

1.10 Rational Functions and Holes

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS hole 空心点 • function 函数 • rational function 有理函数 • vertical asymptote 垂直渐近线
 • leading term 首项

Objective

Build the skills to answer exam questions on **rational functions and holes**.

You must be able to:

- identify a **hole** 空心点 where a factor cancels from numerator and denominator
- find the location (c, L) of a hole

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Holes

A rational function has a **hole** at $x = c$ when a factor $(x - c)$ **cancels** — the numerator's multiplicity of c is at least the denominator's.

■ Location

The hole is at (c, L) , where $L = \lim_{x \rightarrow c} r(x)$ — evaluate the **simplified** function at c .

■ Example

$r(x) = \frac{(x-2)(x+1)}{x-2}$ simplifies to $x+1$ (for $x \neq 2$), so there is a hole at $(2, 3)$.

2 Practice

2.1 State when a rational function has a hole.

[1]

2.2 For $r(x) = \frac{(x-2)(x+1)}{x-2}$, state the location of the hole.

[2]

2.3 State the difference between a hole and a vertical asymptote. [1]

3 Exam-style questions

3.1 A rational function has a hole where a factor [1]

- **A** appears only in the denominator
 - **B** cancels from numerator and denominator
 - **C** is the leading term
 - **D** is constant
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3.2 The x -value of a hole is [1]

- **A** always 0
 - **B** where a common factor is zero
 - **C** a vertical asymptote
 - **D** undefined for all x
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3.3 $r(x) = \frac{(x-3)(x+4)}{x-3}$.

(a) Simplify $r(x)$ (for $x \neq 3$). [1]

(b) State the x -coordinate of the hole. [1]

(c) Find the y -coordinate of the hole. [1]

Solutions

2.1 when a factor cancels from both the numerator and the denominator.

2.2 simplifies to $x + 1$; hole at $(2, 3)$.

2.3 at a hole the function is merely undefined at one point; at an asymptote the values blow up to $\pm\infty$.

3.1 B.

3.2 B.

3.3 (a) $x + 4$. (b) $x = 3$. (c) $3 + 4 = 7$.