

4.12 The Geometric Distribution

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS geometric 几何 • independent 独立 • probability 概率 • random 随机 • random variable 随机变量

Objective

Build the skills to answer exam questions on **the geometric distribution**.

You must be able to:

- describe a **geometric** 几何 random variable as the number of trials until the first success
- use $P(\text{first success on trial } x) = (1 - p)^{x-1} p$
- find the mean, $\frac{1}{p}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The geometric setting

Like a binomial, but instead of a fixed number of trials, a **geometric** random variable counts how many **trials until the first success** (independent trials, constant p).

■ Probability and mean

$$P(X = x) = (1 - p)^{x-1} p, \quad \mu_X = \frac{1}{p}.$$

With $p = 0.2$, the expected number of trials is $\frac{1}{0.2} = 5$.

2 Practice

2.1 State what a geometric random variable counts. [1]

2.2 For $p = 0.25$, find the mean number of trials to the first success. [2]

2.3 Write $P(\text{first success on trial 1})$ in terms of p . [1]

3 Exam-style questions

3.1 A geometric random variable counts the [1]

- **A** number of successes in n trials
 - **B** number of trials until the first success
 - **C** total number of trials
 - **D** number of failures only
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3.2 The mean of a geometric distribution is [1]

- **A** p
 - **B** $\frac{1}{p}$
 - **C** np
 - **D** $\sqrt{p}$
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3.3 A game gives $P(\text{win}) = 0.1$ on each try.

(a) Find the expected number of tries until the first win. [2]

(b) Name the distribution. [1]

Solutions

2.1 the number of trials until the first success.

2.2 $\mu_X = \frac{1}{0.25} = 4.$

2.3 $P(X = 1) = p.$

3.1 B.

3.2 B.

3.3 (a) $\frac{1}{0.1} = 10.$ (b) geometric.