2 Sequences and exponential functions – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
sequence	数列			
arithmetic sequence	等差数列			
common difference	公差			
geometric sequence	等比数列			
common ratio	公比			
exponential function	指数函数			
exponential growth	指数增长			
exponential decay	指数衰减			

Recall

You have seen number patterns and powers before:

- Sequences like $2, 4, 6, 8, \ldots$ (adding) or $2, 4, 8, 16, \ldots$ (doubling).
- Powers like $2^3 = 8$.
- Some things grow faster and faster – like money earning interest.

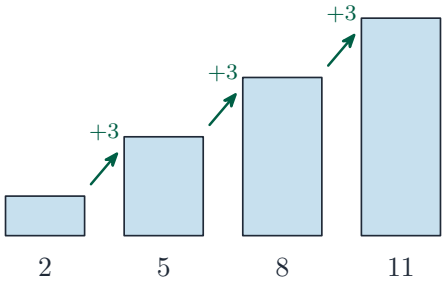
This sheet lines up sequences and exponential growth. Each idea has a worked example.

New ideas

■ 1 Two kinds of sequence

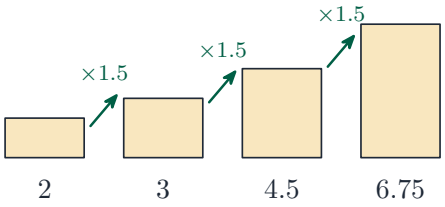
A **sequence** 数列 is an ordered list of numbers. Two important types:

arithmetic: add d each step



$u_n = a + (n-1)d$: equal steps

geometric: multiply by r each step



$u_n = ar^{n-1}$: growing steps

- an **arithmetic sequence** 等差数列 **adds** a fixed **common difference** 公差 d each step,
- a **geometric sequence** 等比数列 **multiplies** by a fixed **common ratio** 公比 r each step.

Worked example. Arithmetic with $a_0 = 3$, $d = 5$: the terms are $3, 8, 13, 18, \ldots$ Geometric with $g_0 = 2$, $r = 3$: $2, 6, 18, 54, \ldots$

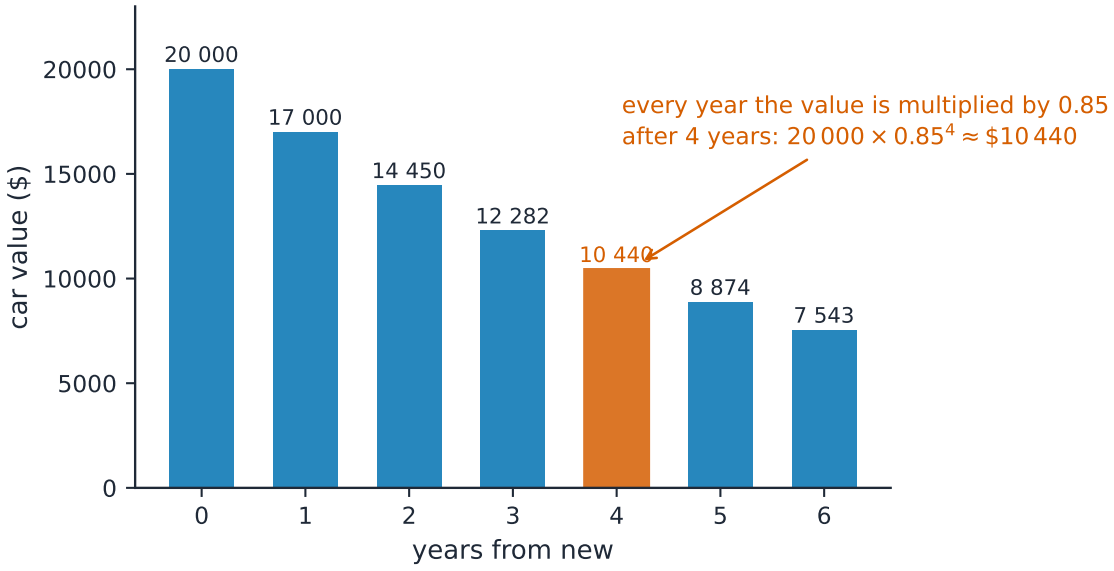
■ 2 Finding a term

For arithmetic, $a_n = a_0 + dn$; for geometric, $g_n = g_0\, r^{\,n}$.

Worked example. $a_4 = 3 + 5(4) = 23$; $g_4 = 2 \cdot 3^4 = 162$.

■ 3 Exponential functions

An **exponential function** 指数函数 is $f(x) = ab^x$ – a starting value a multiplied by the **base** b repeatedly.



- $b > 1$ gives **exponential growth** 指数增长,
- $0 < b < 1$ gives **exponential decay** 指数衰减.

■ 4 Adding versus multiplying

A linear function **adds** the same amount each step; an exponential function **multiplies** by the same factor. That is why an exponential always overtakes a linear one in the end.

Watch out: exponential growth means multiplying by the same factor each step, so it starts slow but soon shoots up far past any straight line – that is the power of "compounding".

Practice

Try these in order. The methods are all above.

2.1 State the difference between an arithmetic and a geometric sequence. [2]

2.2 An arithmetic sequence has $a_0 = 4$ and $d = 3$. Find a_5. [2]

2.3 A geometric sequence has $g_0 = 5$ and $r = 2$. Find g_3. [2]

2.4 For $f(x) = 3 \cdot 2^x$, find $f(4)$. [2]

2.5 State whether $f(x) = 100 \cdot (0.5)^x$ shows growth or decay, and why. [2]