

8.6 Carrying Out a Chi-Square Test for Homogeneity or Independence

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS chi-square 卡方 • test for homogeneity 同质性

Objective

Build the skills to answer exam questions on **carrying out a chi-square test for homogeneity or independence**.

You must be able to:

- compute χ^2 from a two-way table and find the **p-value**
- compare it to α and reach a decision
- state a conclusion about association in context

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The procedure

1. State the hypotheses; check conditions (expected counts ≥ 5).
2. Compute the expected counts, then $\chi^2 = \sum \frac{(O - E)^2}{E}$, with $df = (r - 1)(c - 1)$.
3. Find the **p-value**.
4. Compare to α : a **small** p-value $\rightarrow$ **reject** $H_0 \rightarrow$ the variables are **associated**.

■ Finding an expected count

Each expected count is $\frac{\text{row total} \times \text{column total}}{\text{grand total}}$. For a cell whose row totals 40 and column totals 50, with grand total 200:

$$E = \frac{40 \times 50}{200} = 10.$$

Compute every E , then sum $\frac{(O - E)^2}{E}$.

2 Practice

2.1 State the steps to carry out a chi-square test for a two-way table. [2]

2.2 State what a small p-value indicates about the two variables. [1]

2.3 A test of independence gives a p-value of 0.02 at $\alpha = 0.05$. State the decision. [1]

3 Exam-style questions

3.1 A chi-square test of independence with a small p-value suggests the variables are [1]

- **A** independent
 - **B** associated
 - **C** exactly equal
 - **D** unrelated
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3.2 A chi-square test rejects H_0 when the p-value is [1]

- **A** above α
 - **B** below α
 - **C** exactly zero
 - **D** exactly one
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3.3 A test of independence gives $\chi^2 = 10$, $\text{df} = 2$, p-value 0.007 at $\alpha = 0.05$.

(a) State the decision. [1]

(b) State whether the variables are associated. [1]

(c) State the conclusion in context.

[1]

Solutions

2.1 state hypotheses and check conditions; compute expected counts and χ^2 ; find the p-value; compare to α and conclude.

2.2 the two variables are associated (not independent).

2.3 reject H_0 ($0.02 < 0.05$).

3.1 B.

3.2 B.

3.3 (a) reject H_0 . (b) yes, they are associated. (c) there is convincing evidence of an association between the two variables.