Oscillations

AP Physics C: Mechanics ▪ Unit 7

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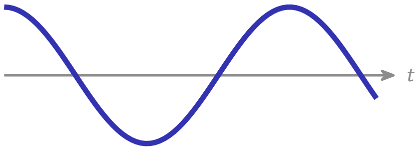
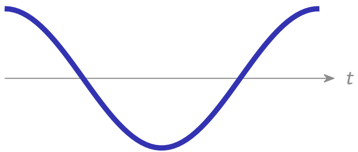


What we will cover

1. Defining SHM as a differential equation
2. Frequency and period
3. Representing and analyzing SHM
4. Energy of an oscillator
5. Simple and physical pendulums



1 Defining SHM

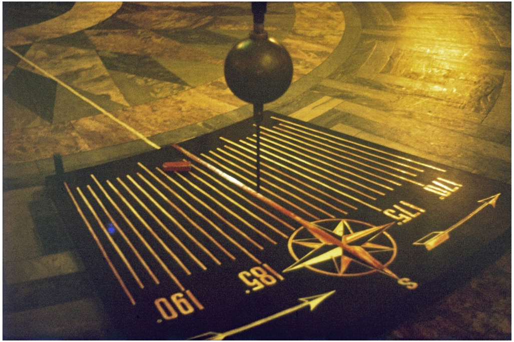
SHM is a differential equation

A **restoring force** 回复力 $F = -kx$ put into $\vec{F} = m\vec{a}$ gives the defining law:

The SHM equation

$$\frac{d^2x}{dt^2} = -\omega^2 x, \qquad \omega = \sqrt{\frac{k}{m}}$$

Any system whose acceleration is $-\omega^2$ times its displacement oscillates in SHM.



Foucault pendulum

What makes a motion simple harmonic

periodic motion 周期运动	anything that repeats; SHM is one special case of it
condition 1	there is an **equilibrium** 平衡位置 position the system returns to
condition 2	the restoring force is proportional to the displacement from it: $F = -kx$
the consequence	$\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x$ – and every solution of that is a sinusoid
the payoff	the period is independent of amplitude: $T = 2\pi\sqrt{m/k}$

> Whenever you can show a restoring force is proportional to displacement, you have proved the motion is SHM and may quote T immediately – that is the marked step.

2 Period

Frequency and period

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$$

The period does not depend on amplitude – a bigger swing just moves faster.

$0.5\,\mathrm{kg}$ on $k = 50\,\mathrm{N/m}$:

$$T = 2\pi\sqrt{\frac{0.5}{50}} \approx 0.63\,\mathrm{s}$$

Pull it 2 or $10\,\mathrm{cm}$ – same period. Heavier mass or softer spring lengthens T. Each system also ignores one obvious variable: a pendulum's period does not depend on its mass, and a spring's does not depend on g.

3 Analyzing SHM

Representing and analyzing SHM

The solution is a cosine; differentiate for v and a:

Position, velocity, acceleration

$$x = A\cos(\omega t + \phi)$$

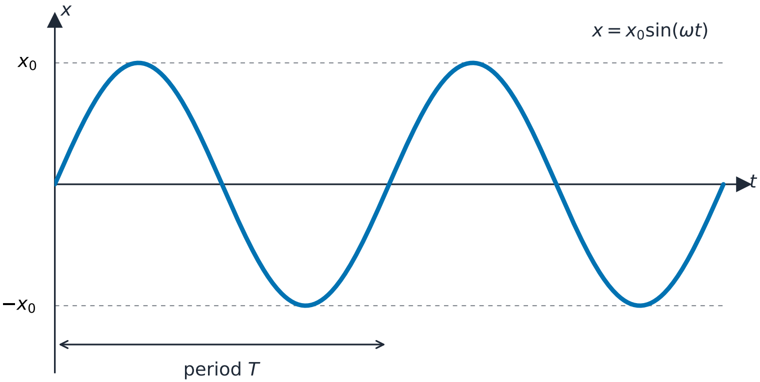
$$v = -A\omega\sin(\omega t + \phi)$$

$$a = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x$$

> $v_{max} = A\omega$ (at the middle);
> $a_{max} = A\omega^2$ (at the extremes).

