AP STATISTICS • EXERCISE SHEET

8.3 Carrying Out a Chi-Square Test for Goodness of Fit

Name: _____ Class: _____ Date: _____ **Total: 8 marks**

KEY WORDS degrees of freedom 自由度 • chi-square 卡方

Objective

Build the skills to answer exam questions on **carrying out a chi-square goodness-of-fit test**.

You must be able to:

- compute $\chi^2 = \sum \dfrac{(O-E)^2}{E}$ with degrees of freedom $= k-1$
- find the **p-value** and reach a decision
- state a conclusion in context

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The procedure

1. State the hypotheses and check conditions (all expected counts ≥ 5).
2. Compute $\chi^2 = \sum \dfrac{(O-E)^2}{E}$, with df $= k-1$ (categories minus 1).
3. Find the **p-value** from the chi-square distribution.
4. Compare to α and conclude **in context**.

A **large** χ^2 gives a **small** p-value — stronger evidence against the claimed distribution.

■ A worked statistic

For observed $(18, 12, 30)$ and expected $(20, 20, 20)$,

$$\chi^2 = \frac{(18-20)^2}{20} + \frac{(12-20)^2}{20} + \frac{(30-20)^2}{20} = 0.2 + 3.2 + 5.0 = 8.4,$$

with df $= 3-1 = 2$. This gives a p-value $\approx 0.015 < 0.05$ – reject the claimed distribution.

2 Practice

2.1 State the degrees of freedom for a goodness-of-fit test with k categories. [1]

2.2 State what a large chi-square value suggests. [1]

2.3 A goodness-of-fit test gives a p-value of 0.03 at $\alpha = 0.05$. State the decision. [1]

3 Exam-style questions

3.1 The degrees of freedom for a goodness-of-fit test with 6 categories is [1]

- **A** 6
- **B** 5
- **C** 4
- **D** 1

3.2 A large chi-square statistic gives a _____ p-value. [1]

- **A** large
- **B** small
- **C** zero
- **D** negative

3.3 A goodness-of-fit test of a die (6 faces) gives $\chi^2 = 15$ with $\text{df} = 5$ and p-value 0.01.

(a) State the degrees of freedom. [1]

(b) State the decision at $\alpha = 0.05$. [1]

(c) State the conclusion in context. [1]

Solutions

2.1 $k - 1$.

2.2 a large difference between observed and expected counts (evidence against H_0).

2.3 reject H_0 ($0.03 < 0.05$).

3.1 B.

3.2 B.

3.3 (a) 5. (b) reject H_0. (c) there is convincing evidence the die does not follow the claimed (fair) distribution.