

Linear Momentum

AP Physics C: Mechanics ■ Unit 4

AP Physics C: Mechanics



Linear Momentum

What we will cover

- 1 Linear momentum
- 2 Impulse and $\vec{F} = d\vec{p}/dt$
- 3 Conservation of momentum
- 4 Elastic and inelastic collisions



1

Momentum

Linear Momentum

Linear momentum

Mass in motion

$$\vec{p} = m\vec{v}$$

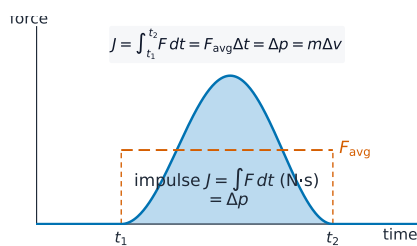
A **vector** 矢量. A **system** 粒子系统's total
= $M\vec{v}_{cm}$.

Momentum grows with **both** mass and speed – a heavy slow object can match a light fast one.



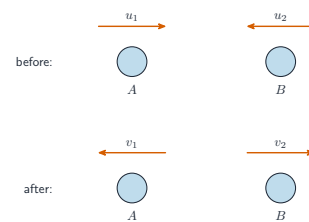
Newton's cradle

Impulse is the area under the force-time curve



Impulse is the area under the force-time curve, equal to the average force times the contact time

A head-on collision



A head-on collision: total momentum before equals total momentum after

2 Impulse

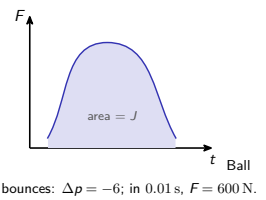
Impulse and Newton's law

Impulse – and the deepest form of $F = ma$

$$\vec{J} = \int \vec{F} dt = \Delta \vec{p}$$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

For a fixed Δp , a longer **contact time** 接触时间 means a **smaller** force – cushioning.



The impulse–momentum theorem, and reading it off a graph

The theorem

The **impulse–momentum theorem**: $\vec{J} = \int \vec{F} dt = \Delta \vec{p}$. Impulse is what a force *does over time*, and it equals the change in momentum.

On a force–time graph

The impulse is the **area** under the curve – so a small force for a long time can match a large force for a short one.

On a momentum–time graph

The net force is the **slope** 斜率 at each instant. Same two quantities, the other way round.

$\vec{F}_{net} = d\vec{p}/dt$ reduces to $m\vec{a}$ only when the mass is constant. When mass changes, go back to the momentum form.

Why impulse explains airbags

the theorem

$\vec{J} = \int \vec{F} dt = \Delta \vec{p}$ – the impulse–momentum theorem 冲量-动量定理

the general law

$\sum \vec{F} = \frac{d\vec{p}}{dt}$, which reduces to $m\vec{a}$ only when the mass is constant

changing mass

the extra $\frac{dm}{dt}\vec{v}$ term appears for rockets and conveyor belts – one of AP's favourite twists

the design idea

the same Δp spread over a **longer** time means a **smaller** force

Airbags, crumple zones, catching gloves and bending your knees are all one physics: they stretch Δt , and $F = \Delta p / \Delta t$ falls in proportion.

3 Conservation

Conservation of momentum

With **zero net external force**, a system is **isolated** 孤立系统 and total momentum is conserved. **Internal forces** 内力 cancel in third-law pairs.

Worked example – recoil

Astronaut 60 kg throws a 2 kg wrench at 10 m/s:

$$0 = 2(10) + 60v \Rightarrow v = -0.33 \text{ m/s}$$



Conserved separately along each axis in 2-D.

4 Collisions

Elastic and inelastic collisions

Momentum is conserved in **every** collision; kinetic energy is not:

- **elastic** 弹性碰撞 – KE conserved (add it as a 2nd equation)
- **inelastic** 非弹性碰撞 – some KE lost
- **perfectly inelastic** 完全非弹性碰撞 – they stick

Worked example (stick)

2 kg at 3 m/s hits a 4 kg at rest:

$$2(3) = 6v_f \Rightarrow v_f = 1 \text{ m/s}$$

KE: 9 → 3 J; the lost 6 J became heat.

Where the kinetic energy goes

Elastic collision

Momentum conserved, and kinetic energy conserved as well. Two equations, so both final velocities can be found.

Inelastic collision

Momentum conserved; kinetic energy **decreases** – it becomes heat, sound and deformation 形变.

Perfectly inelastic collision

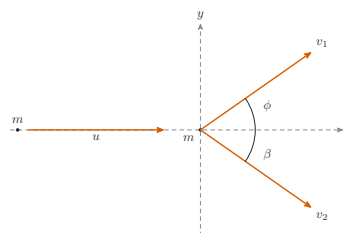
A **perfectly inelastic collision** is the largest possible loss: the objects stick together and share one velocity.

Explosions

Run the same algebra backwards. In an **explosion** 爆炸 the total momentum stays what it was – zero, if the body started at rest – so the pieces fly apart with equal and opposite momenta.

Momentum is conserved in **every** one of these. Kinetic energy is the question, never the assumption.

A glancing collision



A glancing collision, resolved along two perpendicular axes

Exam tips

- Impulse equals the momentum change: $\vec{J} = \int \vec{F} dt = \Delta \vec{p}$, and it is the area under a force–time graph.
- **Momentum is conserved** whenever the net external force is zero – always the go-to for collisions.
- Distinguish **elastic** (kinetic energy conserved) from **inelastic** (kinetic energy decreases) collisions; in a **perfectly inelastic** collision the objects stick and move with one velocity.
- Apply conservation to each component (x and y) separately in 2-D.
- Connect to center of mass: total momentum = $M\vec{v}_{cm}$.

5 Recap

Unit 4 – key takeaways

1 Momentum

$\vec{p} = m\vec{v}$, a vector; system total
 $= M\vec{v}_{cm}$

3 Conservation

zero external force keeps total
momentum constant

2 Impulse

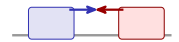
$$\vec{J} = \int \vec{F} dt = \Delta\vec{p}; \vec{F} = d\vec{p}/dt$$

4 Collisions

momentum always conserved; *KE*
only if elastic

Check yourself

- 1 Write Newton's second law in terms of momentum.
- 2 What is the area under a force-time graph?
- 3 An astronaut throws a tool – which way does she drift?
- 4 Which collision also conserves kinetic energy?



Answers 1. $\vec{F} = d\vec{p}/dt$ 2. the impulse 3. opposite the throw 4. elastic