Linear Momentum

AP Physics C: Mechanics ▪ Unit 4

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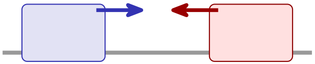
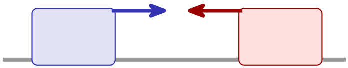


What we will cover

1. Linear momentum
2. Impulse and $\vec{F} = d\vec{p}/dt$
3. Conservation of momentum
4. Elastic and inelastic collisions



1 Momentum

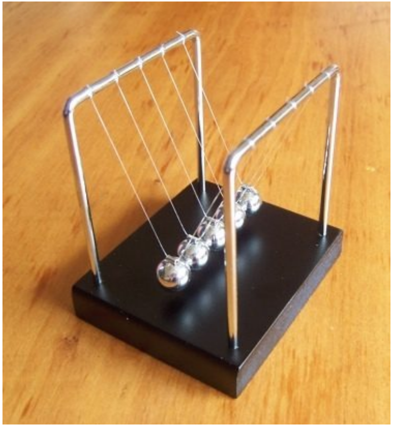
Linear momentum

Mass in motion

$$\vec{p} = m\vec{v}$$

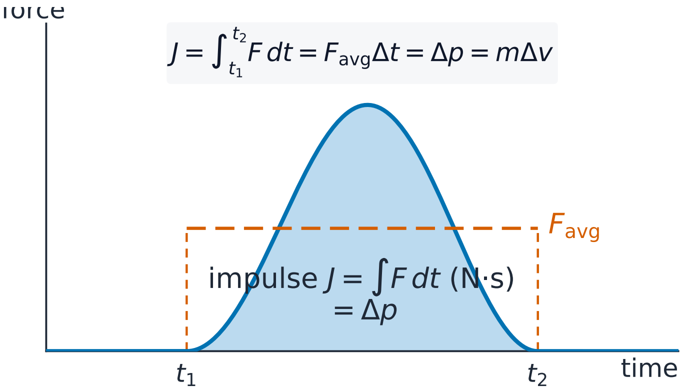
A **vector** 矢量. A **system** 粒子系统's total $= M\vec{v}_{cm}$.

Momentum grows with both mass and speed – a heavy slow object can match a light fast one.



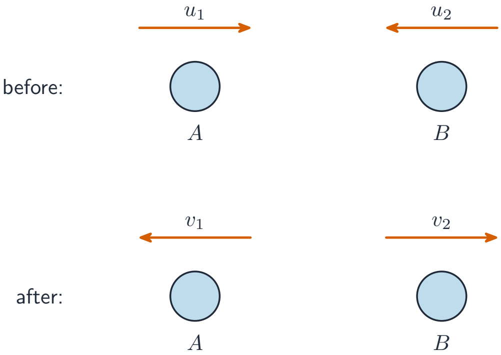
Newtons cradle

Impulse is the area under the force-time curve



Impulse is the area under the force-time curve, equal to the average force times the contact time

A head-on collision



A head-on collision: total momentum before equals total momentum after

2 Impulse

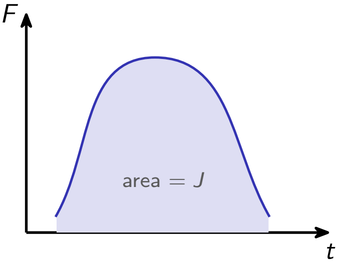
Impulse and Newton's law

Impulse – and the deepest form of $F = ma$

$$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

For a fixed Δp, a longer **contact time** 接触时间 means a smaller force – cushioning.



Ball bounces: $\Delta p = -6$; in $0.01\,\mathrm{s}$, $F = 600\,\mathrm{N}$.

The impulse–momentum theorem, and reading it off a graph

The theorem	The **impulse–momentum theorem**: $\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$. Impulse is what a force *does over time*, and it equals the change in momentum.
On a force–time graph	The impulse is the area under the curve – so a small force for a long time can match a large force for a short one.
On a momentum–time graph	The net force is the **slope** 斜率 at each instant. Same two quantities, the other way round.

$\vec{F}_{\text{net}} = d\vec{p}/dt$ reduces to $m\vec{a}$ only when the mass is constant. When mass changes, go back to the momentum form.

