

3.10 Trigonometric Equations and Inequalities

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS periodicity 周期性 • period 周期

Objective

Build the skills to answer exam questions on **trigonometric equations and inequalities**.

You must be able to:

- solve a simple trigonometric equation on a given interval
- explain why such equations have infinitely many solutions

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Solving trig equations

Solve $\sin x = c$ by finding one solution, then using symmetry and **periodicity** 周期性 to get the rest ($+2\pi n$).

■ Example

$\sin x = \frac{1}{2}$ gives $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ on $[0, 2\pi)$, plus $2\pi n$ for all solutions.

2 Practice

2.1 Solve $\sin x = 0$ on $[0, 2\pi)$. [1]

2.2 State why a trig equation has infinitely many solutions. [1]

2.3 Solve $\cos x = 1$ on $[0, 2\pi)$. [2]

3 Exam-style questions

3.1 The equation $\sin x = \frac{1}{2}$ has [1]

- **A** one solution in total
 - **B** two solutions in $[0, 2\pi)$
 - **C** no solution
 - **D** no solution in $[0, 2\pi)$
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3.2 Trig equations have infinitely many solutions because sine and cosine are [1]

- **A** linear
 - **B** periodic
 - **C** polynomial
 - **D** rational
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3.3 Solve $\sin x = 1$.

(a) State the solution on $[0, 2\pi)$. [1]

(b) State the general solution (all x). [1]

(c) State the amount added each time. [1]

Solutions

2.1 $x = 0$ and $x = \pi$.

2.2 sine and cosine are periodic, so each solution repeats every 2π .

2.3 $\cos x = 1$ at $x = 0$ only on $[0, 2\pi)$.

3.1 B.

3.2 B.

3.3 (a) $x = \frac{\pi}{2}$. (b) $x = \frac{\pi}{2} + 2\pi n$. (c) 2π .