Most of this exam asks you to build or interpret a model, and the credit sits in the sentences around the algebra: which family fits and why, what the parameters mean, and over what domain the model is reasonable. Each solution below writes those out. 得分点多在"选择理由、参数含义、适用范围"的表述，而非计算本身。

Choosing and building a model

A table gives values (1, 6), (2, 12), (3, 24), (4, 48). Decide whether a linear or an exponential model fits, justify the choice, and write the model.

Exponential: $f(x) = 3 \cdot 2^x$.

Test both properties. The differences are 6, 12, 24 – not constant, so it is not linear. The ratios are 12/6 = 2, 24/12 = 2, 48/24 = 2 – constant, so the function changes by a constant FACTOR over equal intervals, which is the defining property of an exponential. So f(x) = ab^x with b = 2. Using the point (1, 6): 6 = a · 2, so a = 3, and $f(x) = 3 \cdot 2^x$. Check against (4, 48): 3 · 16 = 48. Stating the ratio test explicitly is what earns the justification point; a fitted equation with no reason does not.

A ferris wheel of radius 20 m has its centre 25 m above the ground and completes one revolution every 40 seconds. A rider boards at the lowest point at t = 0. Write a model for the rider's height.

$h(t) = 25 - 20\cos(\pi t/20)$, with t in seconds and h in metres.

Identify the four parameters from the description. The midline is the centre height, 25 m. The amplitude is the radius, 20 m. The period is 40 s, so $b = 2\pi/40 = \pi/20$. At t = 0 the rider is at the MINIMUM, and a negative cosine starts at its minimum, so use −20 cos rather than a sine with a phase shift. Hence $h(t) = 25 - 20\cos(\pi t/20)$. Check: at t = 0, h = 25 − 20 = 5 m, the lowest point; at t = 20, h = 45 m, the top. Define the variables and their units in the answer – that is scored separately from the equation.

Rates of change and function behaviour

For $f(x) = x^3 - 6x^2 + 9x$, find the average rate of change on [0, 2] and describe the concavity there using average rates of change.

Average rate of change is 1; the function is concave down on that interval.

f(0) = 0 and f(2) = 8 − 24 + 18 = 2, so the average rate of change is (2 − 0)/(2 − 0) = 1. For concavity in this course's language, compare average rates of change over successive equal sub-intervals. On [0, 1] it is (4 − 0)/1 = 4; on [1, 2] it is (2 − 4)/1 = −2. The average rates are DECREASING, so the graph is concave down on [0, 2]. Phrasing it through decreasing average rates rather than a second derivative is what this exam expects, and it is the wording the rubric matches.

A rational function has $f(x) = (x^2 - 4)/(x^2 - x - 6)$. Identify every asymptote and hole, with justification.

Hole at x = −2; vertical asymptote at x = 3; horizontal asymptote y = 1.

Factor both before doing anything else: the numerator is (x−2)(x+2) and the denominator $x^2 - x - 6$ = (x−3)(x+2). The factor (x+2) is common and cancels, so at x = −2 the function has a HOLE, not an asymptote – the graph is undefined there but approaches a finite value. The factor (x−3) does not cancel, so x = 3 is a vertical asymptote. The degrees of numerator and denominator are equal, so the horizontal asymptote is the ratio of the leading coefficients, y = 1. The distinction that earns the marks is exactly that one: a factor that cancels gives a hole, a factor that does not gives a vertical asymptote.

Exponentials, logarithms and inverses

Solve 3^(2x−1) = 40, giving an exact answer and a decimal to three places.

$x = (\log_3 40 + 1)/2 \approx 2.179$.

Take logarithms of both sides: $(2x - 1)\ln 3 = \ln 40$, so $2x - 1 = \ln 40/\ln 3 = 3.358$, giving $2x = 4.358$ and $x = 2.179$. Exactly, $x = (\ln 40/\ln 3 + 1)/2$. Check by substituting back: $3^{(3.358)} \approx 40$. Present the exact form first – the rubric asks for it and a decimal alone will not earn that point.

A quantity decays exponentially with a half-life of 12 years. Write the model with an initial amount of 500, and find when 80 units remain.

$A(t) = 500(½)^{(t/12)}$; about 31.7 years.

Half-life form is the fastest route: $A(t) = A_0(½)^{(t/H)}$ with $H = 12$, so $A(t) = 500(½)^{(t/12)}$. Set $A = 80$: $(½)^{(t/12)} = 0.16$. Taking logs, $(t/12)\ln(0.5) = \ln(0.16)$, so $t/12 = \ln 0.16/\ln 0.5 = 2.644$ and $t = 31.7$ years. Note the check: 31.7 years is between two and three half-lives (24 and 36 years), and after two half-lives 125 remain while after three, 62.5 – so a value of 80 in between is consistent. That order-of-magnitude check costs one line and catches sign errors.

Given f(x) = 2x/(x − 3), find the inverse function and state its domain.

$f^{-1}(x) = 3x/(x - 2)$, with domain all real x except 2.

Set $y = 2x/(x - 3)$ and solve for x: $y(x - 3) = 2x$, so $yx - 3y = 2x$, so $x(y - 2) = 3y$ and $x = 3y/(y - 2)$. Swapping names, $f^{-1}(x) = 3x/(x - 2)$. The domain of the inverse is the RANGE of the original, and the original has a horizontal asymptote at $y = 2$ which it never reaches – consistent with the inverse being undefined at $x = 2$. Making that connection explicitly, rather than only reading the new denominator, is what the justification asks for.