

1.6 Polynomial Functions and End Behavior

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS end behavior 端点行为 • limit notation 极限记号 • function 函数 • polynomial 多项式
• leading term 首项

Objective

Build the skills to answer exam questions on **polynomial functions and end behavior**.

You must be able to:

- describe the **end behavior** 端点行为 of a polynomial from its leading term
- use **limit notation** 极限记号 such as $\lim_{x \rightarrow \infty} p(x) = \infty$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ End behavior

The **end behavior** of a polynomial is set by its **leading term** $a_n x^n$:

- **even** n , $a_n > 0$: both ends up.
- **odd** n , $a_n > 0$: left end down, right end up.

■ Limit notation

”Output increases without bound as x increases” is written $\lim_{x \rightarrow \infty} p(x) = \infty$. For $p(x) = x^3$: $\lim_{x \rightarrow \infty} p(x) = \infty$ and $\lim_{x \rightarrow -\infty} p(x) = -\infty$.

2 Practice

2.1 State what determines a polynomial’s end behavior.

[1]

2.2 For $p(x) = x^3$, state the end behavior as $x \rightarrow \infty$ and as $x \rightarrow -\infty$.

[2]

2.3 Write the limit notation for "output increases without bound as x increases". [1]

3 Exam-style questions

3.1 The end behavior of a polynomial is determined by its [1]

- **A** constant term
 - **B** leading term
 - **C** middle term
 - **D** zero
-

3.2 For $p(x) = -2x^4$, as $x \rightarrow \infty$ the output [1]

- **A** $\rightarrow \infty$
 - **B** $\rightarrow -\infty$
 - **C** $\rightarrow 0$
 - **D** oscillates
-

3.3 $p(x) = x^3$.

(a) State $\lim_{x \rightarrow \infty} p(x)$. [1]

(b) State $\lim_{x \rightarrow -\infty} p(x)$. [1]

(c) State whether both ends go the same way. [1]

Solutions

2.1 its leading term (degree and leading coefficient).

2.2 as $x \rightarrow \infty$, $p(x) \rightarrow \infty$; as $x \rightarrow -\infty$, $p(x) \rightarrow -\infty$.

2.3 $\lim_{x \rightarrow \infty} p(x) = \infty$.

3.1 B.

3.2 B — an even degree with a negative leading coefficient sends both ends to $-\infty$.

3.3 (a) ∞ . (b) $-\infty$. (c) no (opposite directions).