

4.7 Parametrization of Implicitly Defined Functions

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS parametric 参数 • implicit 隐式

Objective

Build the skills to answer exam questions on **parametrization of implicitly defined functions**.

You must be able to:

- convert an implicit curve to **parametric** 参数 form
- use $\cos^2 t + \sin^2 t = 1$ to parametrize a circle

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Parametrizing an implicit curve

Rewrite an implicit relation as $x = f(t)$, $y = g(t)$. The circle $x^2 + y^2 = r^2$ becomes

$$x = r \cos t, \quad y = r \sin t,$$

which works because $\cos^2 t + \sin^2 t = 1$.

■ Example

$$x^2 + y^2 = 9 \Rightarrow x = 3 \cos t, y = 3 \sin t.$$

2 Practice

2.1 Parametrize $x^2 + y^2 = 16$.

[2]

2.2 State the identity used to parametrize a circle.

[1]

2.3 Verify that $x = 2 \cos t$, $y = 2 \sin t$ satisfies $x^2 + y^2 = 4$. [1]

3 Exam-style questions

3.1 The circle $x^2 + y^2 = 25$ parametrizes as [1]

- **A** $x = 5t$, $y = 5t$
 - **B** $x = 5 \cos t$, $y = 5 \sin t$
 - **C** $x = t^2$, $y = t^2$
 - **D** $x = 25t$, $y = 25t$
-

3.2 Parametrizing a circle uses the identity [1]

- **A** $\sin t + \cos t = 1$
 - **B** $\cos^2 t + \sin^2 t = 1$
 - **C** $\tan t = \sin t / \cos t$
 - **D** $\sec t = 1 / \cos t$
-

3.3 $x^2 + y^2 = 36$.

(a) State r . [1]

(b) Write $x(t)$. [1]

(c) Write $y(t)$. [1]

Solutions

2.1 $x = 4 \cos t$, $y = 4 \sin t$.

2.2 $\cos^2 t + \sin^2 t = 1$.

2.3 $x^2 + y^2 = 4 \cos^2 t + 4 \sin^2 t = 4(\cos^2 t + \sin^2 t) = 4$.

3.1 B.

3.2 B.

3.3 (a) 6. (b) $x = 6 \cos t$. (c) $y = 6 \sin t$.