

1.5 Polynomial Functions and Complex Zeros

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS zero 零点 • multiplicity 重数 • complex 复数 • function 函数 • degree 次数

Objective

Build the skills to answer exam questions on **polynomial functions and complex zeros**.

You must be able to:

- relate a **zero** 零点 a to the linear factor $(x - a)$
- describe the **multiplicity** 重数 of a zero
- use that a degree- n polynomial has exactly n complex zeros (with multiplicity)

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Zeros and factors

If $p(a) = 0$ then a is a **zero** of p , and (for real a) $(x - a)$ is a **factor**.

■ Multiplicity

If a factor $(x - a)$ is repeated n times, the zero has **multiplicity** n . A polynomial of degree n has exactly n complex zeros, counting multiplicities.

■ Example

$p(x) = (x - 2)^2(x + 1)$ has zeros $x = 2$ (multiplicity 2) and $x = -1$ (multiplicity 1); degree 3.

2 Practice

2.1 State the relationship between a zero a and the factor $(x - a)$. [1]

2.2 State how many complex zeros a degree-4 polynomial has (counting multiplicity). [1]

2.3 For $p(x) = (x - 2)^2(x + 1)$, state the zeros and their multiplicities. [2]

3 Exam-style questions

3.1 If $p(a) = 0$, then $(x - a)$ is a [1]

- **A** vertical asymptote
 - **B** factor of p
 - **C** hole
 - **D** leading term
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3.2 A degree-5 polynomial has _____ complex zeros counting multiplicity. [1]

- **A** 1
 - **B** 3
 - **C** 5
 - **D** 10
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3.3 $p(x) = (x - 3)^3(x + 2)$.

(a) State the zeros. [1]

(b) State the multiplicity of $x = 3$. [1]

(c) State the degree. [1]

Solutions

2.1 $(x - a)$ is a factor of p if and only if a is a zero of p .

2.2 4.

2.3 $x = 2$ (multiplicity 2); $x = -1$ (multiplicity 1).

3.1 B.

3.2 C.

3.3 (a) $x = 3$ and $x = -2$. (b) 3. (c) 4.