

Work, Energy, and Power

AP Physics C: Mechanics ■ Unit 3

AP Physics C: Mechanics



Work, Energy, and Power

What we will cover

- 1 Kinetic energy and the work-energy theorem
- 2 Work as an integral
- 3 Potential energy and force
- 4 Conservation of energy
- 5 Power



1 Kinetic energy

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↳ Kinetic energy

Kinetic energy and the work-energy theorem

Energy of motion

$$K = \frac{1}{2}mv^2$$

A **scalar** 标量: $\propto v^2$, so double the speed quadruples K .

Work-energy theorem

$$W_{\text{net}} = \Delta K$$



Coaster energy

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↳ Kinetic energy

Kinetic energy, and the theorem that uses it

the definition $K = \frac{1}{2}mv^2$, measured in **joules** 焦耳

the square doubling the speed **quadruples** K – which is why stopping distance grows so fast with speed

work-energy theorem 动能定理 $W_{\text{net}} = \Delta K$: the net work on an object equals its change in kinetic energy

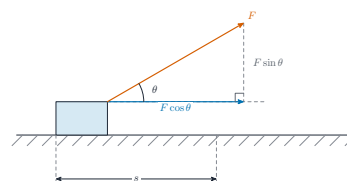
reference frame 参考系 K depends on the observer – a passenger has zero K in the train's frame and a great deal in the platform's

Kinetic energy is a **scalar**: no direction, and never negative. Work can be negative, which is how a force removes energy.

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↳ Kinetic energy

Only the force component along the displacement



Only the component **along** the displacement does work. So a force at right angles does **zero** – the **normal force** 法向力 on a sliding block, gravity on a horizontal move, the tension in a circular swing – and a force opposing the motion does **negative** work.

2 Work

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└─ Work

Work as an integral

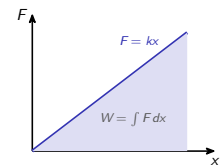
Constant vs variable force

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

$$W = \int \vec{F} \cdot d\vec{r}$$

The integral is the **area** under an F - x graph.

Stretching a spring: $W = \int_0^x kx' dx' = \frac{1}{2} kx^2$.



3 Potential energy

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└─ Potential energy

Potential energy and force

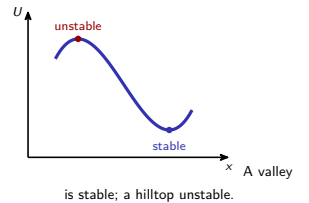
A **conservative force** 保守力 stores **potential energy** 势能:

Two stored forms, and the link

$$U_g = mgy, \quad U_s = \frac{1}{2} kx^2$$

$$F_x = -\frac{dU}{dx}$$

For a spring $-\frac{d}{dx}(\frac{1}{2} kx^2) = -kx$ – Hooke's law drops out.



The three potential energies, and where they come from

near a surface $\Delta U_g = mg\Delta y$ – valid only where g is effectively constant

far from a planet $U_g = -\frac{GMm}{r}$, taking zero at infinity

a spring $U_s = \frac{1}{2} k(\Delta x)^2$, with Δx from **natural length**, not from the floor

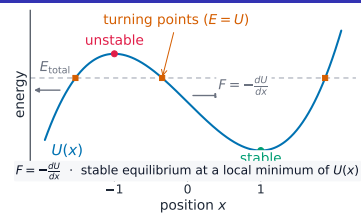
in general a conservative force is minus the **derivative** 导数 of its potential: $F_x = -\frac{dU}{dx}$

several objects add the potential energy of each **pair**, once each

$F = -dU/dx$ means the force points **downhill** on a U - x graph, and a minimum of U is a point of stable equilibrium.

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└─ Potential energy

Potential Energy



The force is $-dU/dx$: it points **downhill** on the curve, and $E = U$ marks the **turning points**. You may put $U = 0$ wherever you like – only **changes** in U matter – so choose the zero that makes the algebra shortest.

4 Conservation

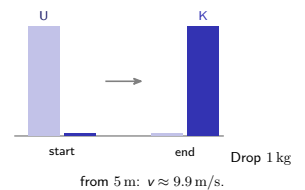
Conservation of energy

With only conservative forces, **mechanical energy is conserved** 机械能守恒:

Before = after

$$K_i + U_i = K_f + U_f$$

A **non-conservative force** 非保守力 like friction removes mechanical energy as heat.