$A = 0.1$, $\omega = 5$: $v_{max} = 0.5\,\mathrm{m/s}$, $a_{max} = 2.5\,\mathrm{m/s^2}$. Velocity peaks a quarter-cycle before position.

Displacement varies sinusoidally with time in simple



Displacement varies sinusoidally with time in simple harmonic motion

Where the speed is greatest, and where the acceleration is

at the ends $x = \pm A$	speed is zero, acceleration is maximum ($a_{\max} = \omega^2 A$) and points back to centre
at $x = 0$	acceleration is zero, speed is maximum ($v_{\max} = \omega A$)
the phase	x, v and a are each a quarter cycle apart – v leads x by 90°, a is 180° out of phase with x
the energy	$E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ – constant, traded back and forth twice per cycle

The two extremes are never at the same place. If your answer has maximum speed *and* maximum acceleration at one point, a sign has gone astray.

4 Energy

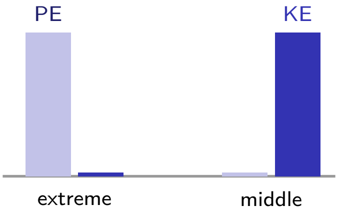
Energy of an oscillator

Energy sloshes between two forms, but the total is constant:

Set by the amplitude

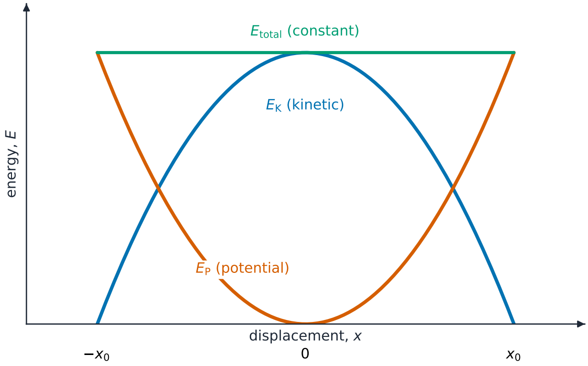
$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2$$

All **potential** 弹性势能 at the extremes; all **kinetic** 动能 in the middle – solve for the speed at any x.



$k = 200$, $A = 0.1$: $E = 1\,\mathrm{J}$ always.

Energy of Simple Harmonic Oscillators



Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example – where is the energy split evenly?

At what displacement is the potential energy exactly half the total?

- $\frac{1}{2}kx^2 = \frac{1}{2}E = \frac{1}{4}kA^2$
- $x^2 = \frac{1}{2}A^2$
- $x = A/\sqrt{2} \approx 0.71A$

Much closer to the end than to the middle – because U grows as x^2, so the outer part of the swing holds most of the potential energy.

5 Pendulums

Simple and physical pendulums

Simple pendulum (small angle)

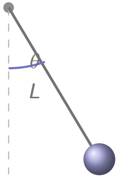
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Depends on L and g only – not mass or amplitude.

Physical pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

$d =$ pivot to centre of mass.



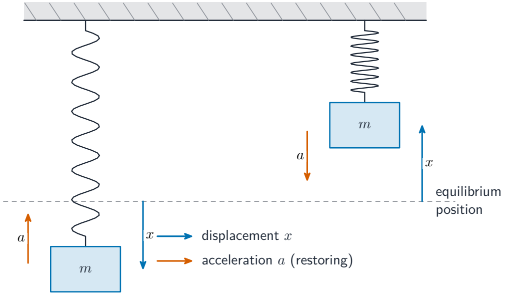
$1\,\mathrm{m}$ pendulum: $T \approx 2.0\,\mathrm{s}$.