Why impulse explains airbags

the theorem	$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$ – the **impulse–momentum theorem** 冲量-动量定理
the general law	$\sum \vec{F} = \dfrac{d\vec{p}}{dt}$, which reduces to $m\vec{a}$ only when the mass is constant
changing mass	the extra $\dfrac{dm}{dt}\vec{v}$ term appears for rockets and conveyor belts – one of AP's favourite twists
the design idea	the same Δp spread over a longer time means a smaller force

Airbags, crumple zones, catching gloves and bending your knees are all one physics: they stretch Δt, and $F = \Delta p/\Delta t$ falls in proportion.

3 Conservation

Conservation of momentum

With zero net external force, a system is **isolated** 孤立系统 and total momentum is conserved. **Internal forces** 内力 cancel in third-law pairs.

Worked example – recoil

Astronaut $60\,\mathrm{kg}$ throws a $2\,\mathrm{kg}$ wrench at $10\,\mathrm{m/s}$:

$$0 = 2(10) + 60v \Rightarrow v = -0.33\,\mathrm{m/s}$$

Conserved separately along each axis in 2-D.

4 Collisions

Elastic and inelastic collisions

Momentum is conserved in every collision; kinetic energy is not:

- **elastic** 弹性碰撞 – *KE* conserved (add it as a 2nd equation)
- **inelastic** 非弹性碰撞 – some *KE* lost
- **perfectly inelastic** 完全非弹性碰撞 – they stick

Worked example (stick)

$2\,\mathrm{kg}$ at $3\,\mathrm{m/s}$ hits a $4\,\mathrm{kg}$ at rest:

$$2(3) = 6v_f \Rightarrow v_f = 1\,\mathrm{m/s}$$

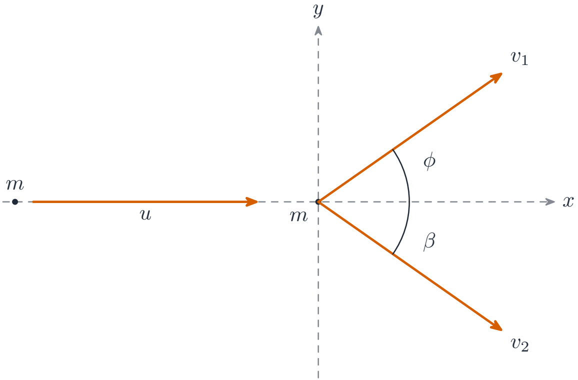
KE: $9 \to 3\,\mathrm{J}$; the lost $6\,\mathrm{J}$ became heat.

Where the kinetic energy goes

Elastic collision	Momentum conserved, and kinetic energy conserved as well. Two equations, so both final velocities can be found.
Inelastic collision	Momentum conserved; kinetic energy decreases – it becomes heat, sound and **deformation** 形变.
Perfectly inelastic collision	A **perfectly inelastic collision** is the largest possible loss: the objects stick together and share one velocity.
Explosions	Run the same algebra backwards. In an **explosion** 爆炸 the total momentum stays what it was – zero, if the body started at rest – so the pieces fly apart with equal and opposite momenta.

Momentum is conserved in *every* one of these. Kinetic energy is the question, never the assumption.

A glancing collision



A glancing collision, resolved along two perpendicular axes

Exam tips

- Impulse equals the momentum change: $\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$, and it is the area under a force–time graph.
- **Momentum is conserved** whenever the net external force is zero – always the go-to for collisions.
- Distinguish **elastic** (kinetic energy conserved) from **inelastic** (kinetic energy decreases) collisions; in a **perfectly inelastic** collision the objects stick and move with one velocity.
- Apply conservation to each component (x and y) separately in 2-D.
- Connect to center of mass: total momentum $= M\vec{v}_{cm}$.

5 Recap

Unit 4 – key takeaways

1 Momentum

$\vec{p} = m\vec{v}$, a vector; system total $= M\vec{v}_{cm}$

2 Impulse

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$; $\vec{F} = d\vec{p}/dt$

3 Conservation

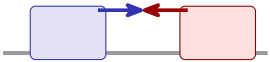
zero external force keeps total momentum constant

4 Collisions

momentum always conserved; KE only if elastic

Check yourself

1. Write Newton's second law in terms of momentum.
2. What is the area under a force-time graph?
3. An astronaut throws a tool – which way does she drift?
4. Which collision also conserves kinetic energy?



Answers **1.** $\vec{F} = d\vec{p}/dt$ **2.** the impulse **3.** opposite the throw **4.** elastic