

Learn these by heart before the exam.

Polynomial and Rational Functions

Rate of change: a polynomial of degree n has a constant n th DIFFERENCE over equal input steps. Linear has constant first differences; quadratic constant second.

End behaviour is set by the leading term. Even degree: both ends agree. Odd degree: ends disagree. Negative leading coefficient flips both.

Zero of multiplicity: odd multiplicity CROSSES the x -axis; even multiplicity touches and turns.

$f(x) = \frac{p(x)}{q(x)}$ vertical asymptote where $q = 0$ and $p \neq 0$; a HOLE where both are 0

Horizontal asymptote: $\deg p < \deg q$ gives $y = 0$; equal degrees give the ratio of leading coefficients;

$\deg p = \deg q + 1$ gives a SLANT asymptote (divide).

$y = a f(b(x - h)) + k$ h and k shift; a scales vertically; b scales horizontally by $1/b$

Even $f(-x) = f(x)$ (symmetric in the y -axis). Odd $f(-x) = -f(x)$ (symmetric about the origin).

Exponential and Logarithmic Functions

Exponential has a constant RATIO over equal input steps; linear has a constant difference. That is how you tell them apart from a table.

$f(x) = ab^x$, $f(x) = ae^{kx}$ growth if $b > 1$ or $k > 0$; decay if $0 < b < 1$ or $k < 0$

$y = \log_b x \iff b^y = x$

$\log(mn) = \log m + \log n$, $\log \frac{m}{n} = \log m - \log n$, $\log m^p = p \log m$

$\log_b x = \frac{\ln x}{\ln b}$

$A = P \left(1 + \frac{r}{n}\right)^{nt}$, $A = Pe^{rt}$

Semi-log plot: exponential data becomes a STRAIGHT line when $\log y$ is plotted against x .

Log and exponential are inverses; reflect one in $y = x$ to get the other. Domain of log is $x > 0$.

Trigonometric and Polar Functions

$s = r\theta$, $180^\circ = \pi \text{ rad}$

$\sin \theta = \frac{y}{r}$, $\cos \theta = \frac{x}{r}$, $\tan \theta = \frac{y}{x}$

$\sin^2 \theta + \cos^2 \theta = 1$, $1 + \tan^2 \theta = \sec^2 \theta$

$y = a \sin(b(x - h)) + k$ amplitude $|a|$, period $\frac{2\pi}{|b|}$, phase shift h , midline $y = k$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$, $\sin 2A = 2 \sin A \cos A$

$\frac{a}{\sin A} = \frac{b}{\sin B}$, $c^2 = a^2 + b^2 - 2ab \cos C$

$x = r \cos \theta$, $y = r \sin \theta$, $r^2 = x^2 + y^2$, $\tan \theta = \frac{y}{x}$

Polar graphs: $r = a$ circle, $r = a \cos \theta$ circle through the origin, $r = a \pm b \cos \theta$ limacon/cardiod,

$r = a \cos(n\theta)$ rose.

r increasing as θ increases means the curve moves AWAY from the origin. Say it that way in a description.

Functions Involving Parameters, Vectors, and Matrices

$x = f(t)$, $y = g(t)$ eliminate t to get the Cartesian form; keep the direction of travel

$\vec{v} = \langle a, b \rangle$, $|\vec{v}| = \sqrt{a^2 + b^2}$, $\theta = \arctan \frac{b}{a}$

$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$, $k\vec{v} = \langle ka, kb \rangle$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$

$\det = ad - bc$, $A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\det$ is the

AREA scale factor; $\det = 0$ means no inverse

Matrix multiplication is NOT commutative: $AB \neq BA$ in general. Order is the transformation order, right to left.

Exam technique

Describe behaviour in words AND in context: 'as x increases without bound, $f(x)$ approaches 3 from below'.

State the domain restriction whenever you invert, take a log, or divide. Examiners look for it.

Show which model fits and WHY: constant difference means linear, constant ratio means exponential, constant second difference means quadratic.