Past-paper walk-through

Sinusoidal Functions ▪ 3.5

eXercise

Questions in this set

- 2026 Q3
- 2025 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2026 ▪ Q3

The figure shows a circular waterwheel that rotates in a counterclockwise direction at a constant rate and completes 1 revolution in 10 seconds. At time $t = 0$, point W on the edge of the waterwheel is at a height of 6 feet directly above the center of the waterwheel. The height of point W from the horizontal line through the center of the waterwheel periodically decreases and increases as the waterwheel moves.
[Diagram of a circular waterwheel partially submerged in water. The wheel has spokes radiating from a labeled "Center" point, and hatched paddles around the rim. Point W is labeled at the top of the wheel. An arrow labeled "Counterclockwise" indicates the direction of rotation. A vertical double-headed arrow labeled "Height of W" shows the vertical distance from the horizontal dashed line through the center up to W. The bottom portion of the wheel is submerged in wavy water. Note: Figure not drawn to scale.]

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The sinusoidal function h models the height of point W from the horizontal line through the center of the waterwheel, in feet, as a function of time t, in seconds. A positive value of $h(t)$ indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

(a) The graph of h and its dashed midline for two full cycles is shown. Five points, $F, G, J, K,$ and P, are labeled on the graph. No scale is indicated, and no axes are presented.

[Graph of h: a sinusoidal (cosine-shaped) curve with a dashed horizontal midline, showing two full cycles. No axes or scale are shown — only the curve and the midline. Point F is labeled at the first (leftmost) maximum of the curve. Point G is labeled where the curve crosses the midline, descending, between the first maximum and the first minimum. Point J is labeled at the minimum of the curve (between the two maxima). Point K is labeled where the curve crosses the midline, ascending, between

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the minimum and the second maximum. Point P is labeled at the second maximum of the curve.]
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.

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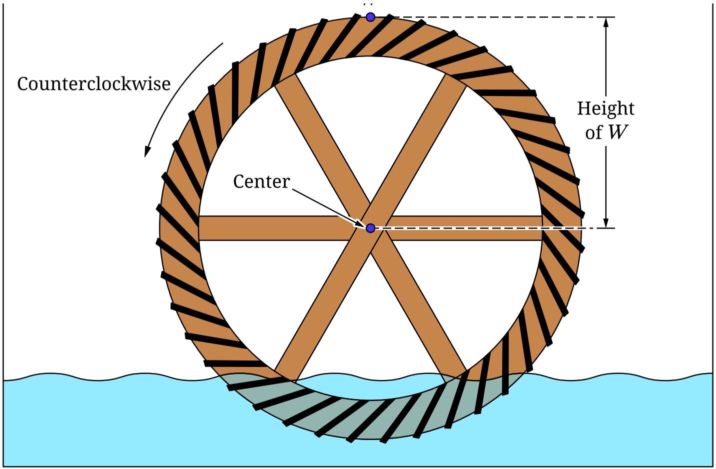

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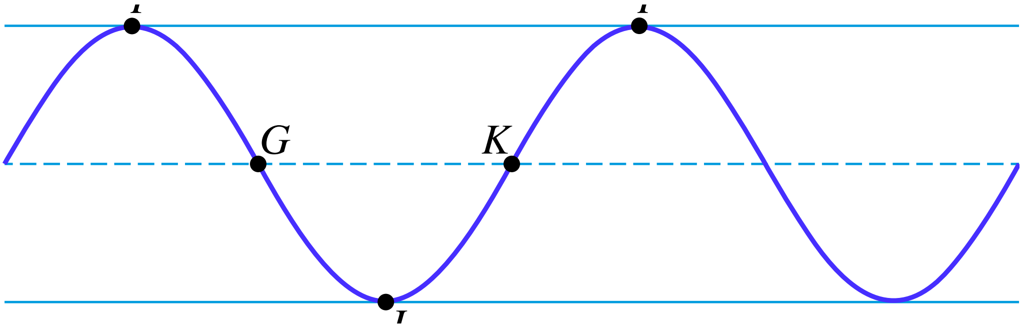

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[Diagram of a circular waterwheel partially submerged in water. The wheel has spokes radiating from a labeled "Center" point, and hatched paddles around the rim. Point W is labeled at the top of the wheel. An arrow labeled "Counterclockwise" indicates the direction of rotation. A vertical double-headed arrow labeled "Height of W" shows the vertical distance from the horizontal dashed line through the center up to W. The bottom portion of the wheel is submerged in wavy water. Note: Figure not drawn to scale.]

The sinusoidal function h models the height of point W from the horizontal line through the center of the waterwheel, in feet, as a function of time t, in seconds. A positive value of $h(t)$

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indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

(b) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

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indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

(c)(i) Refer to the graph of h in part A. The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing. b. h is positive and decreasing. c. h is negative and increasing. d. h is negative and decreasing.

(c)(ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

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For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.
At time $t = 0$ seconds, point X on one vibrating guitar string starts at its highest position, 2 millimeters above its resting position. Then it passes through its resting position and moves to its lowest position, 2 millimeters below the resting position. Point X then passes through its resting position and returns to 2 millimeters above the resting position. This motion occurs 200 times in 1 second.
The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t, in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

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(a) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.

[Graph of the sinusoidal function h over two full cycles, with a dashed horizontal midline and two solid horizontal lines marking the maximum and minimum levels; no numerical axes or scale shown. The curve starts at a maximum, labeled point F, descends crossing the midline at point G, continues down to a minimum labeled point J, rises back up crossing the midline at point K, and reaches the next maximum labeled point P, then the curve continues descending again through the end of the second cycle. Points in left-to-right order: F (first maximum), G (midline, descending), J (minimum), K (midline, ascending), P (second maximum).]

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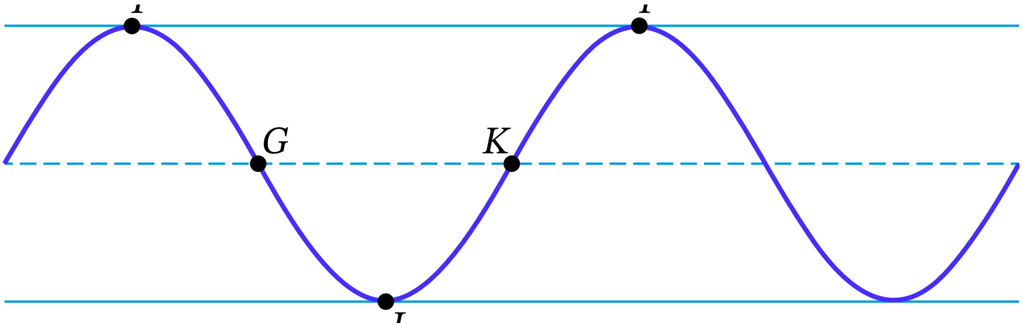

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(b) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

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(c)(i) Refer to the graph of h in part A. The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

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On the interval (t_1, t_2), which of the following is true about h?
a. h is positive and increasing. b. h is positive and decreasing. c. h is negative and increasing. d. h is negative and decreasing.
(c)(ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

Solution 3

(a) Mark scheme

- One full cycle: the period is $\frac{1}{200}$ s (the motion occurs 200 times per second), so a quarter period is $\frac{1}{800}$ s. Taking F at the starting maximum: $F(0,2)$, $G\left(\frac{1}{800},0\right)$ (descending through rest), $J\left(\frac{2}{800},-2\right)$ (minimum), $K\left(\frac{3}{800},0\right)$ (ascending through rest), $P\left(\frac{4}{800},2\right)$ (next maximum). t-coordinates may vary (e.g. shifted by a multiple of $\frac{4}{800}$) as long as they are evenly spaced by $\frac{1}{800}$ and follow this same pattern; correct $h(t)$-coordinates $(2,0,-2,0,2$ for F, G, J, K, $P)$ are worth 1 point, and correct matching t-coordinates are worth 1 point. No supporting work is required.

Solution 3

(b) Mark scheme

- $h(t) = a\sin(b(t+c)) + d$. Since the period is $\frac{1}{200}$ s, $\frac{2\pi}{b} = \frac{1}{200} \Rightarrow b = 400\pi$. The midline is at 0, so $d = 0$. Using $a = 2$: $c = \frac{1}{800}$ (or $c = \frac{1}{800} + \frac{4}{800}k$ for any integer k), giving $h(t) = 2\sin\left(400\pi\left(t + \frac{1}{800}\right)\right)$. (Equivalently, using $a = -2$: $c = -\frac{1}{800}$, giving $h(t) = -2\sin\left(400\pi\left(t - \frac{1}{800}\right)\right)$.) Values of a and d are worth 1 point; values of b and c are worth 1 point. No supporting work is required.

Solution 3

(c)(i)

Mark scheme

- Choice (d): h is negative and decreasing on (t_1, t_2) (the interval between G, where $h = 0$ descending, and J, the minimum).

Solution 3

(c)(ii)

Mark scheme

- The graph of h is concave up on the interval (t_1, t_2), and the rate of change of h is increasing on (t_1, t_2). Both descriptions (concavity AND behavior of the rate of change) are required for the point; combining `concave up'' with` function h is decreasing at an increasing rate" is also acceptable, but claims about the rate of change itself increasing/decreasing at an increasing/decreasing rate are not (that requires calculus).