The pendulum, and why small angles matter

small-angle approximation 小角度近似	$\sin\theta \approx \theta$ in radians – accurate to 1% below about $14°$
the equation	$\tau = I\alpha$ gives $-mgL\sin\theta = I\ddot{\theta}$, which is SHM only after that approximation
simple pendulum	$T = 2\pi\sqrt{L/g}$ – mass cancels, amplitude does not appear
physical pendulum	$T = 2\pi\sqrt{\dfrac{I}{mgd}}$, with d from pivot to centre of mass
natural frequency 固有频率	$f_0 = \dfrac{1}{2\pi}\sqrt{k/m}$; driving at f_0 gives **resonance 共振** and a growing amplitude

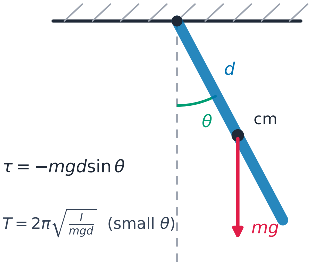
At large angles a pendulum is not simple harmonic and its period grows with amplitude. Every quoted pendulum formula silently assumes the approximation.

Defining Simple Harmonic Motion



Simple harmonic motion 简谐运动 needs exactly two things: an **equilibrium** 平衡位置 point, and a restoring force proportional to the displacement and directed back towards it. Recognise the equation $\ddot{x} = -\omega^2 x$ and you know the motion is sinusoidal – you are not asked to solve it.

Simple and Physical Pendulums, continued



physical pendulum: $T = 2\pi\sqrt{I/mgd}$

> Gravity acting at the center of mass provides a physical pendulum's restoring torque

The phase constant, and one more oscillator

Phase constant	In $x = A\cos(\omega t + \phi)$ the **phase constant** ϕ sets where the motion starts: $\phi = 0$ from the amplitude, $\phi = -\pi/2$ from the equilibrium moving positive.
Reading it off	Two initial conditions – $x(0)$ and $v(0)$ – fix A and ϕ together.
Torsion pendulum	A **torsion pendulum** twists a wire instead of stretching a spring: $\tau = -\kappa\theta$, so $\omega = \sqrt{\kappa/I}$.
The pattern	Every simple harmonic oscillator has the same form: a restoring quantity over an inertial one, square-rooted.

Identify the restoring constant and the inertia, and ω follows – k/m, κ/I, mgd/I for a physical pendulum.

Exam tips

- Identify SHM from a **linear restoring force** $F = -kx$, which gives $\frac{d^2x}{dt^2} = -\omega^2 x$ with $\omega = \sqrt{k/m}$.
- Write the solution $x = A\cos(\omega t + \phi)$ and get v, a by differentiating; period $T = \frac{2\pi}{\omega}$ is amplitude-independent.
- Energy trades between $\frac{1}{2}kx^2$ and $\frac{1}{2}mv^2$, with total $\frac{1}{2}kA^2$.
- For a pendulum, use the small-angle approximation $\sin\theta \approx \theta$ to reach SHM.
- Match phase to the start: released from rest at maximum displacement uses cosine.

6 Recap

Unit 7 – key takeaways

1 SHM

$d^2x/dt^2 = -\omega^2 x$ with $\omega = \sqrt{k/m}$

2 Motion

$x = A\cos(\omega t + \phi)$; $v_{max} = A\omega$, $a_{max} = A\omega^2$

3 Energy

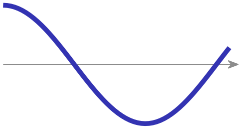
$E = \frac{1}{2}kA^2$; trades between PE and KE

4 Pendulums

$2\pi\sqrt{L/g}$ (simple), $2\pi\sqrt{I/mgd}$ (physical)

Check yourself

1. What differential equation defines simple harmonic motion?
2. Does a mass-spring period depend on the amplitude?
3. Where in the swing is the speed greatest?
4. Does a simple pendulum's period depend on the bob's mass?



Answers **1.** $d^2x/dt^2 = -\omega^2 x$ **2.** no **3.** in the middle **4.** no