

7.9 Carrying Out a Test for the Difference of Two Population Means

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS distribution 分布

Objective

Build the skills to answer exam questions on **carrying out a test for a difference of two means**.

You must be able to:

- compute the two-sample t statistic and find the **p-value**
- compare the p-value to α and reach a decision
- state a conclusion in context

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The procedure

1. State $H_0 : \mu_1 = \mu_2$ and H_a ; check conditions.
2. Compute $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$.
3. Find the **p-value** from the t -distribution.
4. Compare to α and decide (reject or fail to reject).
5. State the conclusion **in context**.

A **large** $|t|$ gives a small p-value — stronger evidence of a difference.

■ A worked test

$\bar{x}_1 = 20, s_1 = 3, n_1 = 25$ and $\bar{x}_2 = 18, s_2 = 4, n_2 = 25$. Then

$$t = \frac{20 - 18}{\sqrt{3^2/25 + 4^2/25}} = \frac{2}{\sqrt{1}} = 2.0.$$

This gives a p-value near 0.05 – borderline evidence of a difference in means.

2 Practice

2.1 State the steps to carry out a two-sample t-test. [2]

2.2 State what a large $|t|$ suggests. [1]

2.3 A two-sample t-test gives a p-value of 0.06 at $\alpha = 0.05$. State the decision. [1]

3 Exam-style questions

3.1 A two-sample t-test compares [1]

- **A** two proportions
 - **B** two means
 - **C** one mean to a fixed value
 - **D** categories
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3.2 A two-sample t-test fails to reject H_0 when the p-value is [1]

- **A** below α
 - **B** above α
 - **C** exactly zero
 - **D** negative
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3.3 A two-sample t-test gives a p-value of 0.01 at $\alpha = 0.05$.

(a) State the decision. [1]

(b) State whether the means differ significantly. [1]

(c) State the conclusion in context.

[1]

Solutions

2.1 state hypotheses and check conditions; compute t ; find the p-value; compare to α and conclude.

2.2 a difference between the two means (a small p-value).

2.3 fail to reject H_0 ($0.06 > 0.05$).

3.1 B.

3.2 B.

3.3 (a) reject H_0 . (b) yes. (c) there is convincing evidence that the two means differ.