $y = x$ line and other unreasonable predictions.
- Reporting a prediction equation that is similar to the equation given in the stem, with 61 inserted for x, satisfies components 1 and 2 if the value of the predicted arm span is between 61.5 and 62.5 inches. Otherwise, neither of those two components is satisfied. This could occur if a student enters the data from the graph into a calculator to compute the least squares regression line, instead of using the formula provided in the stem.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and two parts partially correct

OR

One part essentially correct, one part partially correct, and one part incorrect

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

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Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe how sample data would differ using two different sampling methods; (2) describe the sampling distribution of the sample mean for two different sampling methods; and (3) choose the sampling method that will result in the best estimate of the population mean.

Solution

Part (a):

No, a sample obtained using Method 2 will not be representative of all tortillas made that day. The sample obtained using Method 2 will only represent the tortillas from one production line, not from the entire population because the distributions of diameters for the two production lines are different.

Part (b):

Method 1 was most likely used to select this sample. The bimodal shape in the histogram of sample data indicates that tortillas were selected from both production lines, which is what would happen using Method 1. Method 2 would be likely to produce a unimodal distribution of diameters centered at either 5.9 inches or 6.1 inches.

Part (c):

Method 2 would result in less variability in the sample of 200 tortillas on a given day because the sample comes from only one production line. Because the distributions of diameters are not the same for the two production lines, selecting tortillas from both lines as in Method 1 would result in more variable sample data.

Part (d):

The sampling distribution of the sample mean diameter for samples obtained using Method 1 would be approximately normal with mean 6 inches and standard deviation $\frac{0.11}{\sqrt{200}} \approx 0.0078$ inch.

Part (e):

Method 1 would result in less variability in the sample means over the 365 days, because with Method 2, roughly half of the sample means will be around 5.9 inches and the other half will be around 6.1 inches. With Method 1, however, the sample means will all be very close to 6 inches, as indicated by the standard deviation in part (d).

Part (f):

Method 1 is more likely to produce a sample mean close to 6 inches. Because the sample mean is an unbiased estimator for both methods, the manager should pick the method that would result in less variability in the distribution of the sample mean. Based on the answer to part (e), Method 1 results in less variability in the distribution of the sample mean.

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Question 6 (continued)

Scoring

This question is scored in three sections. Section 1 consists of parts (a), (b), and (c), section 2 consists of part (d), and section 3 consists of parts (e) and (f). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. In part (a) the response says no *AND* either argues that the sample will only be selected from one production line and not the entire population (that is, the sample will only represent one production line) *OR* argues that the tortillas from the two productions lines are different.
2. In part (b) the response chooses Method 1 *AND* refers to a relevant characteristic of the histogram (shape, center, or variability) that matches what would be expected when using Method 1 (or that does not match what would be expected when using Method 2).
3. In part (c) the response chooses Method 2 *AND* either justifies by stating that the sample comes from only one production line (does not come from both production lines) *OR* justifies by comparing the possible range of diameters for the two methods.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Note: If a response includes more than one justification in an individual part, score the weaker of the two justifications. For example, a response for part (a) that says "No, because only one line was selected and because the sample size was too small" does not satisfy the first component because the sample size argument is incorrect.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. The shape is approximately normal.
2. The mean is 6 inches.
3. The standard deviation is $\frac{0.11}{\sqrt{200}}$ inch.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Note: It is not necessary to include units (inches) for the mean or standard deviation.

Section 3 is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. In part (e) the response chooses Method 1 *AND* describes the sampling distribution of the sample mean for Method 2 as having some *sample means* close to the mean of production line A (5.9 inches) and the other *sample means* close to the mean of production line B (6.1 inches).

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Question 6 (continued)

2. In part (f) the response chooses Method 1 *AND*
   - refers to a correct answer in part (e);

   OR
   - describes the sampling distribution of the sample mean for Method 2 as having some *sample means* close to the mean of production line A (5.9, less than 6) and the other *sample means* close to the mean of production line B (6.1, greater than 6);

   OR
   - argues that on a single day, it would be preferable to get a sample with a mean around 6 rather than getting a sample that would have a mean around 5.9 (less than 6) or a mean around 6.1 (greater than 6).

Partially correct (P) if the response includes one of the two components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In part (e), the response must be clear that there is *more than one* mean being described (365 sample means).
  - Correct: The sample means will be around 5.9 or 6.1.
  - Not correct: The sample mean will be around 5.9 or 6.1.

- Parts (e) and (f) are not satisfied if the response does not imply the sample means *vary* from the population means in both methods (or does not imply that the sample mean varies from the population mean when making the single day argument in part (f)). However, if the mistake is made in part (e), do not penalize the response in part (f) for the same mistake.
  - Correct: The sample means will be around 5.9 or 6.1 instead of close to 6.
  - Not correct: The sample means will be 5.9 or 6.1 instead of close to 6.
  - Not correct: The sample means will be 6 instead of around 5.9 or 6.1.

4 Complete Response

All three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and one section incorrect

OR

One section essentially correct and either one or two sections partially correct

OR

All three sections partially correct

1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect