

Past-paper walk-through

Inverse Functions ■ 2.8

eXercise

Questions in this set

- 2026 Q1
- 2025 Q1
- 2024 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 1

2026 ■ Q1

The figure shows the graph of the increasing function f on its domain of $-3 \leq x \leq 3$. [Graph of f : a smooth increasing S-shaped curve on a coordinate grid, x -axis from -3 to 3 , y -axis from -2 to 9 (gridlines at every integer). Labeled points on the curve: $(-3, -2)$, $(-1, 1)$, $(0, 1.8)$, $(1, 3)$, $(2, 5)$, $(3, 9)$. The curve rises slowly near the middle (around $x = 0$) and rises steeply toward both ends.]

The function g is given by $g(x) = -4.792 + \ln(6x - 6)$.

(a)(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

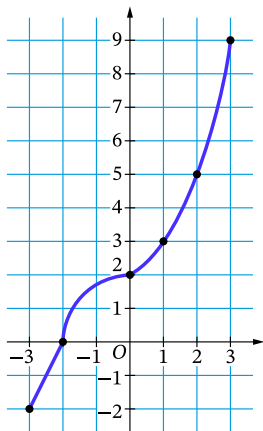
(a)(ii) Find all values of x for which $f(x) = 3$, or indicate that there are no such values.

(b)(i) Find all values of x , as decimal approximations, for which $g(x) = -1.5$, or indicate that there are no such values.

Question 1

- (b)(ii)** Determine the behavior of g as x -values decrease and get arbitrarily close to 1. Express your answer using the mathematical notation of a limit.
- (c)(i)** Is the function f invertible?
- (c)(ii)** Give a reason for your answer in part C (i) based on properties of the function f . Refer to points on the graph of f in your reasoning.

Question 1



Graph of f

Question 1

2025 ■ Q1

The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x .

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

Question 1

(a)(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

(a)(ii) Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

(b)(i) Find all values of x , as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

(b)(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(c)(i) Based on the table, which of the following function types best models function f : linear, quadratic, exponential, or logarithmic?

Question 1

(c)(ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f . Refer to the values in the table in your reasoning.

Question 1

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

- i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ decimal approximation, or indicate that it is not defined. Show the work that le

Solution 1

(a)(i)

Mark scheme

- $h(1) = g(f(1)) = g(1.75)$. From the table, $f(1) = 1.75$. Then $g(1.75) = -0.167(1.75)^3 + (1.75)^2 - 1.834 \approx 0.333$. So $h(1) = 0.333$ (decimal approximation, accurate to three places). The point is earned for the correct decimal approximation of 0.333 with supporting work showing $f(1)$ or $g(1.75)$ or 1.75.

(a)(ii)

Mark scheme

- From the table, $f(0) = 3.5$, so $f^{-1}(3.5) = 0$. No supporting work is required for this point; a response of “0” earns it.

Solution 1

(b)(i)

Mark scheme

- Solve $g(x) = 0$: $-0.167x^3 + x^2 - 1.834 = 0$. The three real solutions (decimal approximations) are $x = -1.233$, $x = 1.578$, $x = 5.643$. No supporting work is required; all three values must be given.

Solution 1

(b)(ii)

Mark scheme

- As x increases without bound, the output values of g decrease without bound (the leading term $-0.167x^3$ dominates and is negative). So $\lim_{x \rightarrow \infty} g(x) = -\infty$. The correct limit statement needs four components: “lim”, “ $x \rightarrow \infty$ ”, the function g , and $-\infty$.

Solution 1

(c)(i)

Mark scheme

- An exponential function best models f .

Solution 1

(c)(ii) Mark scheme

- In the table, the input-value intervals all have length 1, and the ratios of successive output values are constant: $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$. Because the successive output values over equal-length input intervals are proportional (by a constant factor of 0.5), an exponential model is best. The reasoning must reference the table values and show the 0.5 ratio applies to more than one pair of successive outputs; referencing “exponential regression” or r/r^2 values alone is not sufficient.

Question 1

2024 ■ Q1

[Graph of f : x -axis from -4 to 4 , y -axis from -2 to 4 , gridlines at each integer. The curve begins at an open/filled point near $(-3, 3.5)$, drops steeply down through $(-2, 0)$ to a minimum near $(-1.7, -1.4)$, rises up crossing near $x = -1$ and $x = 0$ passing through $(0, 1)$, continues rising to a maximum near $(2, 3.5)$, then decreases sharply ending at a point near $(3.3, -1.6)$. Labeled "Graph of f ".]

The figure shows the graph of the function f on its domain of $-3.5 \leq x \leq 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f . The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

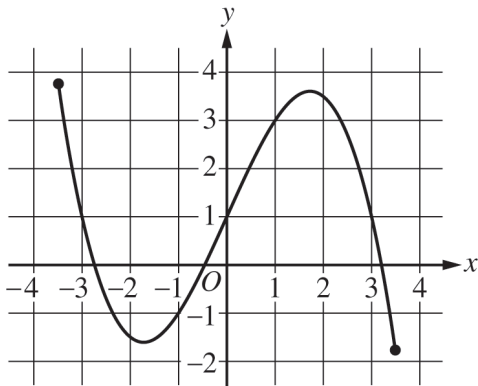
(a)(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

(a)(ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

Question 1

- (b)(i)** Find all values of x , as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.
- (b)(ii)** Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.
- (c)(i)** Determine if f has an inverse function.
- (c)(ii)** Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Question 1



Graph of f

Solution 1

(a)(i)

Mark scheme

- $h(3) = g(f(3)) = g(1) = 2.041$, using $g(x) = 2.916 \cdot (0.7)^x$ and $f(3) = 1$ read from the graph. 1 point for the correct decimal approximation 2.041 (decimals must be correct to three places; the first decimal-presentation error in the whole question is forgiven).

Solution 1

(a)(ii)

Mark scheme

- From the graph, $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$ (the three marked points $(-3, 1)$, $(0, 1)$, $(3, 1)$). 1 point for all three correct values with no incorrect values included.

Solution 1

(b)(i)**Mark scheme**

- Solve $g(x) = 2$: $2.916(0.7)^x = 2 \Rightarrow x = 1.057$ (decimal approximation). 1 point for the correct value with no incorrect values included.

Solution 1

(b)(ii)**Mark scheme**

- As x increases without bound, the output values of g get arbitrarily close to 0, so $\lim_{x \rightarrow \infty} g(x) = 0$. 1 point requires a correct limit statement with all four components: 'lim', ' $x \rightarrow \infty$ ', the function g , and the value 0 (e.g. $\lim_{x \rightarrow \infty} g(x) = 0$).

Solution 1

(c)(i)

Mark scheme

- f does not have an inverse function on its domain $-3.5 \leq x \leq 3.5$. 1 point for the correct answer.

Solution 1

(c)(ii)

Mark scheme

- There are output values of f that are not mapped from unique input values; for example $f(-3) = f(0) = f(3) = 1$. 1 point requires an implicit or explicit reference to the definition of a function AND support by referencing specific function values (e.g. citing $f(-3) = f(0) = 1$); saying only ' f fails the horizontal line test' or ' f is not one-to-one' is NOT sufficient. This point cannot be earned if part (i) has any error.