

1.8 Rational Functions and Zeros

Name: _____ Class: _____ Date: _____

Total: 8 marks

KEY WORDS zeros 零点 • domain 定义域 • function 函数 • rational function 有理函数 • hole 空洞

Objective

Build the skills to answer exam questions on **rational functions and zeros**.

You must be able to:

- find the real **zeros** 零点 of a rational function from its numerator
- recognise which numerator zeros are excluded because they are outside the domain

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Zeros of a rational function

The real zeros of $r(x) = \frac{p(x)}{q(x)}$ are the real zeros of the **numerator** $p(x)$ — provided those inputs are **in the domain** (i.e. the denominator is not also zero there).

■ Example

$r(x) = \frac{x-4}{x+1}$ has a zero at $x = 4$ (and $x = -1$ is excluded from the domain).

2 Practice

2.1 State where the real zeros of a rational function come from. [1]

2.2 For $r(x) = \frac{x-4}{x+1}$, state the zero. [1]

2.3 State why a numerator zero might **not** be a zero of r . [1]

3 Exam-style questions

3.1 The real zeros of a rational function come from the [1]

- **A** denominator
 - **B** numerator
 - **C** leading term
 - **D** degree
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3.2 The function $r(x) = \frac{x-2}{x-2}$ [1]

- **A** has a zero at 2
 - **B** has a hole at 2
 - **C** has an asymptote at 2
 - **D** is undefined everywhere
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3.3 $r(x) = \frac{x^2 - 9}{x - 1}$.

(a) State the zeros of the numerator. [1]

(b) State the zeros of r . [1]

(c) State the value excluded from the domain. [1]

Solutions

2.1 from the real zeros of the numerator (that lie in the domain).

2.2 $x = 4$.

2.3 if it is also a zero of the denominator, it is outside the domain (a hole, not a zero).

3.1 B.

3.2 B.

3.3 (a) $x = 3$ and $x = -3$. (b) $x = 3$ and $x = -3$. (c) $x = 1$.