Work, Energy, and Power

AP Physics C: Mechanics ▪ Unit 3

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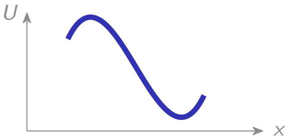
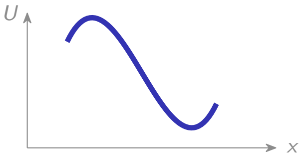


What we will cover

1. Kinetic energy and the work-energy theorem
2. Work as an integral
3. Potential energy and force
4. Conservation of energy
5. Power



1 Kinetic energy

Kinetic energy and the work-energy theorem

Energy of motion

$$K = \frac{1}{2}mv^2$$

A **scalar** 标量; $\propto v^2$, so double the speed quadruples K.

Work-energy theorem

$$W_{net} = \Delta K$$

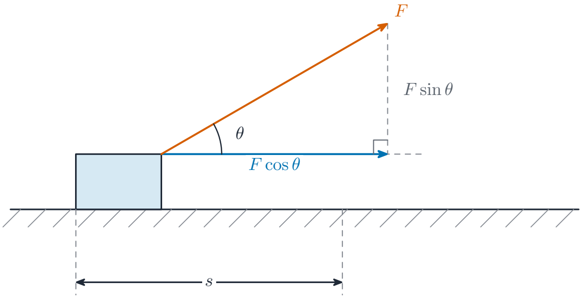
Coaster energy

Kinetic energy, and the theorem that uses it

the definition	$K = \frac{1}{2}mv^2$, measured in **joules 焦耳**
the square	doubling the speed quadruples K – which is why stopping distance grows so fast with speed
work–energy theorem 动能定理	$W_{\text{net}} = \Delta K$: the net work on an object equals its change in kinetic energy
reference frame 参考系	K depends on the observer – a passenger has zero K in the train's frame and a great deal in the platform's

> Kinetic energy is a scalar: no direction, and never negative. Work can be negative, which is how a force removes energy.

Only the force component along the displacement



Only the component along the displacement does work. So a force at right angles does zero – the **normal force** 法向力 on a sliding block, gravity on a horizontal move, the tension in a circular swing – and a force opposing the motion does negative work.

2

Work

Work as an integral

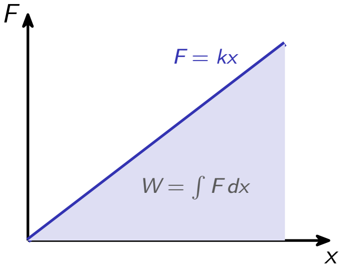
Constant vs variable force

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

$$W = \int \vec{F} \cdot d\vec{r}$$

The integral is the area under an F–x graph.

Stretching a spring: $W = \int_0^x kx'\,dx' = \frac{1}{2}kx^2$.



3 Potential energy

Potential energy and force

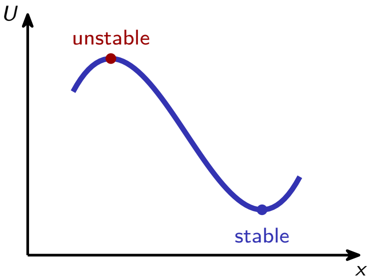
A **conservative force** 保守力 stores **potential energy** 势能:

Two stored forms, and the link

$$U_g = mgy, \qquad U_s = \tfrac{1}{2}kx^2$$

$$F_x = -\frac{dU}{dx}$$

For a spring $-\frac{d}{dx}(\frac{1}{2}kx^2) = -kx$ – Hooke's law drops out.



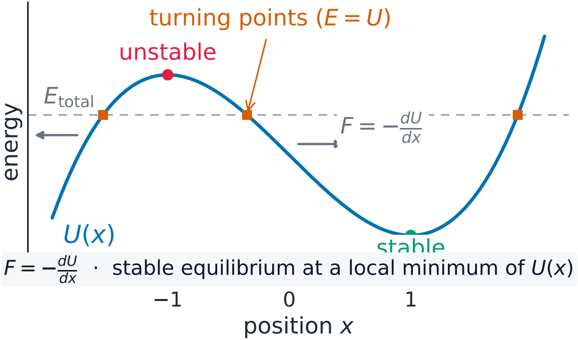
A valley is stable; a hilltop unstable.

The three potential energies, and where they come from

near a surface	$\Delta U_g = mg\,\Delta y$ – valid only where g is effectively constant
far from a planet	$U_g = -\dfrac{GMm}{r}$, taking zero at infinity
a spring	$U_s = \frac{1}{2}k(\Delta x)^2$, with Δx from natural length, not from the floor
in general	a conservative force is minus the **derivative** 导数 of its potential: $F_x = -\dfrac{dU}{dx}$
several objects	add the potential energy of each pair, once each

$F = -dU/dx$ means the force points downhill on a U–x graph, and a minimum of U is a point of stable equilibrium.

Potential Energy



The force is $-dU/dx$: it points downhill on the curve, and $E = U$ marks the turning points. You may put $U = 0$ wherever you like – only *changes* in U matter – so choose the zero that makes the algebra shortest.

4 Conservation

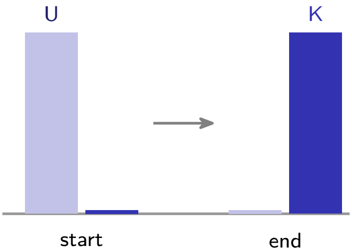
Conservation of energy

With only conservative forces, **mechanical energy is conserved** 机械能守恒:

Before = after

$$K_i + U_i = K_f + U_f$$

A **non-conservative force** 非保守力 like friction removes mechanical energy as heat.



