

1.9 Rational Functions and Vertical Asymptotes

Name: _____ Class: _____ Date: _____

Total: 8 marks

KEY WORDS vertical asymptote 垂直渐近线 • function 函数 • zero 零点 • rational function 有理函数
• leading term 首项

Objective

Build the skills to answer exam questions on **rational functions and vertical asymptotes**.

You must be able to:

- identify a **vertical asymptote** 垂直渐近线 at $x = a$
- describe the behaviour of $r(x)$ near a vertical asymptote

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Vertical asymptotes

A rational function has a **vertical asymptote** at $x = a$ if a is a zero of the **denominator** but **not** of the numerator (or has higher multiplicity in the denominator).

■ Behaviour near it

Near $x = a$, the denominator is close to 0, so the values **increase or decrease without bound**: $\lim_{x \rightarrow a^+} r(x) = \pm\infty$.

For $r(x) = \frac{1}{x-3}$, the vertical asymptote is $x = 3$.

2 Practice

2.1 State the condition for a vertical asymptote at $x = a$. [1]

2.2 For $r(x) = \frac{1}{x-3}$, state the vertical asymptote. [1]

2.3 State what happens to $r(x)$ near a vertical asymptote. [1]

3 Exam-style questions

3.1 A rational function has a vertical asymptote where the _____ is zero (but not the numerator). [1]

- **A** numerator
 - **B** denominator
 - **C** leading term
 - **D** degree
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3.2 Near a vertical asymptote, the function values [1]

- **A** approach 0
 - **B** increase or decrease without bound
 - **C** stay constant
 - **D** equal the mean
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3.3 $r(x) = \frac{x+2}{x-5}$.

(a) State the vertical asymptote. [1]

(b) State the zero. [1]

(c) State the domain restriction. [1]

Solutions

2.1 a is a zero of the denominator but not of the numerator.

2.2 $x = 3$.

2.3 the values increase or decrease without bound.

3.1 B.

3.2 B.

3.3 (a) $x = 5$. (b) $x = -2$. (c) $x \neq 5$.