Energy is always conserved – the question is where it went

mechanical energy 机械能 $E = K + U$; conserved when only conservative forces do work

external work $W_{\text{ext}} = \Delta E_{\text{mech}}$ – work done on the system from outside changes it

nonconservative forces 非保守力 friction and drag convert mechanical energy into **thermal energy** 热能: $\Delta E_{\text{mech}} = -F_f d$

the balance $W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{thermal}}$ – nothing is ever lost, only relabelled

"Energy is lost to friction" scores nothing on its own. Say **how much**, as $F_f d$, and where it went.

5 Power

Power

Power 功率 is the **rate** of doing work:

Two forms

$$P = \frac{dW}{dt}, \quad P = \vec{F} \cdot \vec{v}$$

Instantaneous power varies as force or speed changes.

Worked example

Lift 50 kg at a steady 2 m/s:

$$P = Fv = (490)(2) = 980 \text{ W}$$

Twice as fast $\Rightarrow$ 1960 W – same work per metre, half the time.

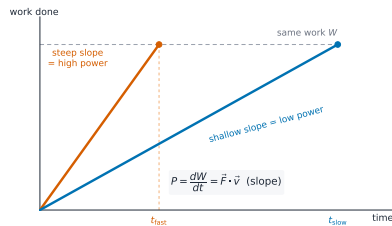
Worked example – the car climbing a hill

A 1200 kg car climbs a road that rises 1.0 m per 20 m travelled, at a steady 15 m/s. Find the power against gravity.

- the climb rate: $15 \times \frac{1.0}{20} = 0.75 \text{ m/s}$ vertically
- $P = mgv_{\text{vertical}} = (1200)(9.8)(0.75)$
- $= 8.8 \text{ kW}$, about 12 horsepower

Power is measured in **watts** 瓦特, $P = dW/dt = \vec{F} \cdot \vec{v}$. Also worth knowing: a block from rest down a frictionless 3.0 m ramp arrives at $v = \sqrt{2gh} = 7.7 \text{ m/s}$, **whatever the ramp's shape**.

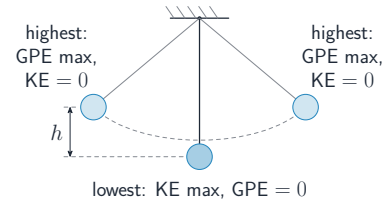
Power is the slope of the work-time graph



Power is the slope of the work-time graph: the same work in less time means more power

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Conservation of Energy, continued



A pendulum trades gravitational potential energy for kinetic energy and back

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The accounting sentence that earns the marks

1. **name the system** say what is inside it – block alone, or block plus Earth, or block plus spring
2. **name the forces** which ones do work on that system, and which are internal
3. **write the balance** $W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{thermal}}$
4. **then substitute** numbers last, once the accounting is on the page

Gravity is an **internal** force if the Earth is in your system (so it has a U) and an **external** one if it is not (so it does work). Both are correct; mixing them in one line is not.

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Exam tips

- Use the **work–energy theorem** $W_{\text{net}} = \Delta KE$ and compute work as $W = \int \vec{F} \cdot d\vec{r}$ for a variable force.
- Read work off a **force–position graph** as the area under the curve.
- Power is $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$; watch instantaneous vs average.
- Split forces into conservative (define a potential energy) and non-conservative (dissipate energy).
- Choose energy methods over kinematics when the force varies or the path is complex.

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Recap

Work, Energy, and Power

Unit 3 – key takeaways

1 Work

$W = \int \vec{F} \cdot d\vec{r}$; the area under an F - x graph

2 Energy

$K = \frac{1}{2}mv^2$; net work equals ΔK

3 Potential

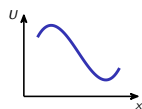
$F_x = -dU/dx$; valley stable, hill-top unstable

4 Conservation

$K + U$ constant; $P = dW/dt = \vec{F} \cdot \vec{v}$

Check yourself

- 1 What integral gives the work done by a variable force?
- 2 How do you get the force from a potential-energy function?
- 3 Is a potential-energy valley stable or unstable equilibrium?
- 4 Write instantaneous power as a dot product.



Answers 1. $\int \vec{F} \cdot d\vec{r}$ 2. $F_x = -dU/dx$ 3. stable 4. $P = \vec{F} \cdot \vec{v}$