

4.8 Mean and Standard Deviation of Random Variables

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS mean (expected value) 期望值 • random 随机 • probability 概率 • random variable 随机变量

Objective

Build the skills to answer exam questions on **the mean and standard deviation of random variables**.

You must be able to:

- calculate the **mean** (expected value) 期望值 $\mu_X = \sum x_i \cdot P(x_i)$
- calculate the **standard deviation** $\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 \cdot P(x_i)}$
- interpret these parameters in context

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Mean (expected value)

$$\mu_X = \sum x_i \cdot P(x_i)$$

— a probability-weighted average of the values.

■ Standard deviation

$$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 \cdot P(x_i)}.$$

■ Example

X: 0 (prob 0.5), 1 (prob 0.3), 2 (prob 0.2):

$$\mu_X = 0(0.5) + 1(0.3) + 2(0.2) = 0.7.$$

2 Practice

2.1 Write the formula for the mean of a discrete random variable. [1]

2.2 X takes 1 (prob 0.5) and 3 (prob 0.5). Find the mean. [2]

2.3 State what the mean of a random variable is also called. [1]

3 Exam-style questions

3.1 The expected value of X is [1]

- **A** $\sum x_i$
 - **B** $\sum x_i \cdot P(x_i)$
 - **C** $\sum P(x_i)$
 - **D** the largest value of x
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3.2 The mean of a random variable is also called its [1]

- **A** median
 - **B** expected value
 - **C** mode
 - **D** variance
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3.3 X takes 2 (prob 0.4) and 5 (prob 0.6).

(a) Find the mean. [2]

(b) State what the probabilities sum to. [1]

Solutions

2.1 $\mu_X = \sum x_i \cdot P(x_i).$

2.2 $\mu_X = 1(0.5) + 3(0.5) = 2.$

2.3 the expected value.

3.1 B.

3.2 B.

3.3 (a) $\mu_X = 2(0.4) + 5(0.6) = 0.8 + 3 = 3.8.$ (b) 1.