Drop $1\,\mathrm{kg}$ from $5\,\mathrm{m}$: $v \approx 9.9\,\mathrm{m/s}$.

Energy is always conserved – the question is where it went

mechanical energy 机械能	$E = K + U$; conserved when only conservative forces do work
external work	$W_{ext} = \Delta E_{mech}$ – work done *on* the system from outside changes it
nonconservative forces 非保守力	friction and drag convert mechanical energy into **thermal energy** 热能: $\Delta E_{mech} = -F_f d$
the balance	$W_{ext} = \Delta K + \Delta U + \Delta E_{thermal}$ – nothing is ever lost, only relabelled

> "Energy is lost to friction" scores nothing on its own. Say how much, as $F_f d$, and where it went.

5 Power

Power

Power 功率 is the rate of doing work:

Two forms

$$P = \frac{dW}{dt}, \qquad P = \vec{F} \cdot \vec{v}$$

Instantaneous power varies as force or speed changes.

Worked example

Lift $50\,\mathrm{kg}$ at a steady $2\,\mathrm{m/s}$:

$$P = Fv = (490)(2) = 980\,\mathrm{W}$$

Twice as fast $\Rightarrow 1960\,\mathrm{W}$ – same work per metre, half the time.

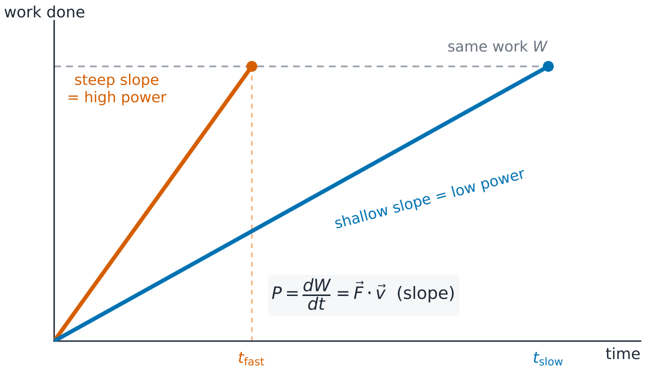
Worked example – the car climbing a hill

A 1200 kg car climbs a road that rises 1.0 m per 20 m travelled, at a steady 15 m/s. Find the power against gravity.

- the climb rate: $15 \times \dfrac{1.0}{20} = 0.75$ m/s vertically
- $P = mgv_{\text{vertical}} = (1200)(9.8)(0.75)$
- $= 8.8$ kW, about 12 horsepower

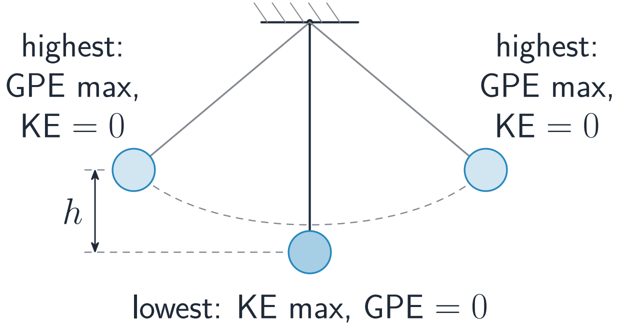
Power is measured in **watts** 瓦特, $P = dW/dt = \vec{F} \cdot \vec{v}$. Also worth knowing: a block from rest down a frictionless 3.0 m ramp arrives at $v = \sqrt{2gh} = 7.7$ m/s, whatever the ramp's shape.

Power is the slope of the work-time graph



Power is the slope of the work-time graph: the same work in less time means more power

Conservation of Energy, continued



A pendulum trades gravitational potential energy for kinetic energy and back

The accounting sentence that earns the marks

1. name the system	say what is inside it – block alone, or block plus Earth, or block plus spring
2. name the forces	which ones do work *on* that system, and which are internal
3. write the balance	$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{thermal}}$
4. then substitute	numbers last, once the accounting is on the page

Gravity is an internal force if the Earth is in your system (so it has a U) and an external one if it is not (so it does work). Both are correct; mixing them in one line is not.

Exam tips

- Use the **work–energy theorem** $W_{net} = \Delta KE$ and compute work as $W = \int \vec{F} \cdot d\vec{r}$ for a variable force.
- Read work off a **force–position graph** as the area under the curve.
- Power is $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$; watch instantaneous vs average.
- Split forces into conservative (define a potential energy) and non-conservative (dissipate energy).
- Choose energy methods over kinematics when the force varies or the path is complex.

6 Recap

Unit 3 – key takeaways

1 Work

$W = \int \vec{F} \cdot d\vec{r}$; the area under an F–x graph

2 Energy

$K = \frac{1}{2}mv^2$; net work equals ΔK

3 Potential

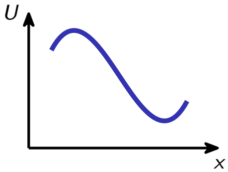
$F_x = -dU/dx$; valley stable, hill-top unstable

4 Conservation

$K + U$ constant; $P = dW/dt = \vec{F} \cdot \vec{v}$

Check yourself

1. What integral gives the work done by a variable force?
2. How do you get the force from a potential-energy function?
3. Is a potential-energy valley stable or unstable equilibrium?
4. Write instantaneous power as a dot product.



Answers **1.** $\int \vec{F} \cdot d\vec{r}$ **2.** $F_x = -dU/dx$ **3.** stable **4.** $P = \vec{F} \cdot \vec{v}$