

6.2 Constructing a Confidence Interval for a Population Proportion

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS confidence interval 置信区间 • margin of error 误差幅度

Objective

Build the skills to answer exam questions on a **confidence interval for a population proportion**.

You must be able to:

- construct a confidence interval $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$
- use the appropriate **critical value** z^*
- interpret the interval

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The confidence interval

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}},$$

where the **margin of error** is z^* times the standard error. For 95% confidence, $z^* = 1.96$.

■ Interpretation

"We are 95% confident that the true proportion p lies within the interval."

■ Example

$$\hat{p} = 0.6, n = 100, 95\%: \text{ME} = 1.96 \sqrt{\frac{0.6(0.4)}{100}} = 1.96(0.049) = 0.096.$$

2 Practice

2.1 Write the formula for a confidence interval for a proportion. [1]

2.2 State the z^* value for a 95% confidence interval. [1]

2.3 A poll gives $\hat{p} = 0.5$ with $n = 100$. Find the 95% margin of error. [2]

3 Exam-style questions

3.1 A confidence interval for p has the form [1]

- **A** $\hat{p} \pm \text{ME}$
 - **B** $\hat{p} \times \text{ME}$
 - **C** $\hat{p}/\text{ME}$
 - **D** ME only
-

3.2 For a 95% confidence interval, z^* is about [1]

- **A** 1.28
 - **B** 1.645
 - **C** 1.96
 - **D** 2.58
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3.3 $\hat{p} = 0.4$, $n = 100$, 95% confidence.

(a) Find the margin of error. [2]

(b) Give the confidence interval. [1]

Solutions

2.1 $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$

2.2 1.96.

2.3 $\text{ME} = 1.96 \sqrt{\frac{0.5(0.5)}{100}} = 1.96(0.05) = 0.098.$

3.1 A.

3.2 C.

3.3 (a) $\text{ME} = 1.96 \sqrt{\frac{0.4(0.6)}{100}} = 1.96(0.049) = 0.096.$ (b) (0.304, 0.496).