

7.2 Constructing a Confidence Interval for a Population Mean

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS degrees of freedom 自由度

Objective

Build the skills to answer exam questions on a **confidence interval for a population mean**.

You must be able to:

- construct the interval $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$
- use degrees of freedom $n - 1$
- compute the margin of error

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The confidence interval

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}},$$

where $\frac{s}{\sqrt{n}}$ is the **standard error**, the **margin of error** is $t^* \frac{s}{\sqrt{n}}$, and the degrees of freedom are $n - 1$.

■ Example

$$\bar{x} = 50, s = 10, n = 25, t^* = 2.06: \text{SE} = \frac{10}{5} = 2, \text{ME} = 2.06 \times 2 = 4.12.$$

2 Practice

2.1 Write the confidence-interval formula for a mean. [1]

2.2 State the degrees of freedom for a one-sample t interval with $n = 25$. [1]

2.3 $\bar{x} = 50$, $s = 10$, $n = 25$, $t^* = 2.06$. Find the margin of error. [2]

3 Exam-style questions

3.1 A confidence interval for a mean uses the _____ critical value. [1]

- **A** z^*
 - **B** t^*
 - **C** p
 - **D** χ^2
-

3.2 The degrees of freedom for a one-sample t interval are [1]

- **A** n
 - **B** $n - 1$
 - **C** $n + 1$
 - **D** $2n$
-

3.3 $\bar{x} = 80$, $s = 12$, $n = 36$, $t^* = 2.03$.

(a) Find the standard error $\frac{s}{\sqrt{n}}$. [1]

(b) Find the margin of error.

[2]

Solutions

2.1 $\bar{x} \pm t^* \frac{s}{\sqrt{n}}.$

2.2 $n - 1 = 24.$

2.3 $\text{ME} = 2.06 \times \frac{10}{\sqrt{25}} = 2.06 \times 2 = 4.12.$

3.1 B.

3.2 B.

3.3 (a) $\frac{12}{\sqrt{36}} = 2.$ (b) $\text{ME} = 2.03 \times 2 = 4.06.$