

4.11 The Inverse and Determinant of a Matrix

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS determinant 行列式 • inverse 逆矩阵 • matrix 矩阵

Objective

Build the skills to answer exam questions on **the inverse and determinant of a matrix**.

You must be able to:

- compute the **determinant** 行列式 of a 2×2 matrix
- decide whether a matrix has an **inverse** 逆矩阵

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Determinant and inverse

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\det(A) = ad - bc$.

A has an inverse if and only if $\det(A) \neq 0$, and then

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

■ Example

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}: \det = 2(1) - 1(1) = 1, \text{ so } A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

2 Practice

2.1 State the determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. [1]

2.2 State when a matrix has an inverse. [1]

2.3 Find the determinant of $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$. [2]

3 Exam-style questions

3.1 $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ equals [1]

- **A** $ad + bc$
 - **B** $ad - bc$
 - **C** $ab - cd$
 - **D** $a + d$
-

3.2 A 2×2 matrix has an inverse if and only if [1]

- **A** it is square
 - **B** $\det \neq 0$
 - **C** it is diagonal
 - **D** all entries are positive
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3.3 $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(a) Find $\det(A)$. [1]

(b) State whether A is invertible. [1]

(c) Find A^{-1} .

[1]

Solutions

2.1 $ad - bc$.

2.2 when its determinant is non-zero.

2.3 $3(4) - 2(1) = 12 - 2 = 10$.

3.1 B.

3.2 B.

3.3 (a) $2(3) - 0 = 6$. (b) yes, since $\det = 6 \neq 0$. (c) $\begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$.